PARALLEL AND DISTRIBUTED ALGORITHMS
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PRAM ALGORITHMS: MERGING AND GRAPH COLORING

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AN OPTIMAL MERGING

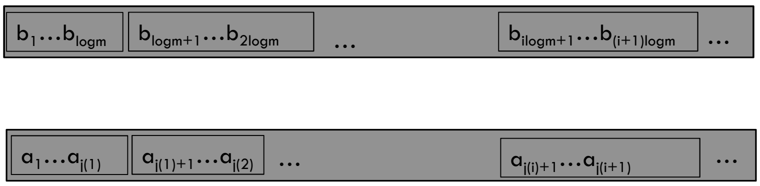
We have seen a parallel merging in the last class which takes O(log n) time with n processors.

What would be a cost-optimal algorithm to perform the merging?

The technique is based on a general strategy which is called partitioning.

- As opposed to Divide and Conquer, the crux lies here in suitably partitioning the problem which helps combining to get the result easy.

OPTIMAL MERGE-PARTITIONING



Spawn k(m)=m/log m processors.

Each processor divides the array B into subarrays, B_i, where $|B_i|$=log m.

It finds rank of b_{ilogm} in A using the binary search algorithm, and let j(i)=rank(b_{ilogm}:A).

The configurations of A_i and B_i shown above.

EXAMPLE

Let A=(4,6,7,10,12,15,18,20), B=(3,9,16,21)

m=4, k(m)=4/log4=2.

B=((3,9),(16,21))

rank(9,A)=3.

Thus A=((4,6,7),(10,12,15,18,20))

Note each element of A_1 and B_1 is larger than each element in A_0 or B_0.

Hence, we can merge A and B by merging separately the pairs (A_0,B_0) and (A_1,B_1).

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PROOF

Let $rank(b_{ilogm}:A)=j(i)$

Thus we have: $a_{j(i)}<b_{ilogm}<a_{j(i)+1}$.

This result implies that:

$b_{ilogm+1}>b_{ilogm}>a_{j(i)}$ (thus each element of B_i is larger than each element of A_{i-1})

Likewise, $a_{j(i)+1}>b_{ilogm}$ (thus each element of A_i is larger than each element of B_{i-1})

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TIMING ANALYSIS

The finding of j(i)'s takes O(n) time, since the binary search is applied to all the elements of A in parallel.

Thus the total number of operations required to execute this step (of doing the binary search and partitioning) is O((logn)xm/log(m))=O(m+n)

Consider the merging of two arrays each of length n.

After the partitioning step we end up with an independent set of merging sub-problems.

- This outcome is the essence of partitioning.
- We would like to handle each merging subproblem in O(logn) time, st the algorithm stays cost optimal.

THE MERGING SUBPROBLEMS

Consider the merging subproblem corresponding to (A_i,B_i). Recall $|B_i|$=logn, for all indices i.

If $|A_i|$=O(log n), we can merge the pair (A_i,B_i) using an optimal sequential algorithm in O(log n) time.

Otherwise, we apply the previous algorithm to partition A_i into blocks each of which is of size O(log n) (in this case A_i plays the role of B, and B_i plays the role of A!)

- This step will take O(log log n) time using $O(|A_i|)$ operations.m
- Thus we can make each of the subsequences to be of length O(log n)
- Then we apply the best sequential algorithm to merge the subsequences in O(logn) using number of processors $\Sigma_i \left(\frac{|A_i|}{logn}\right) = n/logn$, the number of resources also does not increase asymptotically.