Introduction to Syntax Analysis

The Second Phase of Front-End

Syntax Analysis

- The syntactic or the structural correctness of a program is checked during the syntax analysis phase of compilation.
- Structural properties of language constructs can be specified in different ways.
- Different styles of specification are useful for different purpose.

Different Formalisms

- Syntax diagram (SD),
- Backus-Naur form (BNF), and
- Context-free grammar (CFG).

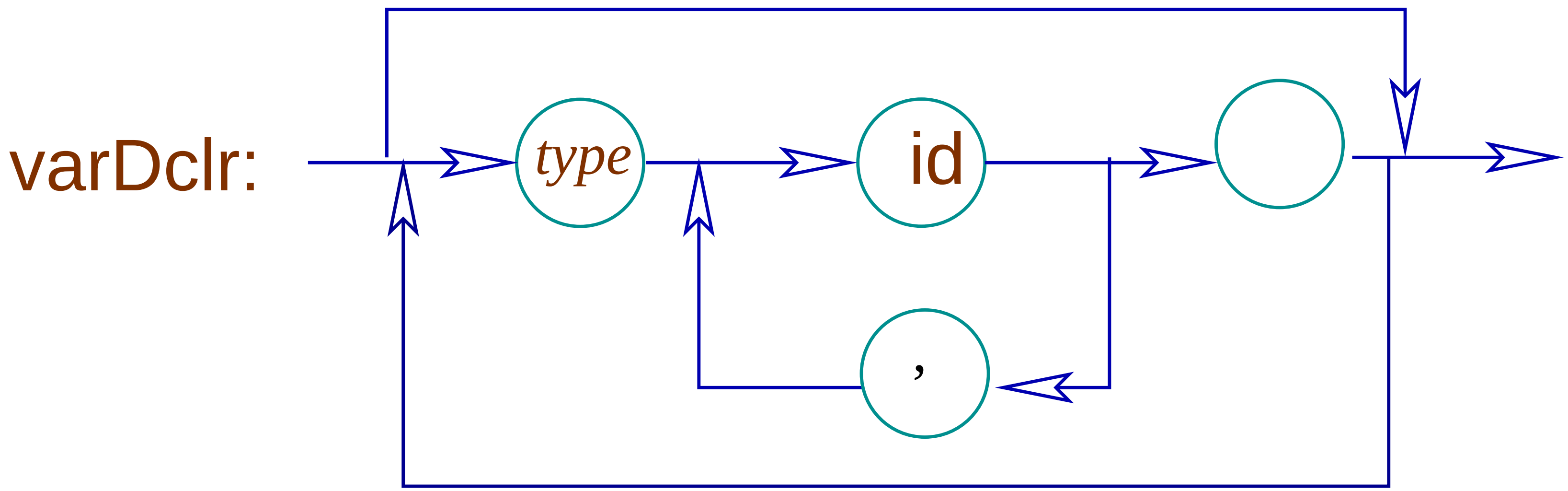
Example

We take an example of simple variable declaration in C language[a].

```
int a, b, c;
float x, y;
```

[a] This part of syntax can be expressed as a regular expression. But we shall treat them as a context-free language.

Syntax Diagram



Exercise

How will the diagram change if the variables are one or multidimensional arrays?

Context-Free Grammar

$$
\begin{aligned}
< \mathsf{VDP} > &\rightarrow \varepsilon \mid < \mathsf{VD} >< \mathsf{VD\_OPT} > \\
< \mathsf{VD} > &\rightarrow < \mathsf{TYPE} > \mathrm{id} < \mathsf{ID\_OPT} > \\
< \mathsf{ID\_OPT} > &\rightarrow \varepsilon \mid , \mathrm{id} < \mathsf{ID\_OPT} > \\
< \mathsf{VD\_OPT} > &\rightarrow ; \mid ; < \mathsf{VD} >< \mathsf{VD\_OPT} > \\
< \mathsf{TYPE} > &\rightarrow \mathrm{int} \mid \mathrm{float} \mid \cdots
\end{aligned}
$$

Exercise

Modify the grammar so that the variables are one or multidimensional arrays?

Backus-Naur Form

$$\begin{aligned} <\textsf{VDP}> \ &::= \ \varepsilon \mid <\textsf{VD}>; \{ <\textsf{VD}> ; \} \\ <\textsf{VD}> \ &::= \ <\textsf{TYPE}> \mathrm{id} \{ , \mathrm{id} \} \end{aligned}$$

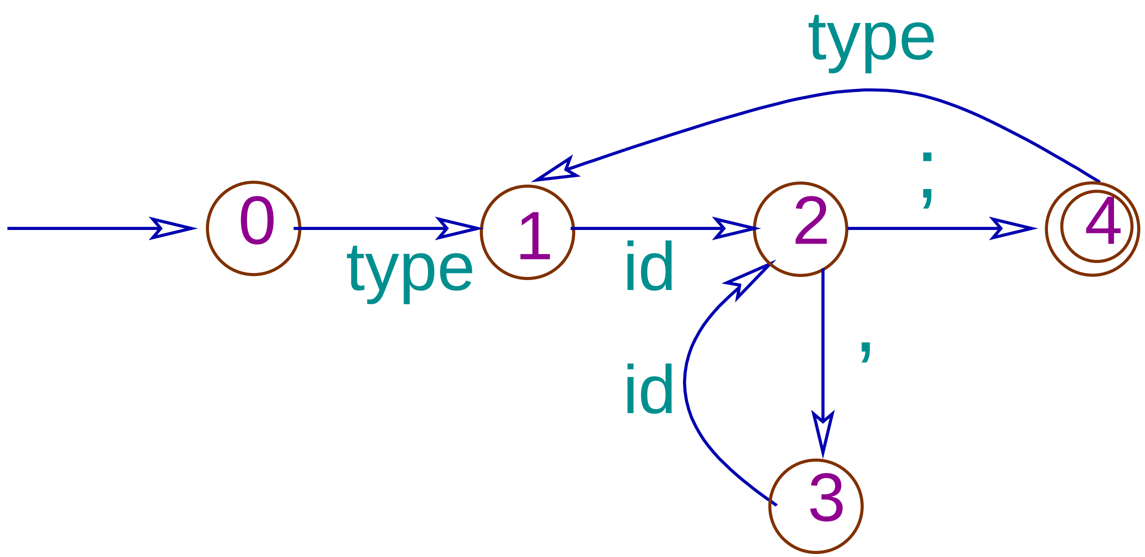
This formalism is a mixture of CFG and regular expression. Here Kleene closure x^* is written as $\{x\}$.

Exercise

Introduce multidimensional arrays in Bacus-Naur form.

Note

Our variable declaration is actually a regular language with the following state transition diagram:



Exercise

Is the variable declaration with **multidimensional arrays** a regular language?

Note

- Why go for context-free grammar. Why regular expression is not good enough.
- Consider arithmetic expressions (AE) with integer constants (IC), identifiers (ID) and four basic operators `+ - * /`.
- There are regular expressions corresponding to ID and IC.

