Linear Time Invariant (LTI) System

Concepts of Convolution & Correlation

LTI System : Concept of Convolution

LTI System

h(n) → system unit sample response

I/p	O/p
$\delta(n)$	$h(n)$
$\delta(n-k)$	$h(n-k)$
$x(k)\,\delta(n-k)$	$x(k)\,h(n-k)$
$\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$	$\sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$
$x(n)$	$y(n)$

Convolution $y(n) = x(n) * h(n)$

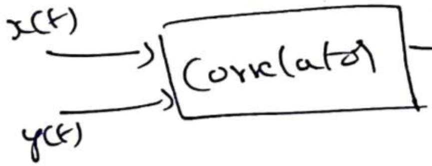
$$y(n) = \sum_k x(k)\,h(n-k)$$

$$\sum_k h(k)\,x(n-k)$$

Freq Domain

$$Y(\omega) = H(\omega)\,X(\omega)$$

Correlation



$c(t) = \int x(t)\,y(t+\tau)$

$c(n) = \sum_{k=-\infty}^{\infty} x(n)\,y(n+k)$

CONV $y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$

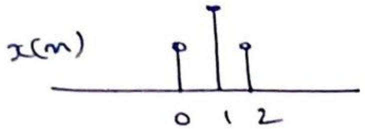
CORR $y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n+k)$

Correlation :

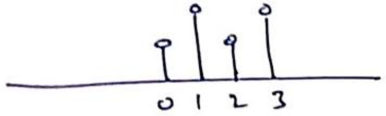
$c(n) = \sum_{k=-\infty}^{+\infty} x(k)h(k-n)$

