Chapter 4

Fuzzy Proposition

Main difference between classical proposition and fuzzy proposition is in the range of their truth values. The proposition value for classical proposition is either true or false but in case of fuzzy proposition the range is not confined to only two values it varies from 2 to n. For example speed may be fast, very fast, medium, slow, and very slow. In fuzzy logic the truth value of fuzzy proposition is also depend on an additional factor known as degree of truth whose value is varies between 0 and 1. For example

p: Speed is Slow
T(p) = 0.8, if p is partly true
T(p) = 1, if p is absolutely true
T(p) = 0, if p is totally false

So, we can say that fuzzy proposition is a statement p which acquires a fuzzy truth value T(p) ranges from(0 to1).

Different types of Fuzzy Propositions

1. Unconditional and unqualified propositions

The canonical form of this type of fuzzy proposition is
p:V is F
Where, V is a variable which takes value v from a universal set U. F is a fuzzy set on U that represents a given inaccurate predicate such as fast, low, tall etc.
For example:
p: Speed (V) is high (F)
T(p) = 0.8, if p is partly true
T(p)=1, if p is absolutely true
T(p)=0, if p is totally false
Where,
T(p) = $\mu_F(v)$ membership grade function indicates the degree of truth of v belongs to F, its value ranges from 0 to 1.

2. Unconditional and qualified propositions

The canonical form of this type of fuzzy proposition is
p:V is F is S
Where, *V* and *F* have the same meaning and *S* is a fuzzy truth qualifier
Example:
Speed is high is very true

3. Conditional and unqualified propositions

The canonical form of this type of fuzzy proposition is
p: if X is A, then Y is B

Where, *X*, *Y* are variables in universes U1 and U2
A, *B* are fuzzy sets on *X*, *Y*

Example:
p: if speed is High, then risk is Low

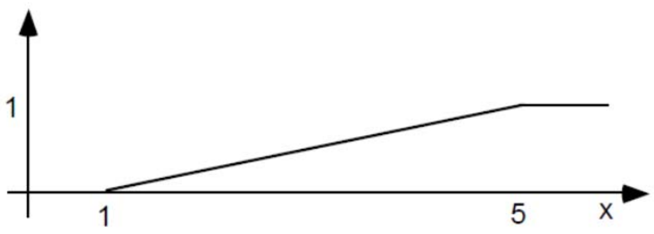
4. Conditional and Qualified Propositions

The canonical form of this type of fuzzy proposition is
p: (if X is A, then Y is B) is S
Where, all variables have same meaning as previous declare

Example:
p: if speed is high than risk is low is true.

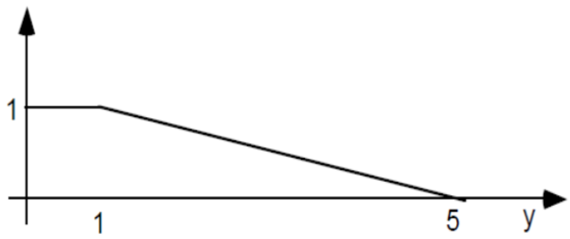
Fuzzy implication

Consider the implication statement
"if pressure is high then volume is small "
The membership function of the fuzzy set A, big pressure, illustrated in the figure



$$A(u) = \begin{cases} 1 & \text{if } u \geq 5 \\ 1 - \dfrac{5-u}{4} & \text{if } 1 \leq u \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

The membership function of the fuzzy set B, small volume, can be interpreted as (see figure)



$$B(v) = \begin{cases} 1 & \text{if } v \leq 1 \\ 1 - \dfrac{v-1}{4} & \text{if } 1 \leq v \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

If p: x is A

Where A is a fuzzy set, for example, big pressure

And q: y is B

For example, small volume

Then we define the fuzzy implication $A \rightarrow B$ as a fuzzy relation.

"IF X is A than Y is B", can be represent by relation

$$R = (A \times B) \cup (\bar{A} \times Y)$$

Where, A and B are two fuzzy sets with membership function μ_A and μ_B respectively

And Y is universe of discourse same as B but membership value for all is1.