Exercise

A regular expression corresponding to AE is as follows:

$$(\mathrm{IC}|\mathrm{ID})((\texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{/})(\mathrm{IC}|\mathrm{ID}))^*.$$

Why it is not good enough?

Note

- SD is good for human understanding and visualization.
- The BNF is very compact. It is used for theoretical analysis and also in automatic parser generating software.
- But for most of our discussion we shall consider structural specification in the form of a context-free grammar (CFG).

Note

There are non-context-free structural features of a programming language that are handled outside the formalism of grammar.

- Variable declaration and use: `... int sum ... sum = ...`, this is of the form $xwywz$ and is not context-free.
- Matching of actual and formal parameters of a function, matching of print format and the corresponding expressions etc.

Specification to Recognizer

The syntactic specification of a programming language, written as a context-free grammar can be be used to construct its parser by synthesizing a push-down automaton (PDA)[a].

[a] This is similar to the synthesis of a scanner from the regular expressions of the token classes.

Context-Free Grammar

- A context-free grammar (CFG) G is defined by a 4-tuple of data (Σ, N, P, S), where Σ is a finite set of terminals, N is a finite set of non-terminals. P is a finite subset of $N \times (\Sigma \cup N)^*$. Elements of P are called production or rewriting rules.
- The forth element S is a distinguished member of N, called the start symbol (axiom) of the grammar.

Derivation and Reduction

- If $p = (A, \alpha) \in P$, we write it as $A \rightarrow \alpha$ ("A produces α" or "A can be replaced by α").
- If $x = uAv \in (\Sigma \cup N)^*$, then we can rewrite x as $y = u\alpha v$ using the rule $p \in P$. Similarly, $y = u\alpha v$ can be reduced to $x = uAv$.
- The first process is called derivation and the second process is called reduction.

Language of a Grammar

- The language of a grammar G is denoted by $L(G) \subseteq \Sigma^*$.
- $x \in \Sigma^*$ is an element of $L(G)$, if starting from the start symbol S, a finite sequence of rewriting[a] can produce x.
- The sequence of derivation of x may be written as $S \rightarrow x$[b].

[a] In other word x can be reduced to the start symbol S.

[b] In fact it is the reflexive-transitive closure of the single step derivation. We abuse the same notation.

Sentence and Sentential Form

- Any $\alpha \in (N \cup \Sigma)^*$ derivable from the start symbol S is called a sentential form of the grammar.
- If $\alpha \in \Sigma^*$, i.e. $\alpha \in L(G)$, then α is called a sentence of the grammar.

Parse Tree

Given a grammar $G = (\Sigma, N, P, S)$, the parse tree of a sentential form x of the grammar is a rooted ordered tree with the following properties:

- The root of the tree is labeled by the start symbol S.
- The leaf nodes from left two right are labeled by the symbols of x.

Parse Tree

- Internal nodes are labeled by non-terminals so that if an internal node is labeled by $A \in N$ and its children from left to right are $A_1 A_2 \cdots A_n$, then $A \rightarrow A_1 A_2 \cdots A_n \in P$.
- A leaf node may be labeled by ε is there is a $A \rightarrow \varepsilon \in P$ and the parent of the leaf node has label A.

Example

Consider the following grammar for arithmetic expressions:

$G = (\{\texttt{id}, \texttt{ic}, (,), +, -, *, /\}, \{E, T, F\}, P, E)$.

The set of production rules, P, are,

$$
\begin{aligned}
E &\rightarrow E + T \mid E - T \mid T \\
T &\rightarrow T * F \mid T/F \mid F \\
F &\rightarrow \texttt{id} \mid \texttt{ic} \mid (E)
\end{aligned}
$$

Example

Two derivations of the sentence $\texttt{id} + \texttt{ic} * \texttt{id}$ are,

d_1: $E \rightarrow E + T \rightarrow E + T * F \rightarrow E + F * F \rightarrow T + F * F \rightarrow F + F * F \rightarrow F + \texttt{ic} * F \rightarrow \texttt{id} + \texttt{ic} * F \rightarrow \texttt{id} + \texttt{ic} * \texttt{id}$

d_2:
$E \rightarrow E + T \rightarrow T + T \rightarrow F + T \rightarrow \texttt{id} + T \rightarrow \texttt{id} + T * F \rightarrow \texttt{id} + F * F \rightarrow \texttt{id} + \texttt{ic} * F \rightarrow \texttt{id} + \texttt{ic} * \texttt{id}$

It is clear that a derivation sequence of a sentential form need not be unique.

Leftmost and Rightmost Derivations

- A derivation is leftmost if at every step the leftmost nonterminal of a sentential form is rewritten to get the next sentential form.
- Similarly, a rightmost derivation is defined similarly.
- Any string derivable unrestricted, can also be derived by leftmost or rightmost derivation, (context-free).

Ambiguous Grammar

A grammar G is said to be ambiguous if there is a sentence $x \in L(G)$ that has two distinct parse trees.

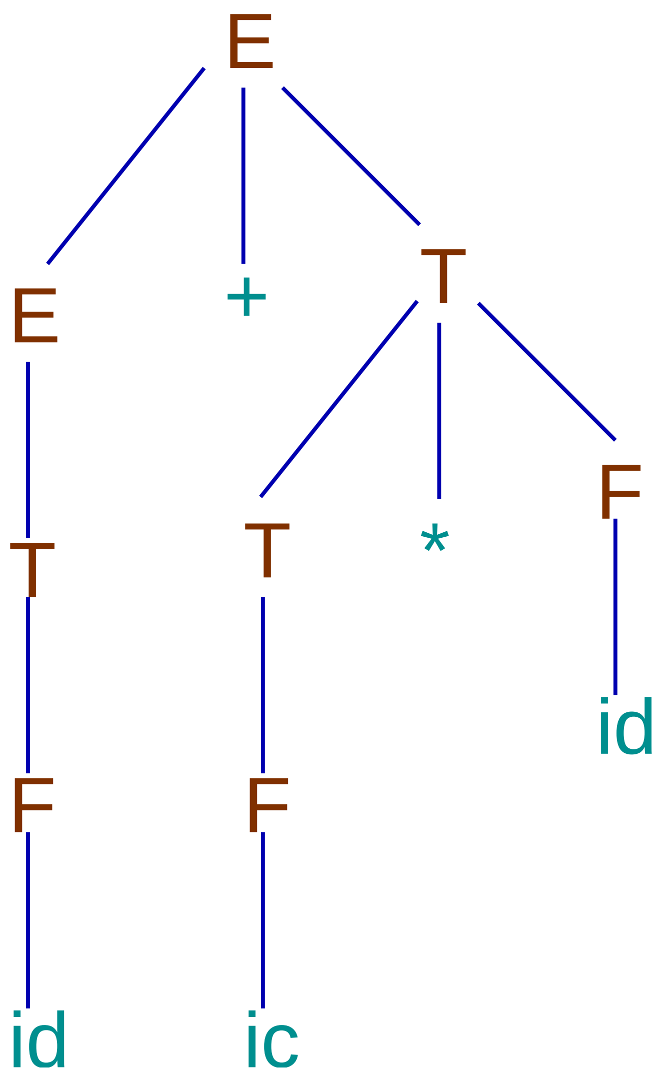
Example

Our previous grammar of arithmetic expressions is unambiguous. Following is an ambiguous grammar for the same language: $G' = (\{\texttt{id}, \texttt{ic}, (,), +, -, *, /\}, \{E\}, P, E)$. The production rules are,

