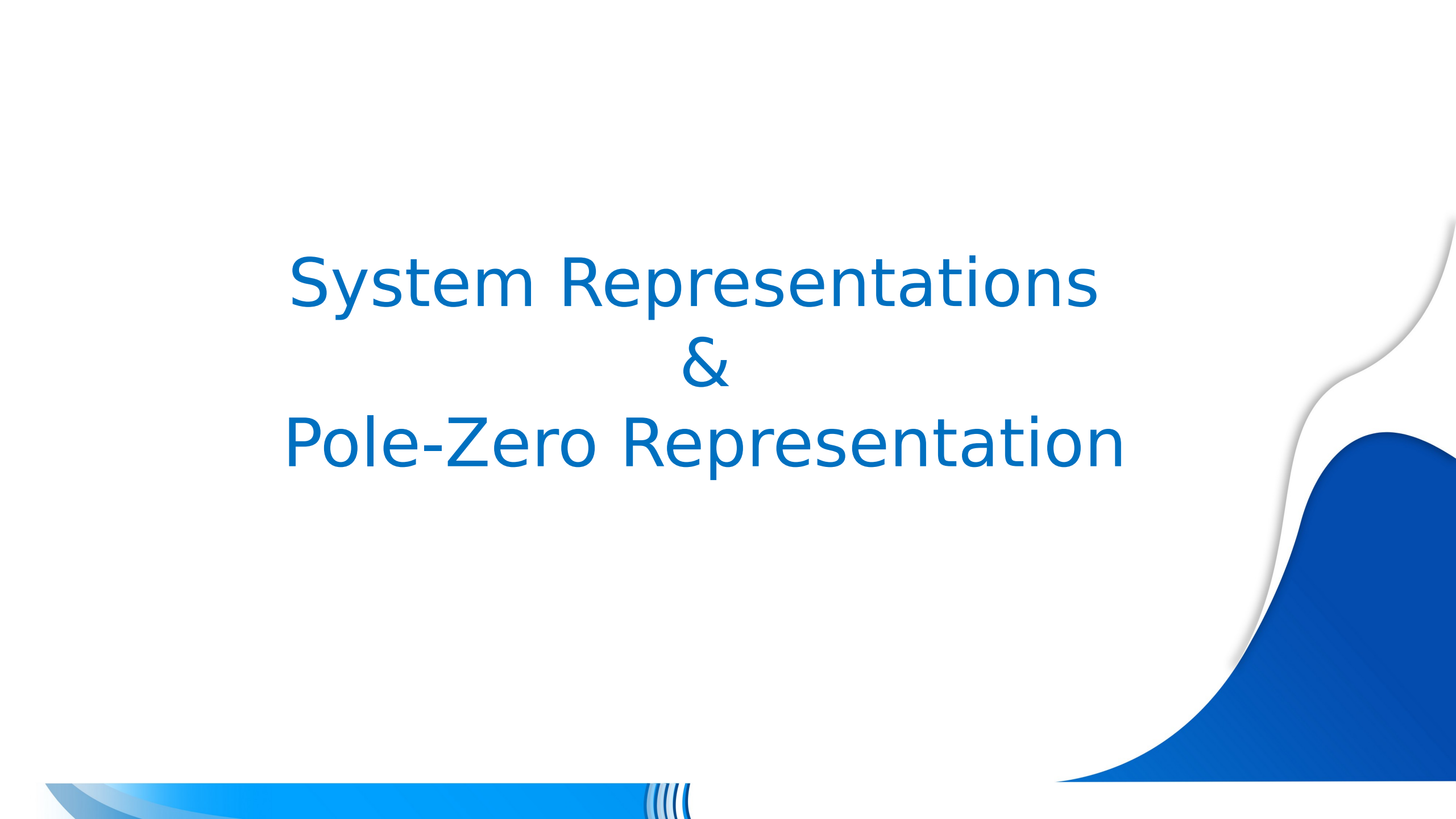


# System Representations & Pole-Zero Representation

The slide features a white background with blue decorative elements. On the right side, there is a large, abstract blue shape that resembles a stylized wave or a mountain peak. At the bottom, there is a horizontal blue bar with a small, white, curved graphic element on its right end.

