

School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Quiz I (19 August 2026)

Answer All Questions

Marks: 15

1. Answer with a brief justification (no formal proof is required). **[10]**
   Note: $\mathbb{R}$ is the set of real numbers and (a, b) is an open interval on the real line.
   (a) Show that there is a *bijection* from $(0, 1) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
   (b) Show that there is a *bijection* from $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$.
   (c) Show that there is a *bijection* from $(0, 1) \to \mathbb{R}$.
   (d) Is the collection of the finite subsets of $\{0, 1\}^*$, $\mathcal{P}\{0, 1\}^*_{fin} = \{S \subset \{0, 1\}^* : S \text{ is finite}\}$, uncountable?
   (e) Let L_1 and L_2 be two co-Turing recognizable (co-r.e.) languages. Is $L_1 \cap L_2$ is also co-Turing recognizable (co-r.e.)?

2. Show that $L_{fin} = \{< M >: M \text{ is an encoding of a Turing machine, and } L(M) \text{ is finite}\}$ is not *Turing recognizable* (showing just undecidable is not enough). **[5]**