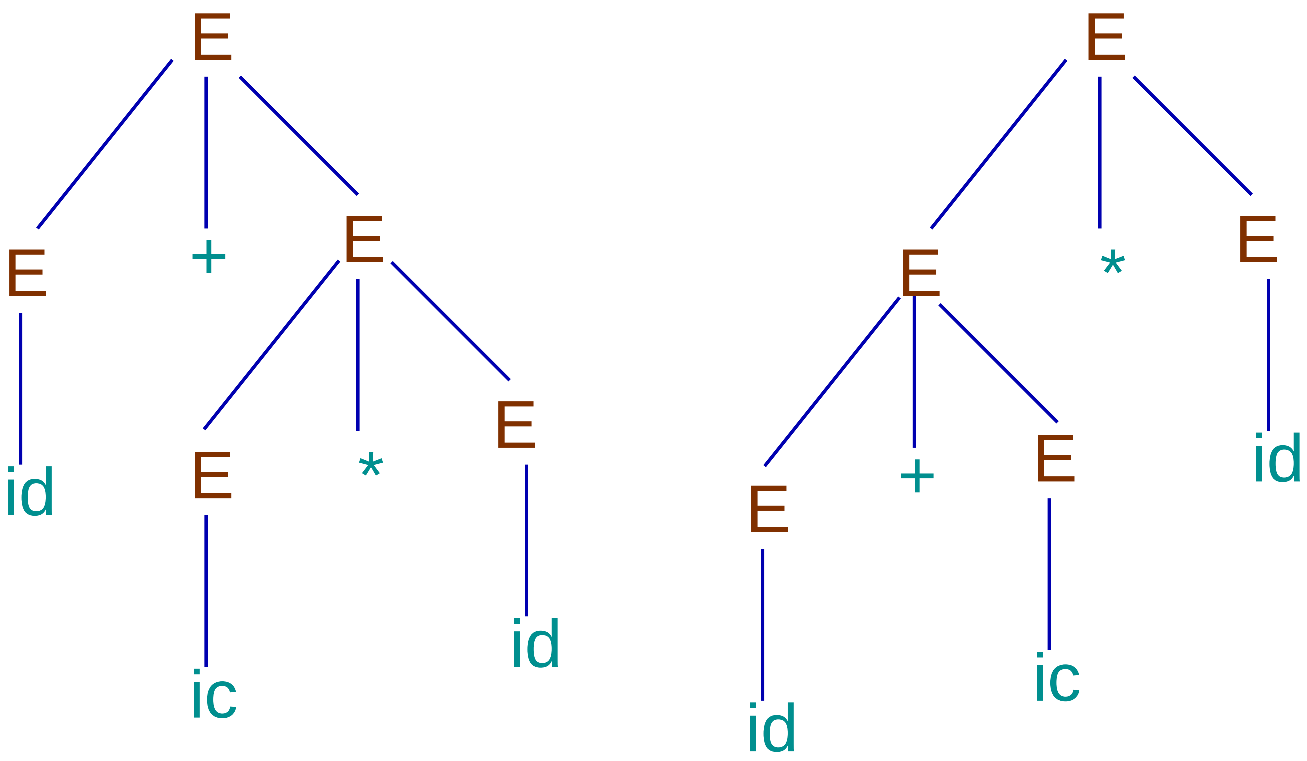
$$
\begin{array}{rcl}
E & \rightarrow & E + E \mid E - E \mid E * E \mid E/E \mid \\
  &   & \texttt{id} \mid \texttt{ic} \mid (E)
\end{array}
$$

Number of non-terminals may be less in an ambiguous grammar.

Unique Parse Tree



Non-Unique Parse Tree



Note

- Leftmost(rightmost) derivation is unique in an unambiguous grammar, but not in case of an ambiguous grammar.
- d_3: $E \rightarrow E + E \rightarrow \texttt{id} + E \rightarrow \texttt{id} + E * E \rightarrow \texttt{id} + \texttt{ic} * E \rightarrow \texttt{id} + \texttt{ic} * \texttt{id}$

  d_4: $E \rightarrow E * E \rightarrow E + E * E \rightarrow \texttt{id} + E * E \rightarrow \texttt{id} + \texttt{ic} * E \rightarrow \texttt{id} + \texttt{ic} * \texttt{id}$
- The length of derivation with an ambiguous grammar may be shorter.

`if-else` Ambiguity

Consider the following production rules:

$$S \rightarrow \texttt{if}(E)S \mid \texttt{if}(E)\ S\ \texttt{else}\ S \mid \cdots$$

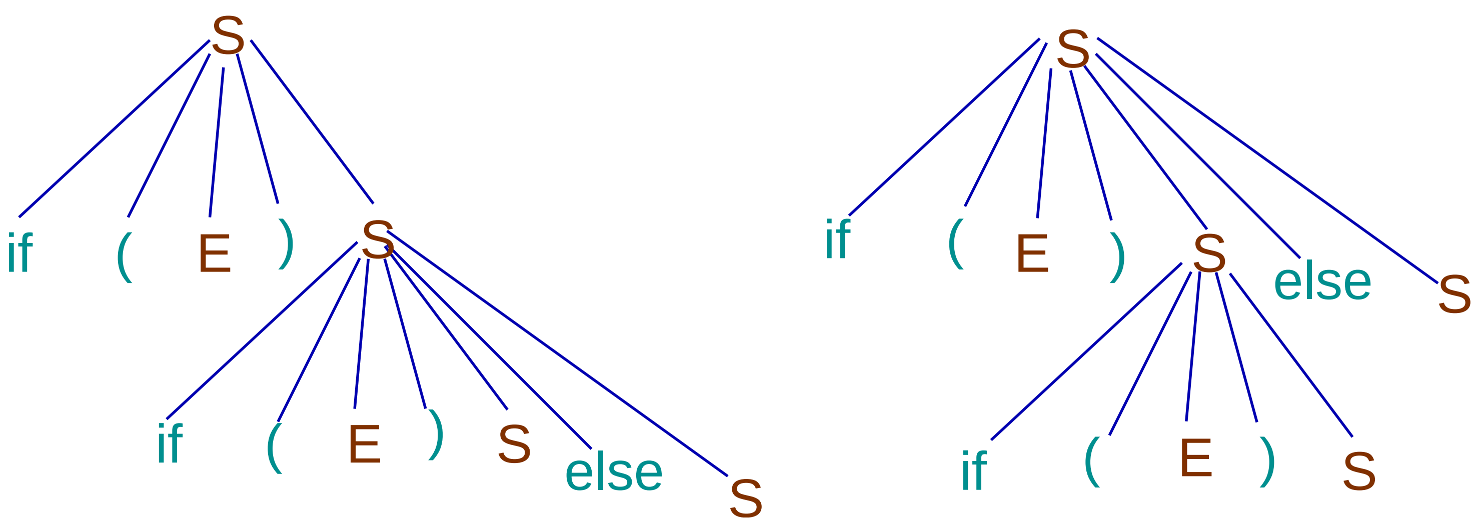
A statement of the form

`if(E1) if(E2) S2 else S3`

can be parsed in two different ways. Normally we associate the `else` to the nearest `if`[a].

