Introduction to Soft Computing

Solution to Practice Sheet FL-2

Topic:

- *Fuzzy relations*
- *Fuzzy propositions*
- *Fuzzy implications*
- Fuzzy inferences

1) If $\boldsymbol{x\ is\ A\ then\ y\ is\ B\ else\ y\ is\ C}$. The output of the given fuzzy rule is

    (a) a fuzzy set
    (b) a crisp set
    (c) a fuzzy relation
    (d) a membership function

2) The cardinality of the given set $A = \{1, 2, 3, 4, 5\}$

    (a) 2
    (b) 5
    (c) 4
    (d) 1

3) The cardinality of the fuzzy set on any universe is

    (a) infinity
    (b) 0
    (c) 1
    (d) -1

4) Given a crisp set $A = \{1, 2, 3, 4\}$. Find the relation matrix for the relation $R: \{(a, b)\ |b = a + 1, \quad a, b \in A\}$

(a)

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(b)

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(c)

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(d)

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

5) Given that "$x\ is\ Sweet$" with $T(x) = 0.8$ and "$y\ is\ Sweet$" with $T(y) = 0.6$. The Fuzzy truth value of "$If\ \ x\ is\ Sweet\ then\ y\ is\ Sweet$" is

(a) 0.4
(b) 0.2
(c) 0.8
(d) 0.6

6) Let $P = \begin{bmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{bmatrix}$ and $Q = \begin{bmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{bmatrix}$ Find R where

$R = P \circ Q$ using max-min composition

(a) $R = \begin{bmatrix} 0.8 & 0.3 & 0.5 & 0.5 \\ 1 & 0.2 & 5 & 0.7 \\ 0.5 & 0.4 & 0.5 & 0.5 \end{bmatrix}$

(b) $R = \begin{bmatrix} 0.8 & 0.3 & 0.5 & 0.5 \\ 0.5 & 0.2 & 0.8 & 0.7 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$

(c) $R = \begin{bmatrix} 1 & 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 1 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$

(d) $R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0.8 & 0.7 \\ 0.5 & 0.5 & 0 & 0.5 \end{bmatrix}$

7) $(\sim(P \land Q) \Longrightarrow R) \land P \land Q$ is equivalent to

(a) $(P \land Q)$
(b) $(P \land Q) \lor R$
(c) P
(d) $(\sim P \lor Q)$

8) If $\boldsymbol{x \text{ is } A \text{ then } y \text{ is } B \text{ else } y \text{ is } C}$, then the relation R is equivalent to

(a) $(A \times B) + (B \times C)$
(b) $(A \times B) \cup (\bar{A} \times C)$
(c) $(A \times B) \rightarrow (B \times C)$
(d) $(A \times C) \cup (B \times C)$

9) What are the applications of Fuzzy Inference Systems?

(a) Wireless services, heat control and printers

(b) Restrict power usage, telephone lines and sort data

(c) Simulink, boiler and CD recording

(d) Automatic control, decision analysis and data classification

10) "Generalized Modus Tollen (GMT)" rule which is as follows:

If x is A Then y is B

y is B'

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x is A'

A' can be calculated as

(a) $A' = B' \circ R\ (x, y)$

(b) $A' = (A \times B) \cup (\bar{A} \times Y)$

(c) $A' = A \circ R\ (x, y)$

(d) $A' = (A \times B) \cup (\bar{A} \times X)$

11) Zadeh's max-min rule is defined as:

(a) $R = \int_{X \times Y} \mu_A(x) * \mu_B(y) \,|(x, y)$

(b) $R = \bar{A} \cup B = \int_{X \times Y} (1 - \mu_A(x)) \vee \mu_B(x) \,|(x, y)$

(c) $R = \bar{A} \cup B = \int_{X \times Y} 1 \wedge (1 - \mu_A(x) + \mu_B(y)) \,|(x, y)$

(d) $R = \bar{A} \cup (A \cap B) = \int_{X \times Y} (1 - \mu_A(x)) \vee (\mu_A(x) \wedge \mu_B(y)) \,|(x, y)$

12) Larsen rule is defined as:

(a) $R = A \times B = \int_{X \times Y} \mu_A(x) \wedge \mu_B(y) \,|(x, y)$

(b) $R = A \times B = \int_{X \times Y} \mu_A(x) . \mu_B(y) \,|(x, y)$

(c) $R = A \times B = \int_{X \times Y} \mu_A(x) \odot \mu_B(y) \,|(x, y)$

(d) $R = A \times B = \int_{X \times Y} \mu_A(x) \hat{\bullet} \mu_B(y) \,|(x, y)$

13) Find the results of the fuzzy operations as instructed in the following:

$R = A \times B$ where

$A = \left\{\frac{0.1}{x_1}, \frac{0.2}{x_3}, \frac{0.5}{x_5}\right\}$

$B = \left\{\frac{0.6}{x_2}, \frac{0.8}{x_3}, \frac{1.0}{x_6}\right\}$

(a)

X	x_1	x_2	x_3	x_5	x_6
x_1	0.0	0.1	0.1	0.0	0.1
x_2	0.0	0.0	0.0	0.0	0.0
x_3	0.0	0.2	0.2	0.0	0.2
x_5	0.0	0.5	0.5	0.0	0.5
x_6	0.0	0.0	0.0	0.0	0.0

(b)

X	x_1	x_2	x_3	x_5	x_6
x_1	0.0	0.1	0.1	0.0	0.1
x_2	0.0	0.0	0.0	0.0	0.0
x_3	0.0	0.1	0.1	0.0	0.1
x_5	0.0	0.5	0.5	0.0	0.5
x_6	0.0	0.0	0.0	0.0	0.0

(c)

X	x_1	x_2	x_3	x_5	x_6
x_1	0.0	0.1	0.1	0.0	0.1
x_2	0.0	0.2	0.2	0.0	0.2
x_3	0.0	0.2	0.2	0.0	0.2
x_5	0.0	0.5	0.5	0.0	0.5
x_6	0.0	0.0	0.0	0.0	0.0

(d)

X	x_1	x_2	x_3	x_5	x_6
x_1	0.0	0.1	0.1	0.0	0.1
x_2	0.0	0.0	0.0	0.0	0.0
x_3	0.0	0.2	0.2	0.0	0.2
x_5	0.0	0.5	0.5	0.0	0.5
x_6	0.0	0.2	0.2	0.0	0.2

14) Let X= {a, b, c, d} Y= {1, 2, 3, 4}

and A= {(a,0.0), (b,0.8), (c,0.6), (d,1.0)}

B= {(1,0.2), (2,1.0), (3,0.8), (4,0.0)}

C= {(1,0.0), (2,0.4), (3,1.0), (4,0.8)}

Determine the implication relation: $\textbf{\textit{If x is A then y is B}}$

(a)

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

(b)

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.4 & 0.6 & 0.6 & 0.4 \\ 0.2 & 0.1 & 0.8 & 0 \end{bmatrix} \end{array}$$

(c)

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0.4 & 1 & 0.8 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.2 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

(d)

1 2 3 4

$$\begin{matrix} a \\ b \\ c \\ d \end{matrix}\begin{bmatrix} 0 & 0.4 & 1 & 0.8 \\ 0 & 0.2 & 0.2 & 0.2 \\ 0 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$