# System Representations

- Time Domain Representation  
 $x(n), y(n), h(n)$
- Frequency Domain Representation  
 $X(W), Y(W), H(W)$
- Z-Transform Representation  
 $X(Z), Y(Z), H(Z)$
- Difference Equation Representation  
 $(a_k, b_k)$
- Pole Zero Representation  
 $H(Z) = Y(Z)/X(Z)$

# System Representations

$$h(n)$$

$$H(\omega)$$

$$H(z)$$

$$(a_k, b_k) \quad \checkmark$$

$$\text{Poles \& zeros} \quad \checkmark$$

$$y(n) = \sum_{k=0}^N a_k \delta(n-k) + \sum_{k=0}^m b_k x(n-k)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^m b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}}$$

# Z-Transform & Difference Equations

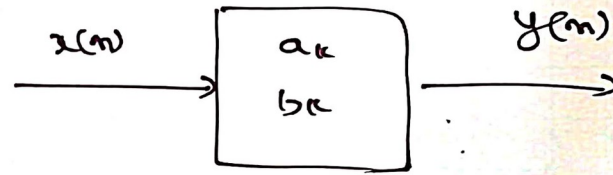
z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad ; \quad z = Re^{j\omega}$$

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$$X(z) \Big|_{\substack{z=e^{j\omega} \\ R=1}} \Rightarrow \text{DFT} [x(n)]$$

$$\begin{aligned} x(n) &\Rightarrow x(t) \\ x(n-1) &= z^{-1} x(t) \end{aligned}$$



$$y(n) = a_1 y(n-1) + a_2 y(n-2) + \dots + a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots + b_M x(n-M)$$

$$Y(z) = a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) + \dots + a_N z^{-N} Y(z) + b_0 X(z) + b_1 z^{-1} X(z) + \dots + b_M z^{-M} X(z)$$

$$= Y(z) \left[ a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N} \right] + X(z) \left[ b_0 + b_1 z^{-1} + \dots + b_M z^{-M} \right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^M b_k z^{-k}}{1 - \sum_{k=1}^N a_k z^{-k}} = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}}$$

# Poles & Zeros of the System

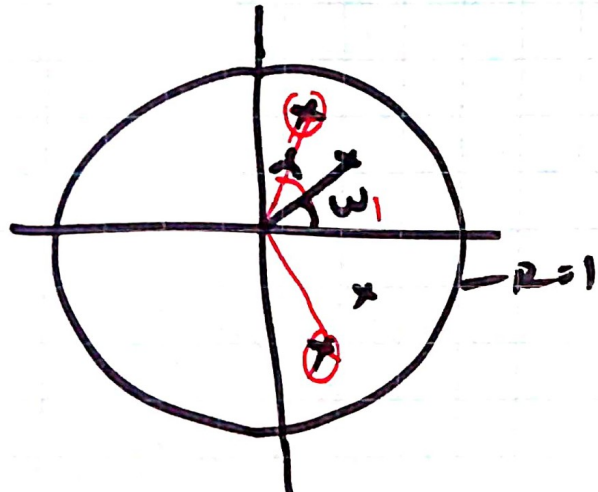
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$$H(z) = \frac{\sum_{k=0}^{\infty} b_k z^{-k}}{\sum_{k=0}^{\infty} a_k z^{-k}}$$