[a] C compiler gives you a warning to disambiguate using curly braces.

`if-else` Ambiguity



`if-else` Modified

Consider the following production rules:

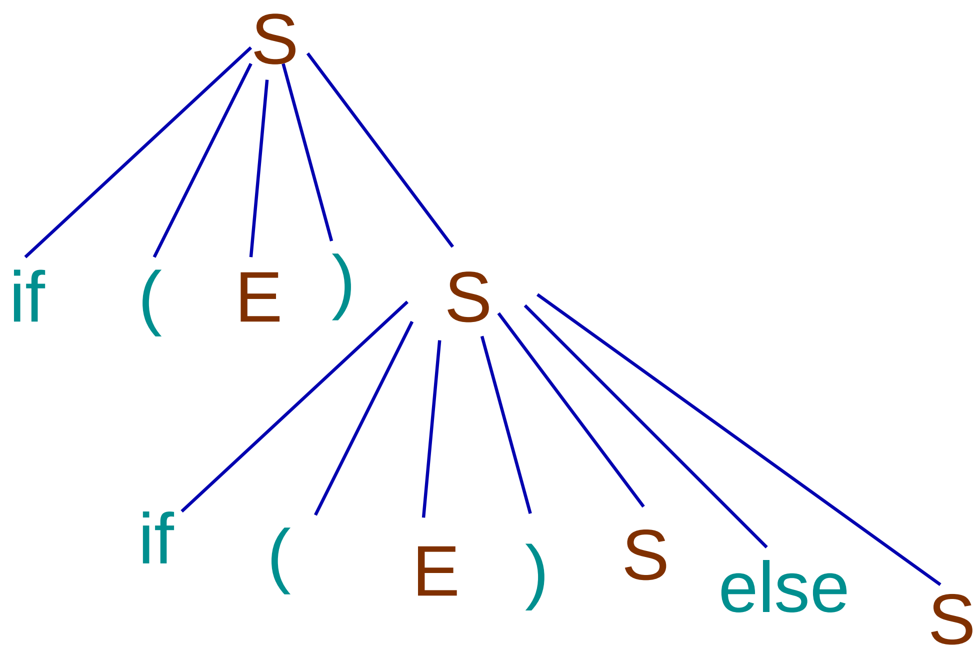
$$S \rightarrow \texttt{if}(E)S \mid \texttt{if}(E)\ ES\ \texttt{else}\ S \mid \cdots$$

$$ES \rightarrow \texttt{if}(E)\ ES\ \texttt{else}\ ES \mid \cdots$$

We restrict the statement that can appear in then-part. Now following statement has unique parse tree.

`if(E1) if(E2) S2 else S3`

if-else Unambiguous



Note

Consider the following grammar G_1 for arithmetic expressions:

$$
\begin{array}{rcl}
E & \rightarrow & T + E \mid T - E \mid T \\
T & \rightarrow & F * T \mid F / T \mid F \\
F & \rightarrow & \texttt{id} \mid \texttt{ic} \mid (E)
\end{array}
$$

Is $L(G) = L(G_1)$? What difference does the grammar make?

Problem

Consider another version of the grammar G_2:

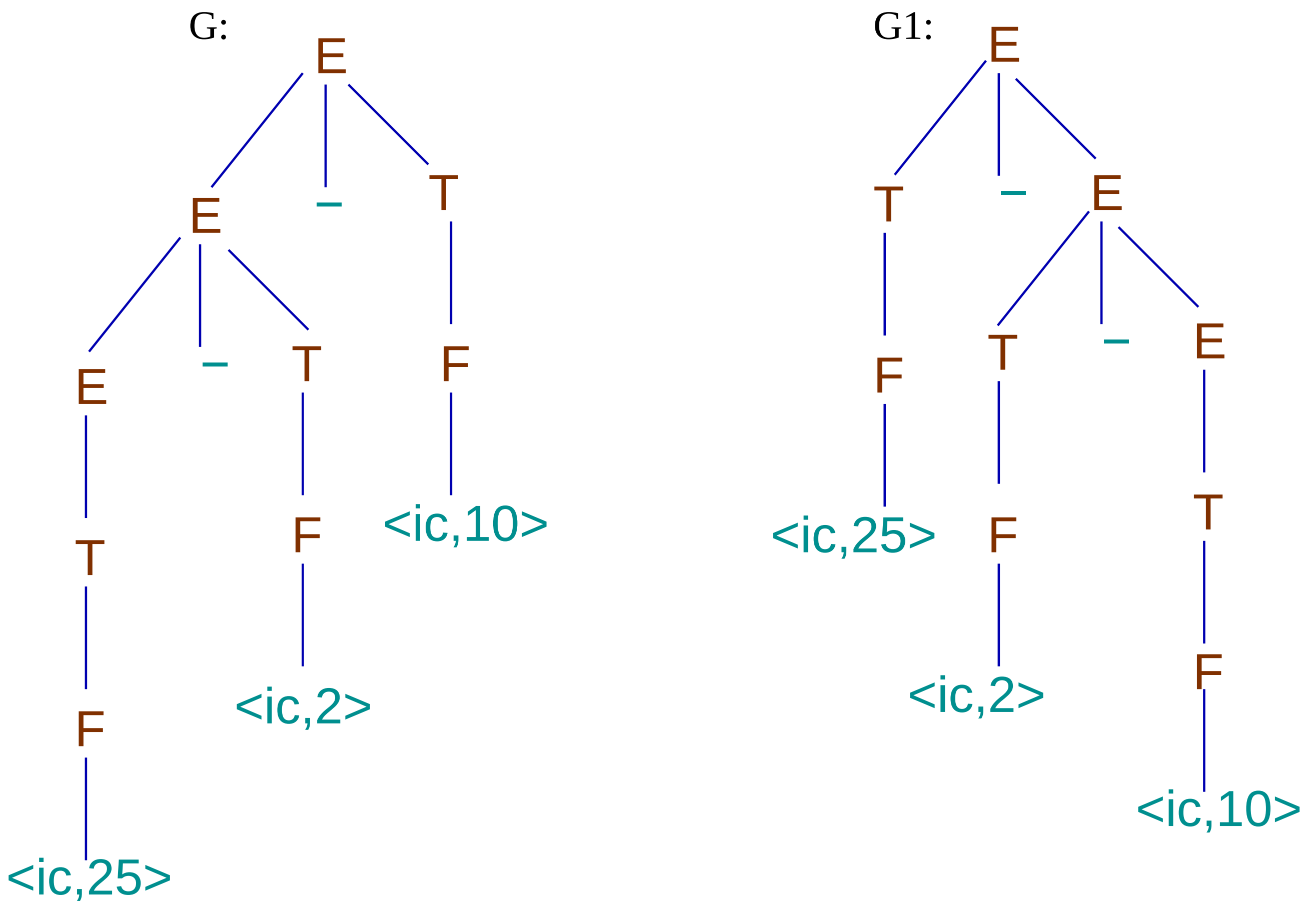
$$
\begin{array}{rcl}
E & \rightarrow & E * T \mid E/T \mid T \\
T & \rightarrow & T + F \mid T - F \mid F \\
F & \rightarrow & \texttt{id} \mid \texttt{ic} \mid (E)
\end{array}
$$

What is the difference in this case? Is $L(G) = L(G_2)$.

