

School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Quiz II (28 October 2026)

Answer All Questions

Total Marks: 20

1. Define the class of *primitive recursive* functions. **Marks: 4**

2. Describe **any eight(8)** of the following functions as *primitive recursive functions*. **Marks:** $[\mathbf{8 \times 2 = 16}]$

   Do not use the R and C operators used in the reading material. You may assume that the i^{th} function in the question list is *primitive recursive* to prove that the j^{th} function is *primitive recursive* provided $i < j$.
   Assume that the following three functions are *primitive recursive*.

   - *Predecessor Function*:
     $P : \mathbb{N}_0 \to \mathbb{N}_0 : n \mapsto \begin{cases} 0 & \text{if } n = 0 \\ n-1 & \text{if } n > 0. \end{cases}$
   - *Is zero*:
     $Z : \mathbb{N}_0 \to \mathbb{N}_0 : n \mapsto \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n > 0. \end{cases}$
   - *Not zero*:
     $NZ : \mathbb{N}_0 \to \mathbb{N}_0 : n \mapsto \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n > 0. \end{cases}$

   (a) $Add_5 : \mathbb{N}_0 \to \mathbb{N}_0 : n \mapsto n + 5$

   (b) $Mult_5 : \mathbb{N}_0 \to \mathbb{N}_0 : n \mapsto 5n$

   (c) $exp_5 : \mathbb{N}_0 \to \mathbb{N}_0 : n \mapsto 5^n$

   (d) $Add : \mathbb{N}_0 \times \mathbb{N}_0 \to \mathbb{N}_0 : (n, m) \mapsto (n + m)$

   (e) $Sum : \mathbb{N}_0 \to \mathbb{N}_0 : n \mapsto 0 + 1 + \cdots + n.$

   (f) $Mult : \mathbb{N}_0 \times \mathbb{N}_0 \to \mathbb{N}_0 : (n, m) \mapsto (n \times m)$

   (g) $Fact : \mathbb{N}_0 \to \mathbb{N}_0 : n \mapsto \begin{cases} 1 & \text{if } n = 0 \\ 1 \times \cdots \times n & \text{if } n > 0. \end{cases}$

   (h) $rSub : \mathbb{N}_0 \times \mathbb{N}_0 \to \mathbb{N}_0 : (n, m) \mapsto \begin{cases} 0 & \text{if } m \leq n \\ m - n & \text{if } m > n. \end{cases}$

(i) $Sub : \mathbb{N}_0 \times \mathbb{N}_0 \to \mathbb{N}_0 : (n, m) \mapsto \begin{cases} 0 & \text{if } n \leq m \\ n - m & \text{if } n > m. \end{cases}$

(j) $diff : \mathbb{N}_0 \times \mathbb{N}_0 \to \mathbb{N}_0 : (n, m) \mapsto \begin{cases} m - n & \text{if } n < m \\ n - m & \text{if } n \geq m. \end{cases}$

(k) $Eq : \mathbb{N}_0 \times \mathbb{N}_0 \to \mathbb{N}_0 : (n, m) \mapsto \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m. \end{cases}$

(l) $Gt : \mathbb{N}_0 \times \mathbb{N}_0 \to \mathbb{N}_0 : (n, m) \mapsto \begin{cases} 1 & \text{if } n > m \\ 0 & \text{if } n \leq m. \end{cases}$

(m) Remainder $rem : \mathbb{N}_0 \times \mathbb{N}_0 \to \mathbb{N}_0 : (n, m) \mapsto \begin{cases} 0 & \text{if } n = 0 \\ rem(n-1, m) + 1 & \text{if } \mathrm{rem}(n-1, m) + 1 < m \\ 0 & \text{if } \mathrm{rem}(n-1, m) + 1 = m. \end{cases}$

(n) Quotient. $quo : \mathbb{N}_0 \times \mathbb{N}_0 \to \mathbb{N}_0 : (n, m) \mapsto \begin{cases} 0 & \text{if } n = 0 \text{ or } m = 0 \\ quo(n-1, m) & \text{if } \mathrm{rem}(n-1, m) + 1 < m \\ quo(n-1, m) + 1 & \text{if } \mathrm{rem}(n-1, m) + 1 = m \end{cases}$