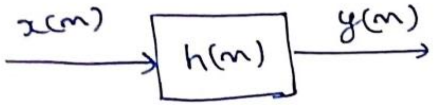
Graphical Interpretation of Convolution

$x(m) = 1, 2, 1 \quad (m = 0, 1, 2)$



$h(m) = 1, 2, 1, 2 \quad (m = 0, 1, 2, 3)$





$$y(m) = x(m) * h(m)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(m-k)$$

$$y(0) = \sum_{k} x(k)\, h(-k)$$

$$y(1) = \sum x(k)\, h(1-k)$$

$$y(2) = \sum x(k)\, h(2-k)$$

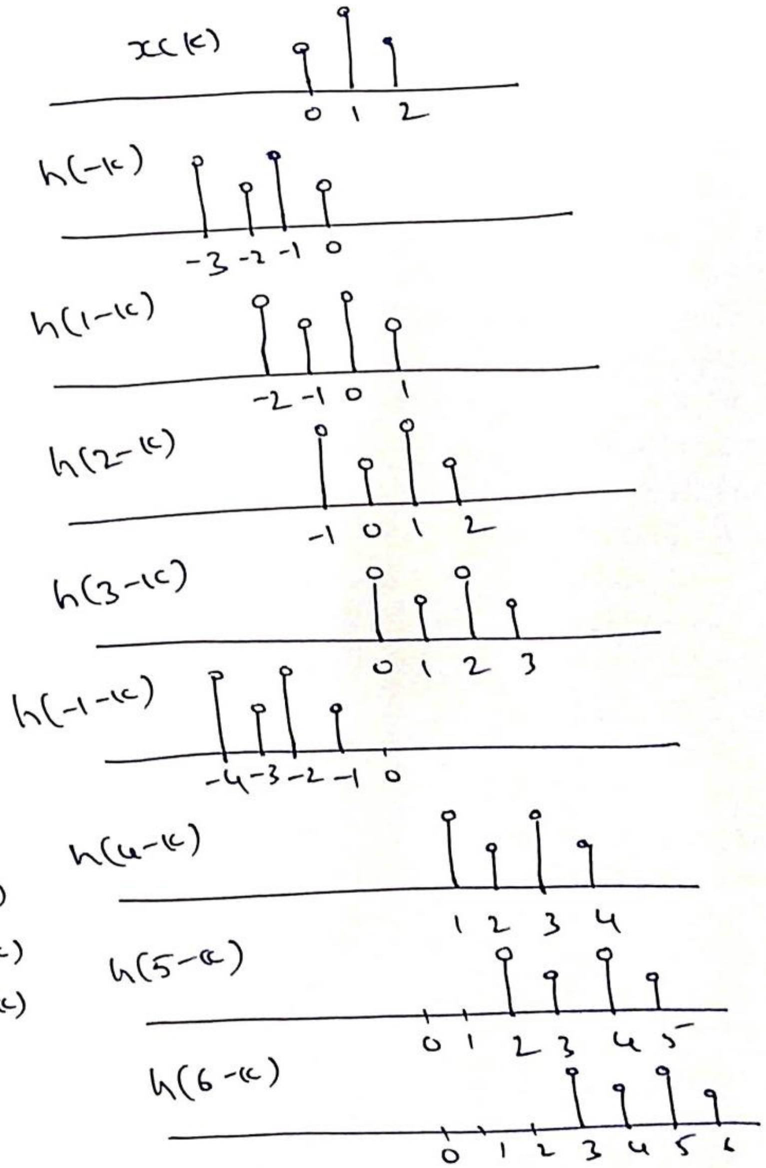
$$y(-1) = \sum x(k)\, h(-1-k)$$

$$y(3) = \sum x(k)\, h(3-k)$$

$$y(4) = \sum x(k)\, h(4-k)$$

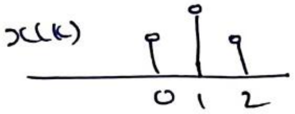
$$y(5) = \sum x(k)\, h(5-k)$$

$$y(6) = \sum x(k)\, h(6-k)$$



Graphical Interpretation of Convolution

$y(0) = \sum x(k)\, h(-k) = 1$



$y(1) = \sum x(k)\, h(1-k) = 4$

$y(2) = \sum x(k)\, h(2-k) = 6$