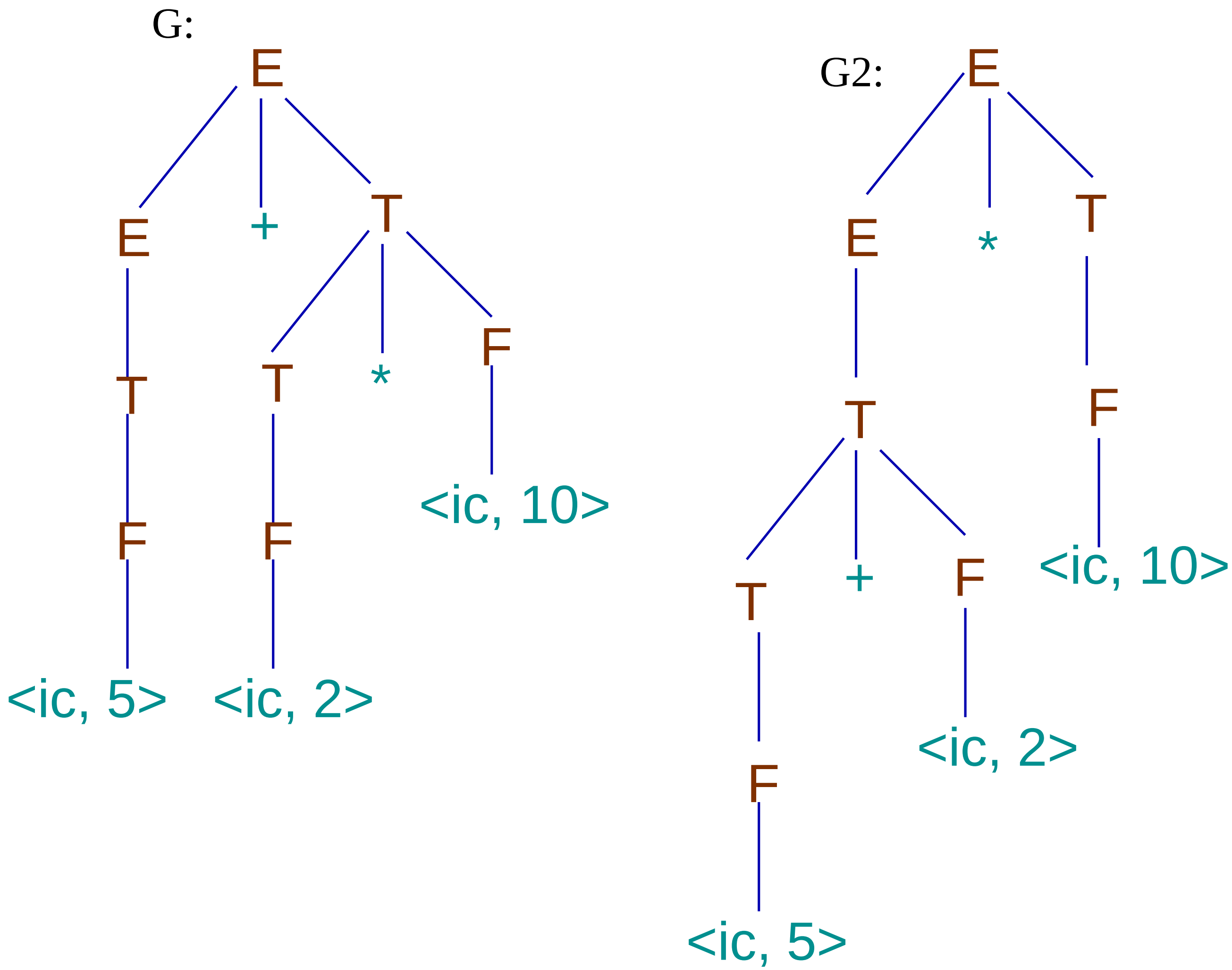
Problem

- Construct parse trees and abstract syntax trees corresponding to the input `25-2-10` for G and G_1.
- Similarly, construct parse trees and abstract syntax trees corresponding to the input `5+2*10` for G and G_2.
- In both the cases find evaluation orders.

G and G_1



G and G_2



- G: $(25 - 2) - 10 = 13$
  G_1: $25 - (2 - 10) = 33$
- G: $5 + (2 * 10) = 25$
  G_2: $(5 + 2) * 10 = 70$

A Few Important Transformations

Useless Symbols

A grammar may have useless symbols that can be removed to produce a simpler grammar. A symbol is useless if it does not appear in any sentential form producing a sentence.

Useless Symbols

We first remove all non-terminals that does not produce any terminal string; then we remove all the symbols (terminal or non-terminal) that does not appear in any sentential form. These two steps are to be followed in the given order[a].

[a]As an example (HU), all useless symbols will not be removed if done in the reverse order on the grammar $S \to AB \mid a$ and $A \to a$.

ε-Production

If the language of the grammar does not have any ε, then we can free the grammar from ε-production rules. If ε is in the language, we can have only the start symbol with ε-production rule and the remaining grammar free of it.

Example

$$
\begin{aligned}
S &\rightarrow 0A0 \mid 1B1 \mid BB \\
A &\rightarrow C \\
B &\rightarrow S \mid A \\
C &\rightarrow S \mid \varepsilon
\end{aligned}
$$

All non-terminals are nullable.

Example

After removal of ε-productions.

$$
\begin{aligned}
S &\rightarrow 0A0 \mid 1B1 \mid BB \mid 00 \mid 11 \mid B \mid \varepsilon \\
A &\rightarrow C \\
B &\rightarrow S \mid A \\
C &\rightarrow S
\end{aligned}
$$

Unit Production

A production of the form $A \to B$ may be removed otherwise the attributes of B is to be propagated to A.

Normal Forms

A context-free grammar can be converted into different normal forms e.g. Chomsky normal form etc. These are useful for some decision procedure e.g. CKY algorithm. But are not of much importance for compilation.

Left and Right Recursion

A CFG is called left-recursive if there is a non-terminal A such that $A \Rightarrow^* A\alpha$ after a finite number of steps. It is necessary to remove left-recursion for a top-down parser[a].

[a]The right recursion can be similarly defined. It does not have so much problem as we do not read input from right to left, but in a bottom-up parser the stack size may be large due to right-recursion.