zeros

poles

if  $a_k = 0$   
if  $b_k = 0$



Real of complex conjugate

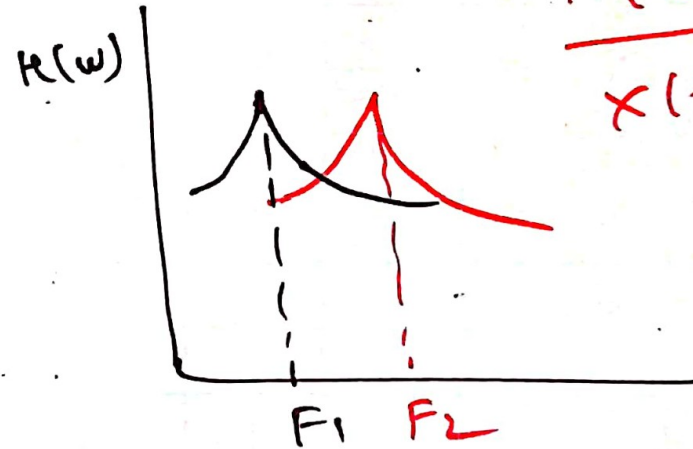
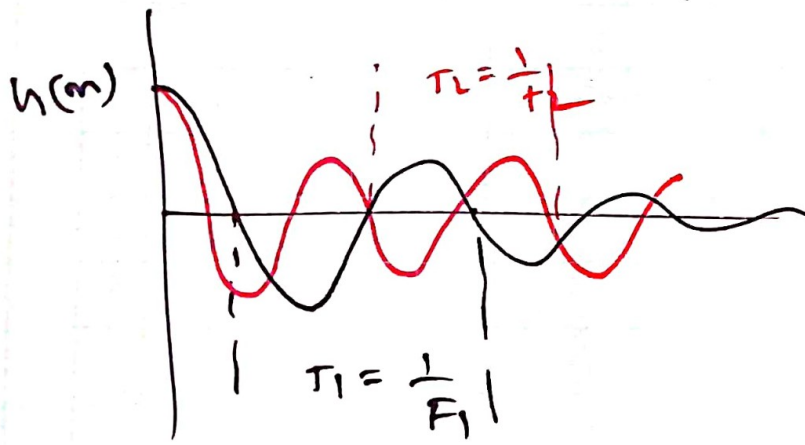
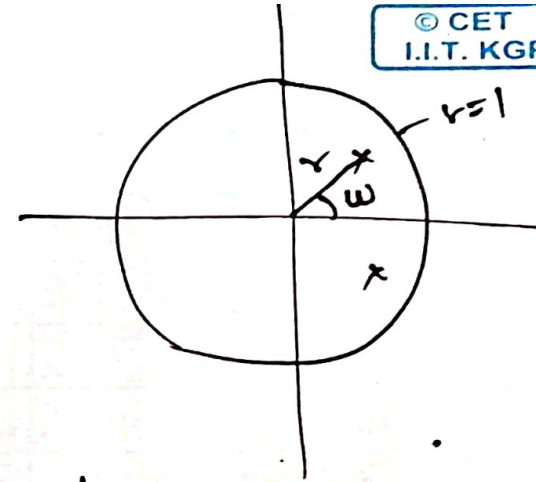
$$\gamma = e^{-\pi BT}$$
$$\omega = 2\pi fT$$

# Poles & Zeros of the System

Poles & zeros of the System

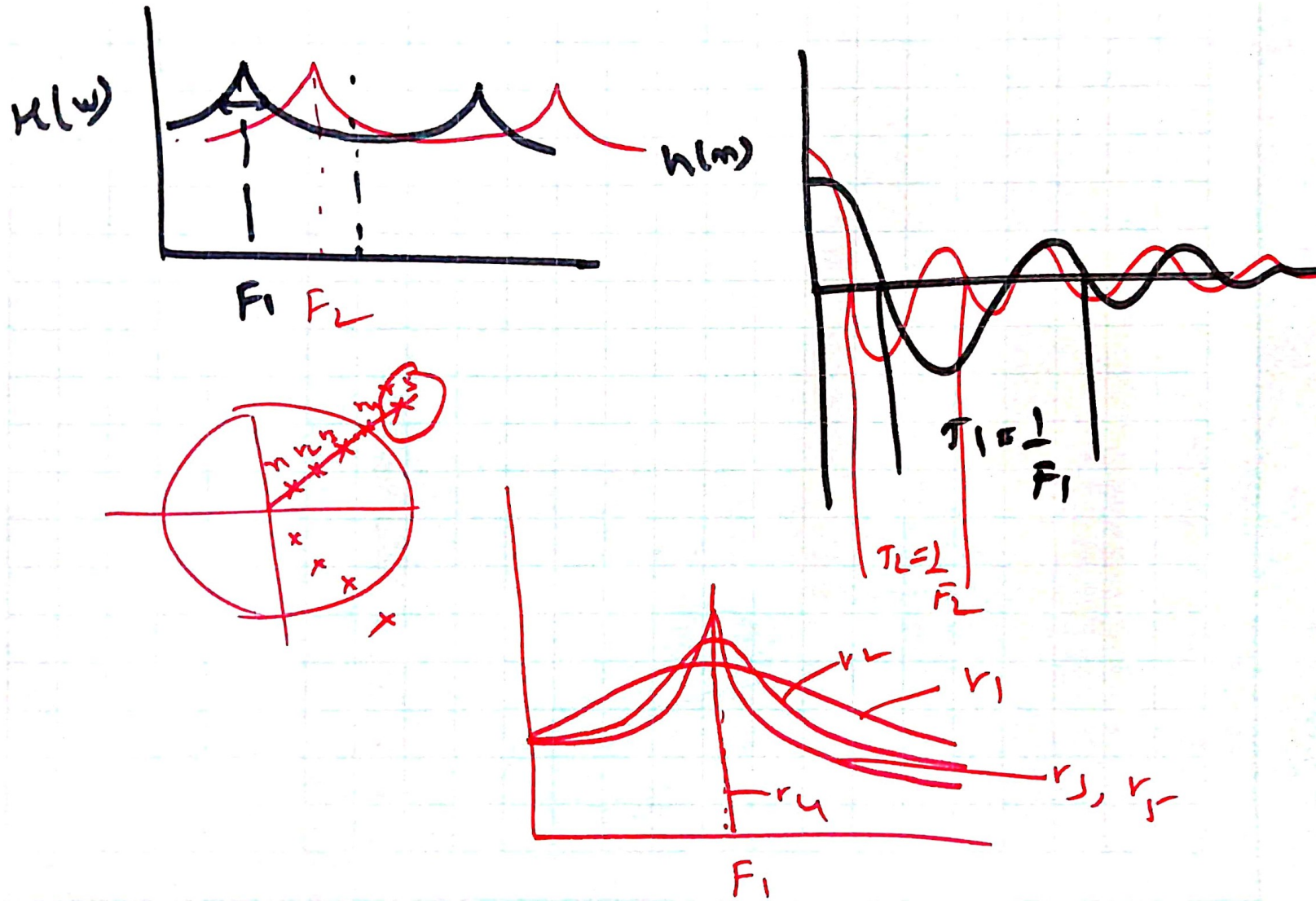
$$\gamma = e^{-\pi B T} ; \omega = 2\pi f T$$