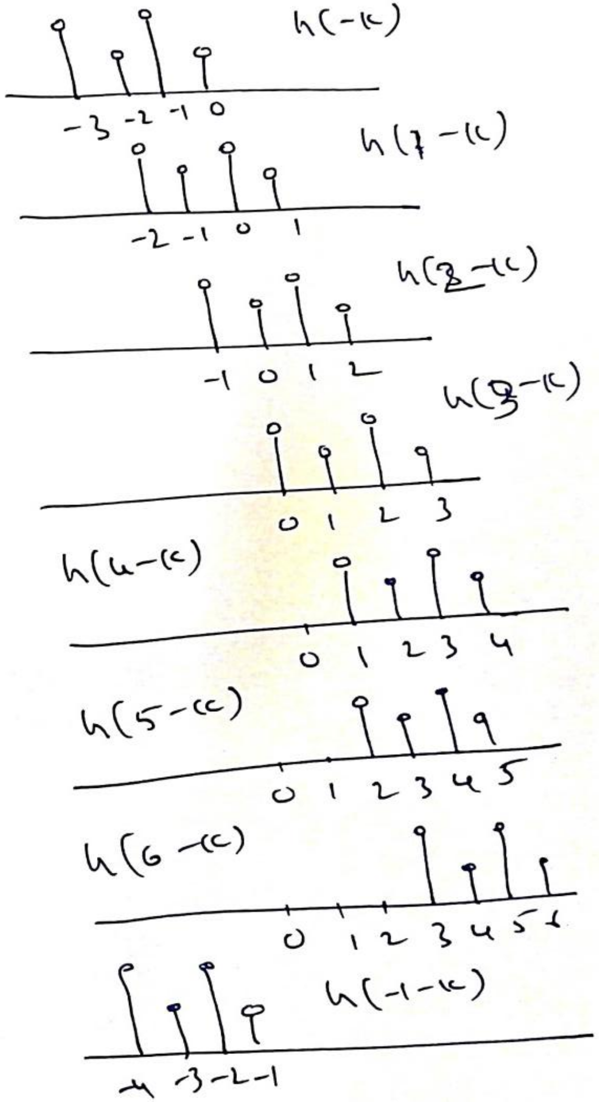
$y(3) = \sum x(k)\, h(3-k) = 6$

$y(4) = \sum x(k)\, h(4-k) = 5$

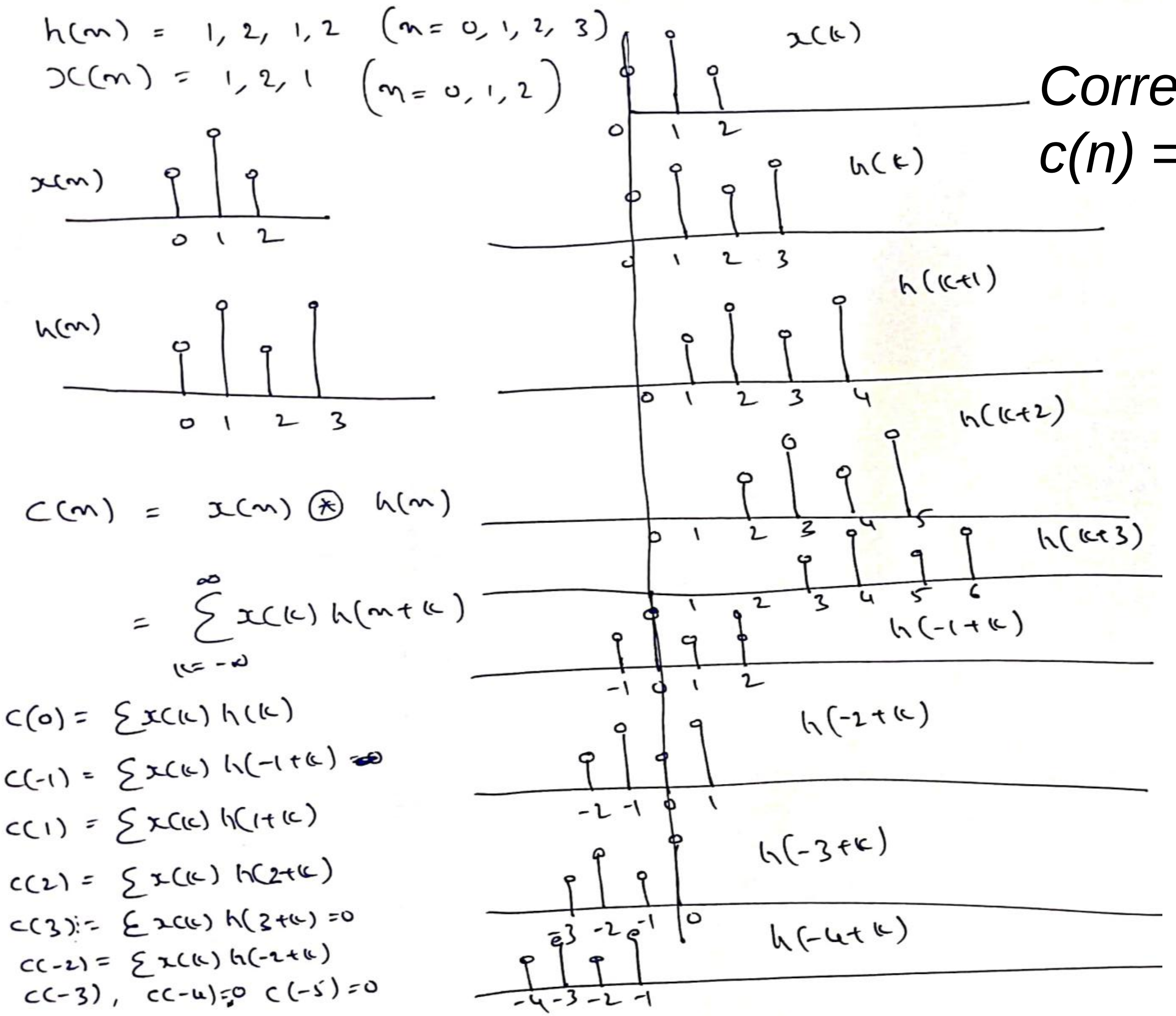
$y(5) = \sum x(k)\, h(5-k) = 2$

$y(6) = \sum x(k)\, h(6-k) = 0$

$y(-1) = \sum x(k)\, h(-1-k) = 0$



Graphical Interpretation of Correlation



Correlation :

$c(n) = \sum_{k=-\infty}^{+\infty} x(k)h(k-n)$

Computational Complexity of Correlation

$c(0) = 6$ $\quad c(2) = 1$ $\quad c(-1) = -6$ $\quad c(-3) = 2$

$c(1) = 4$ $\quad c(3) = 0$ $\quad c(-2) = 5$ $\quad c(-4) = 0$

computations

Length of $x(n)$ —— N

Length of $h(n)$ —— M

$\left.\begin{array}{l} N \\ M \end{array}\right\} N > M$

multiplications $\approx N(N+M-1)$

Auto-Correlation

Autocorrelation

$$R(n) = \sum_{k=-\infty}^{\infty} x(k)\, x(k+n)$$