Immediate Left-Recursion

A left-recursion is called immediate if a production rule of the form $A \rightarrow A\alpha$ is present in the grammar. It is easy to eliminate an immediate left-recursion. We certainly have production rules of the form

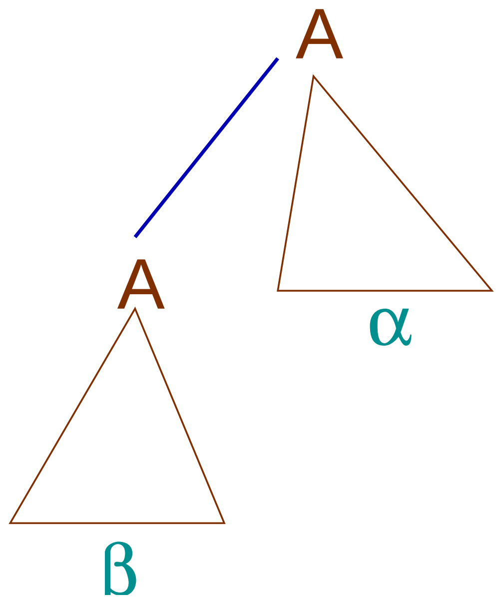
$$A \;\rightarrow\; A\alpha_1 \mid \beta$$

where the first symbol of β does not produce A as the first symbol[a].

[a] Otherwise A will be a useless symbol.

Parse Tree

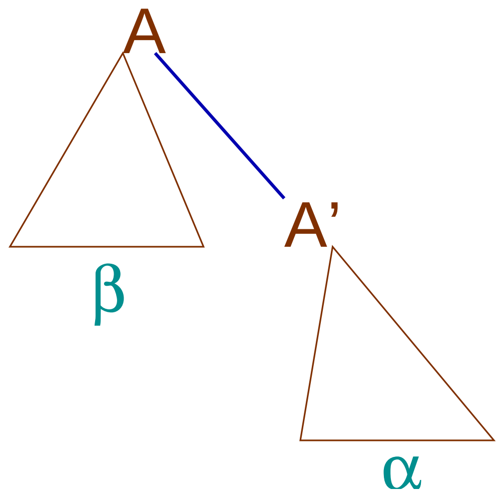
The parse tree with this pair of production rules looks as follows:



The yield is $\beta\alpha$.

Rotation

We can rotate the parse tree to get the same yield, but without the left-recursion.



The new rules are $A \to \beta A'$ and $A' \to \alpha A' \mid \varepsilon$.

Removal of Immediate Left-Recursion

The original grammar is

$$\begin{aligned} A &\rightarrow A\alpha_1 \mid A\alpha_k \mid \cdots \mid A\alpha_k \\ A &\rightarrow \beta_1 \mid \beta_2 \mid \cdots \mid \beta_l \end{aligned}$$

The transformed grammar is

$$\begin{aligned} A &\rightarrow \beta_1 A' \mid \beta_2 A' \mid \cdots \mid \beta_l A' \\ A' &\rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \cdots \mid \alpha_k A' \mid \varepsilon \end{aligned}$$

Example

Original grammar:

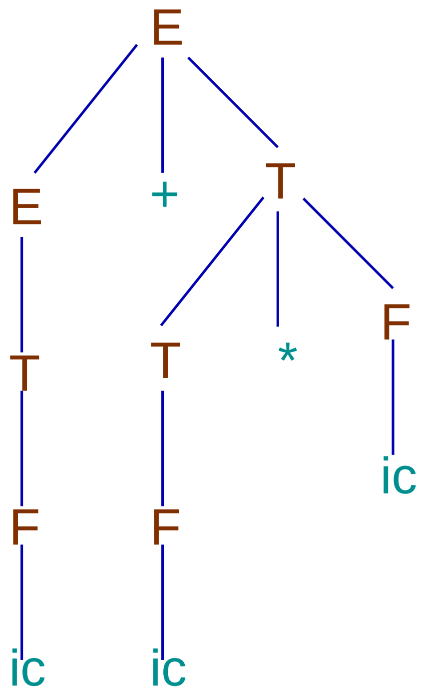
$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T * F \mid F \\
F &\rightarrow (E) \mid ic
\end{aligned}
$$

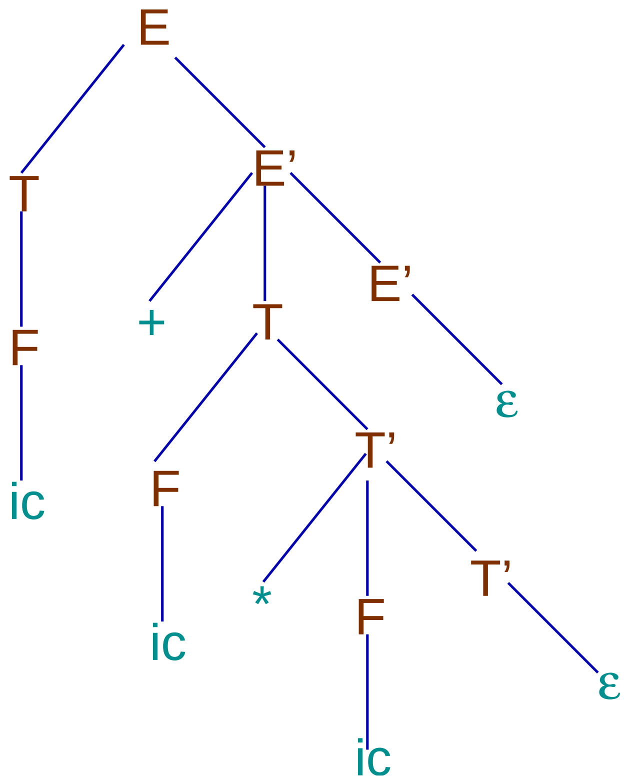
The transformed grammar is

$$
\begin{array}{lllllll}
E & \rightarrow & TE' & & E' & \rightarrow & +TE' \mid \varepsilon \\
T & \rightarrow & FT' & & T' & \rightarrow & *FT' \mid \varepsilon \\
F & \rightarrow & (E) \mid ic & & & &
\end{array}
$$

Change in the Parse Tree

Consider the input `ic+ic*ic`:





Removal of Indirect Left-Recursion

Consider the following grammar:

$$
\begin{aligned}
A &\rightarrow Aab \mid Ba \mid Cb \mid b \\
B &\rightarrow Aa \mid Db \\
C &\rightarrow Ab \mid Da \\
D &\rightarrow Bb \mid Ca
\end{aligned}
$$

The grammar has indirect left-recursion: $A \rightarrow Ba \rightarrow Aaa$ etc.

Removal of Indirect Left-Recursion

- First we order the non-terminals: $A_1 < A_2 < \cdots < A_n$.
- Following algorithm eliminates direct and indirect left-recursions.

Algorithm

for $i = 1$ to n

  for $j = 1$ to $i - 1$

    replace rule of the form $A_i \rightarrow A_j\gamma$

    by $A_i \rightarrow \delta_1\gamma \mid \cdots \mid \delta_k\gamma$, where

    $A_j \rightarrow \delta_1 \mid \cdots \mid \delta_k$ are the current

    A_j productions

  remove immediate left-recursion of

  A_i-productions.