Complex Conjugate poles  $r < 1$

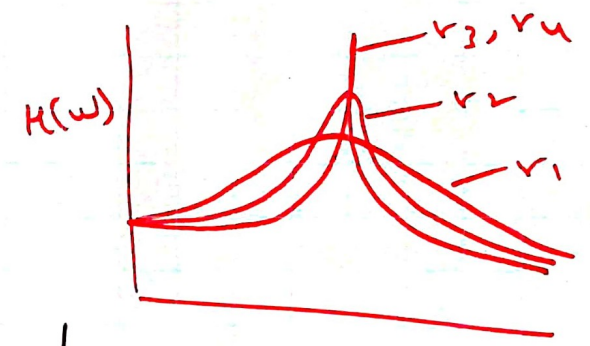
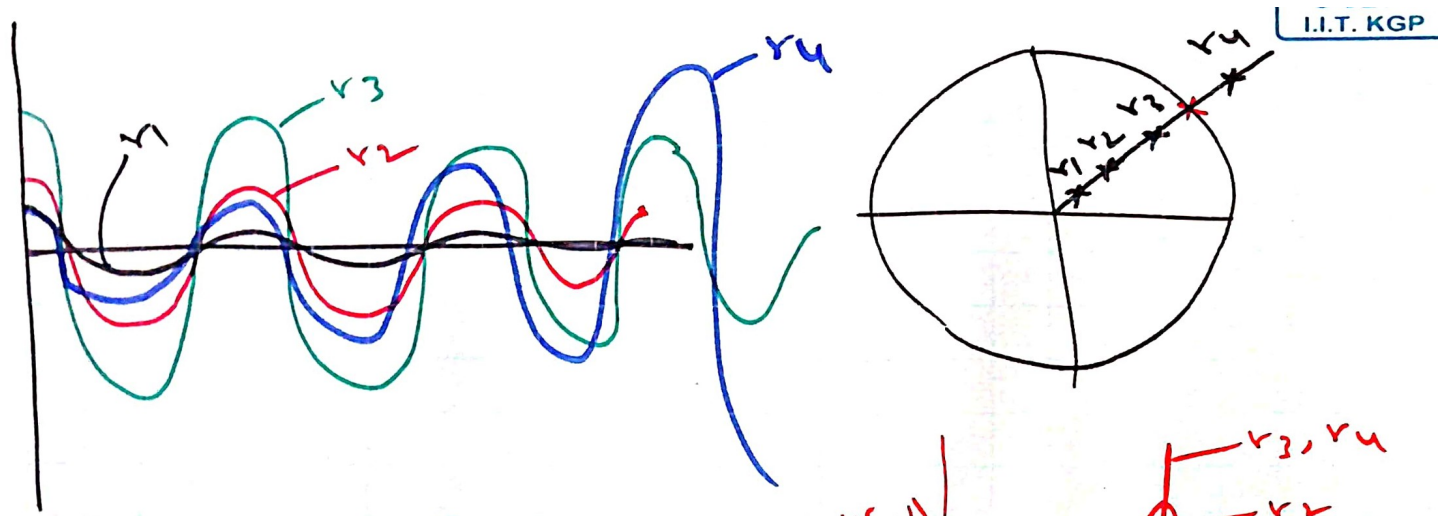


$$\frac{X(z) = 0}{X(z) = \infty}$$

# Poles & Zeros of the System



# Poles & Zeros of the System



Poles on Real axis