The membership function of R is given by

$$\mu_R(x, y) = \max(\min(\mu_A(x), \mu_B(y)), 1 - \mu_A(x))$$

Example:

If X is A than Y is B

Now , suppose

X = {a,b,c}

Y = {1,2,3}

A = { (a,0) (b,0.5) (c,1) }

B = { (1,1) (2,0.3) (3,0.8)}

$$R = (A \times B) \cup (\bar{A} \times Y)$$

A X B =	1	2	3
a	0	0	0
b	0.5	0.3	0.5
c	1	0.3	0.8

$\bar{A}$ X B =	1	2	3
a	1	1	1
b	0.5	0.5	0.5
c	0	0	0

R =	1	2	3
a	1	1	1
b	0.5	0.5	0.5
c	1	0.3	0.8

Fuzzy Inference

Fuzzy inference is the process of obtaining new knowledge through existing knowledge. Knowledge is most commonly represent in the form of rules or proposition for example
"if x is A then y is B" (Where A and B are linguistic values defined by fuzzy sets on universes of discourse X and Y). A rule is also called a fuzzy implication.
"x is A" is called the antecedent or premise and "y is B" is called the consequence or conclusion.
The two important inferring processes are –

- Generalized modus Ponens (GMP)
- Generalized modus Tollens (GMT)

GMP

p: If X is A than Y is B (Analytically known)

q: If X is A' (Analytically known)

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than Y is B' (Analytically unknown)

Where, A, B, A', B' are fuzzy terms.
A' and B' are some predicate with different linguistic hedges. To compute the membership function of B' the min – max composition of fuzzy set A' with R(x,y) which is known as implication rule is used.

$$B' = A' o R(x,y)$$

In terms of membership function

$$\mu_{B'}(y) = \max(\min(\mu_{A'}(x), \mu_R(x,y)))$$

where $\mu_{B'}(y)$, $\mu_{A'}(A')$, $\mu_R(x,y)$ are membership function of B', A' and implication relation respectively.

GMT

p: If X is A than Y is B (Analytically known)

q: If Y is B’ (Analytically known)

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than X is A’ (Analytically unknown)

Where, A, B, A’, B’ are fuzzy terms.

$$A' = B' o\, R(x, y)$$

In terms of membership function

$$\mu_{A'}(y) = \max(\min(\mu_{B'}(x), \mu_R(x, y)))$$

where $\mu_{B'}(y)$, $\mu_{A'}(A')$, $\mu_R(x,y)$ are membership function of B’, A’ and implication relation respectively.

Example:

Let us consider the following logical implication,
p: If Force is huge, acceleration is large
q: If Force is huge

Acceleration is large

Let L(Large), VL(Very Large), H(Huge), VH(Very Huge) indicate the associate fuzzy variable sets.
Now let us also assume,
Force, X = {1, 2, 3, 4, 5, 6}
Acceleration, Y= {10, 20, 30, 40, 50}
L = {(3, 0.9), (4, 1), (5, 0.3)}
VL = {(5, 0.9), (6, 1)}
H = {(20, 1), (30, 0.6)}
VH = {(10, 1), (20, 0.3)}
R(x, y) = max $\{H \times S, H \times Y\}$

$H \times L$ =

	10	20	30	40	50
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0.9	0.6	0	0
4	0	1	0.6	0	0
5	0	0.3	0.3	0	0
6	0	0	0	0	0

$$\bar{H} \times Y = \begin{array}{c|ccccc} & 10 & 20 & 30 & 40 & 50 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 1 \\ 3 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 6 & 1 & 1 & 1 & 1 & 1 \end{array}$$

$$R(x, y) = \begin{array}{c|ccccc} & 10 & 20 & 30 & 40 & 50 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 1 \\ 3 & 0.2 & 0.9 & 0.6 & 0.2 & 0.2 \\ 4 & 0 & 1 & 0.5 & 0 & 0 \\ 5 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 6 & 1 & 1 & 1 & 1 & 1 \end{array}$$

Now we apply the composition rule,

$$VL = VH \circ R(x, y)$$

$$= [0.9 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0] \times$$

$$\begin{array}{c|ccccc} & 10 & 20 & 30 & 40 & 50 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 1 \\ 3 & 0.2 & 0.8 & 0.5 & 0.2 & 0.2 \\ 4 & 0 & 1 & 0.5 & 0 & 0 \\ 5 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 6 & 1 & 1 & 1 & 1 & 1 \end{array}$$

$\therefore VL = [0.9 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.9]$

References:

1. http://reference.wolfram.com/applications/fuzzylogic/Manual/8.html

2. http://if.kaist.ac.kr/lecture/cs670/textbook

3. Introduction to ANN and Fuzzy System by Yu Hen Hu (2001)