Removal of Indirect Left-Recursion

- In the first iteration of the outer loop ($i = 1$), immediate left recursions of A_1 are removed.
- After this iteration any production rule of the form $A_1 \rightarrow A_l\beta$ has $l > 1$.
- Similarly after the $(i-1)^{th}$ iteration of the outer-loop, for no A_k, ($k = 1, \cdots, i-1$), there is any production rule of the form $A_k \rightarrow A_l\gamma$, where $k \geq l$.

Removal of Indirect Left-Recursion

- In the i^{th} iteration, the inner loop exposes any recursion of A_i through A_js, $j = 1, \cdots, i-1$.
- It progressively transforms $(j = 1, \cdots, i-1)$ every production $A_i \rightarrow A_j\beta$, until $j \geq i$.
- Then the outer loop removes the immediate left recursions of A_i.

Example

Let $A < B < C < D$. In the first-pass $(i = 1)$ of the outer loop, the immediate recursion of A is removed.

$$
\begin{array}{rcl}
A & \rightarrow & BaA' \mid CbA' \mid bA' \\
A' & \rightarrow & abA' \mid \varepsilon \\
B & \rightarrow & Aa \mid Db \\
\cdots & \cdots & \cdots
\end{array}
$$

Example

In the second-pass ($i = 2$) of the outer loop, $B \rightarrow Aa$ are replaced and immediate left-recursions on B are removed.

$$
\begin{array}{lcl}
A & \rightarrow & BaA' \mid CbA' \mid bA' \\
A' & \rightarrow & abA' \mid \varepsilon \\
B & \rightarrow & BaA'a \mid CbA'a \mid bA'a \mid Db \\
\cdots & \cdots & \cdots
\end{array}
$$

Example

$$
\begin{aligned}
A &\rightarrow BaA' \mid CbA' \mid bA' \\
A' &\rightarrow abA' \mid \varepsilon \\
B &\rightarrow DbB' \mid bA'aB' \mid CbA'aB' \\
B' &\rightarrow aA'aB' \mid \varepsilon \\
C &\rightarrow Ab \mid Da \\
\cdots & \;\;\cdots \;\;\cdots
\end{aligned}
$$

Example

In the third-pass ($i = 3$) of the outer loop,

$$
\begin{array}{rcl}
A & \rightarrow & BaA' \mid CbA' \mid bA' \\
A' & \rightarrow & abA' \mid \varepsilon \\
B & \rightarrow & DbB' \mid bA'aB' \mid CbA'aB' \\
B' & \rightarrow & aA'aB' \mid \varepsilon \\
C & \rightarrow & BaA'b \mid CbA'b \mid bA'b \mid Da \\
\cdots & \cdots & \cdots
\end{array}
$$

Example

$$
\begin{array}{rcl}
A & \rightarrow & BaA' \mid CbA' \mid bA' \\
A' & \rightarrow & abA' \mid \varepsilon \\
B & \rightarrow & DbB' \mid bA'aB' \mid CbA'aB' \\
B' & \rightarrow & aA'aB' \mid \varepsilon \\
C & \rightarrow & DbB'aA'b \mid bA'aB'aA'b \mid CbA'aB'aA'b \\
 & & CbA'b \mid bA'b \mid Da \\
\cdots & \cdots & \cdots
\end{array}
$$

Left Factoring

- More than one production rules of a non-terminal, with the same prefix at the right hand side, creates the problem of rule selection in a top-down parser.
- The grammar is transformed by left factoring so that the prefixes of the right-hand of different rules of a non-terminal are different.

Example

If we have production rules of the form $A \rightarrow xB\alpha$, $A \rightarrow xC\beta$, $A \rightarrow xD\gamma$, we transform them to $A \rightarrow xE$ and $E \rightarrow B\alpha \mid C\beta \mid D\gamma$, where $x \in \Sigma^*$.

Substitution

- The left factor may not be visible due to the presence of non-terminals.
- It may be necessary to substitute the leftmost non-terminals of the right-hand sides of production rules.

Example

- Let $A \rightarrow Bb \mid Cd$, $B \rightarrow abB \mid b$, $C \rightarrow adC \mid d$ before substitution.
- After the substitution we get, $A \rightarrow abBb \mid bb \mid adCd \mid dd$, $B \rightarrow abB \mid b$, $C \rightarrow adC \mid d$.
- Now the rules of A can be factored.

Parsing

- Using the grammar as a specification, a parser tries to construct the parse tree corresponding to the input (a program to compile). This construction may be top-down or bottom-up.
- The top-down parsing may be viewed as a pre-order construction and the bottom-up parsing as a post-order construction of the parse tree.

Top-Down Parsing

- A top-down parser starts from the start symbol (S) to generate the input string of tokens (x).
- When a top-down parser tries to build the subtree of an internal node, the non-terminal A present at the node is known.
- It decides the appropriate production rule of A using the information from the input.

Top-Down Parsing

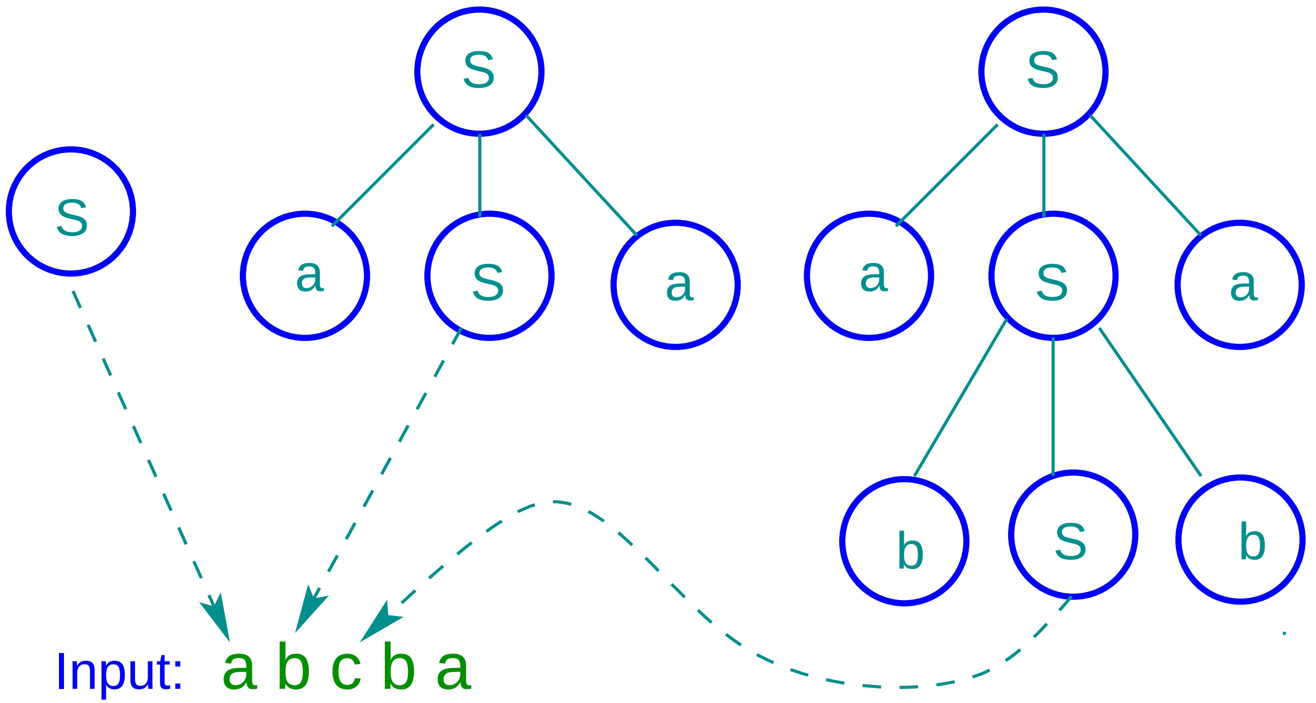
- The node is expanded to its children and they are labeled by the symbols of the chosen production rule of A.
- The parser continues the construction of the tree from the left child (left to right) of A.
- If the left child is a terminal it matches with the leftmost token of the token stream.

Top-Down Parsing

- Once a terminal is matched with the token, the parser continues with the next pre-order node.
- For a context-free grammar the choice of the appropriate rule of a non-terminal, may not be deterministic. And it may be necessary to backtrack.

Top-Down Parsing

Consider the grammar: $S \rightarrow aSa \mid bSb \mid c$



Non-Determinism

- The situation will be different if the rule $S \rightarrow c$ is replaced by $S \rightarrow a$ or $S \rightarrow b$ or $S \rightarrow \varepsilon$.
- Looking at fixed number of incoming tokens we cannot decide the rule to expand S.

Note: Top-Down

- Input is always read (consumed) from left-to-right.
- A snapshot of a top-down parser on an input x is as follows.
- A part of the input u has already been generated (tokens consumed) i.e. $x = uv$ and the parser has the sentential form $uA\alpha$.

Note: Top-Down

- The parser tries to decide the correct rule for A to get the next sentential form.
- It always expands the leftmost variable, following the leftmost derivation.
- The choice of rule depends on the initial part of the remaining input.
- A choice of production rule may lead to a dead-end and backtracking.

Example

Consider the following grammar:

$$S \rightarrow aSa \mid bSb \mid a \mid b$$

Given a sentential form aabaSabaa and the remaining portion of the input `ab`$\cdots$ it is impossible to decide by seeing one or two or any finite number of input symbols, whether to use the first or the third production rule to generate '`a`' of the input.

Example

Consider the following grammar:

$$S \rightarrow aSa \mid bSb \mid c$$

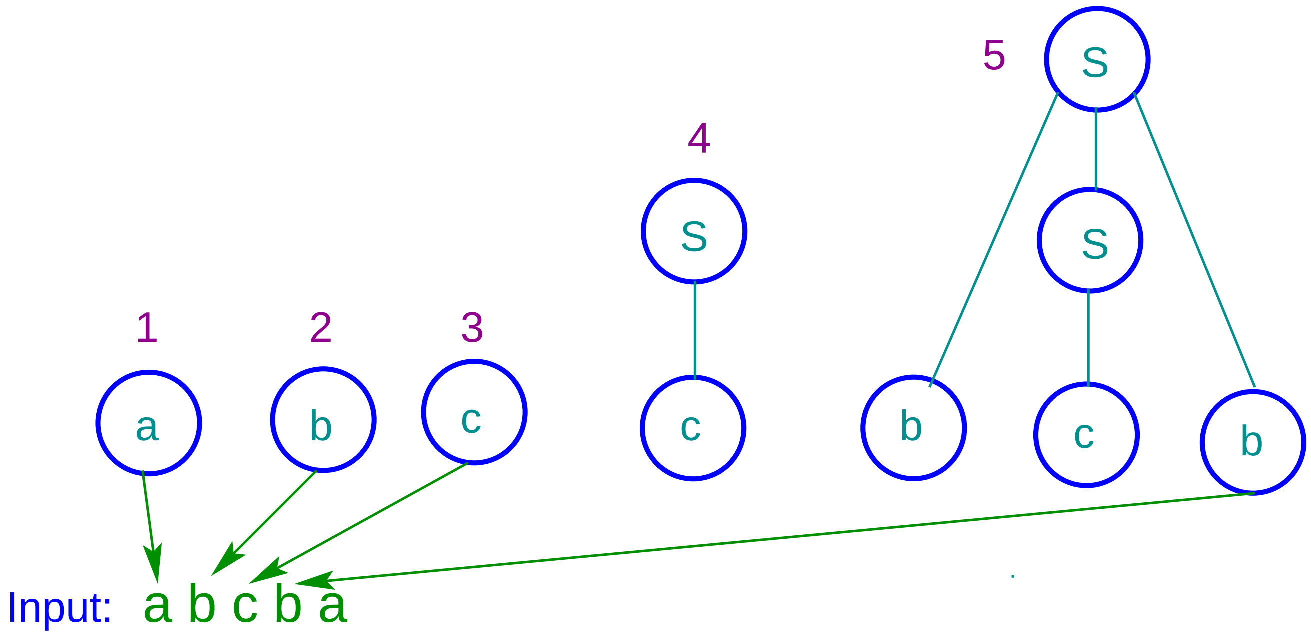
Given a sentential form aabaSabaa and the remaining portion of the input `abc`$\cdots$, it is clear from the first element of input that the first production rule is to be applied to get the next sentential form.

Bottom-Up Parsing

- A bottom-up parser starts from the input x and tries to reduce it to the start symbol S.
- The internal nodes of the syntax-tree are constructed in post-order.
- The root of a subtree is constructed after its children are constructed and labeled (already known).
- Each Token is a sub-tree of label 1.

Bottom-Up Parsing

Consider the grammar: $S \rightarrow aSa \mid bSb \mid c$



Note: Bottom-Up

- In a bottom-up parser on the input x, the parsing proceeds as follows:
- The current sentential form is αv where $\alpha \in \Sigma \cup N$, and the remaining portion of the input is v. If $x = uv$, then $\alpha \Rightarrow^* u$.
- At this point the parser tries to find a β so that $\alpha' \beta v' = \alpha v$, $A \rightarrow \beta \in P$ and $\alpha' A v'$ is the previous sentential form.

Note: Bottom-Up

There may be more than one such choices possible, and some of them may be incorrect. If β is always a suffix of α, then we are following a sequence of right-most derivation in reverse order (reductions).

Example

Consider the grammar:

$$E \rightarrow E + E \mid E * E \mid ic$$

Given the input `ic+ic*ic···`, many reductions are possible and in this case all of them will finally lead to the start symbol. The previous sentential form can be any one of the following three, and there are many more: `E+ic*ic···`, `ic+E*ic···`, `ic+ic*E···` etc. The first one is the right sentential form.