

# Memory Management

# Memory management

- We have seen how CPU can be shared by a set of processes
  - Improve system performance
  - Process management
- Need to keep several process in memory
  - Share memory
- Learn various techniques to manage memory
  - Hardware dependent

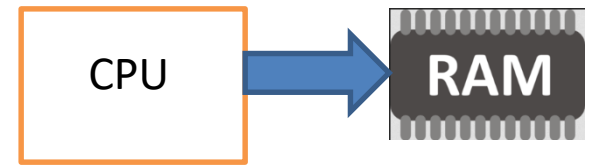
# Memory management

## *What are we going to learn?*

- ***Basic Memory Management:*** logical vs. physical address space, protection, contiguous memory allocation, paging, segmentation, segmentation with paging.
- ***Virtual Memory:*** background, demand paging, performance, page replacement, page replacement algorithms (FCFS, LRU), allocation of frames, thrashing.

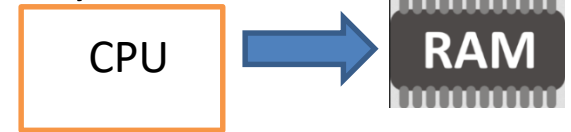
# Background

- Program must be brought (from disk) into memory
- Fetch-decode-execute cycle
- Memory unit only sees a stream of addresses + read requests, or address + data and write requests
- Sequence of memory addresses generated by running program



# Logical vs. Physical Address Space

**Logical address** – generated by the CPU; also referred to as **virtual address**



**Physical address** – address seen by the memory unit

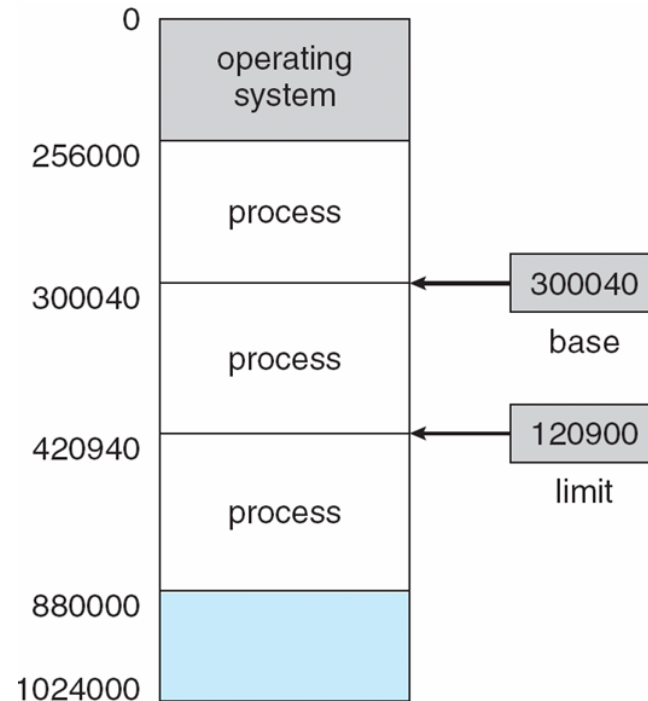
- **Logical address space** is the set of all logical addresses generated by a program
- **Physical address space** is the set of all physical addresses generated by a program

# Background

## Multiple processes resides in memory

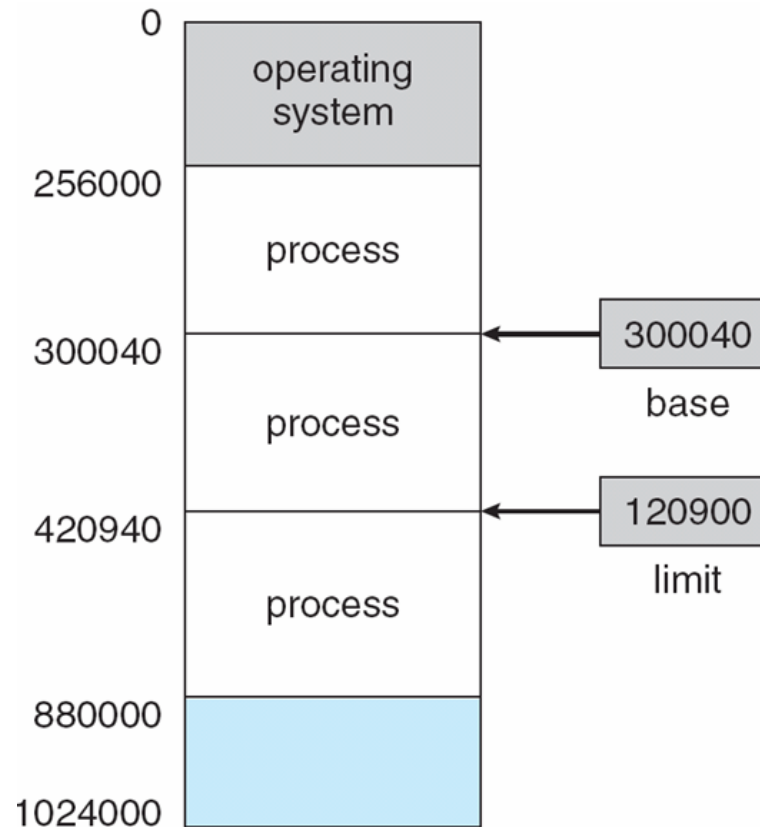
- Protection of memory required to ensure correct operation

1. Protect OS
2. Protect user processes

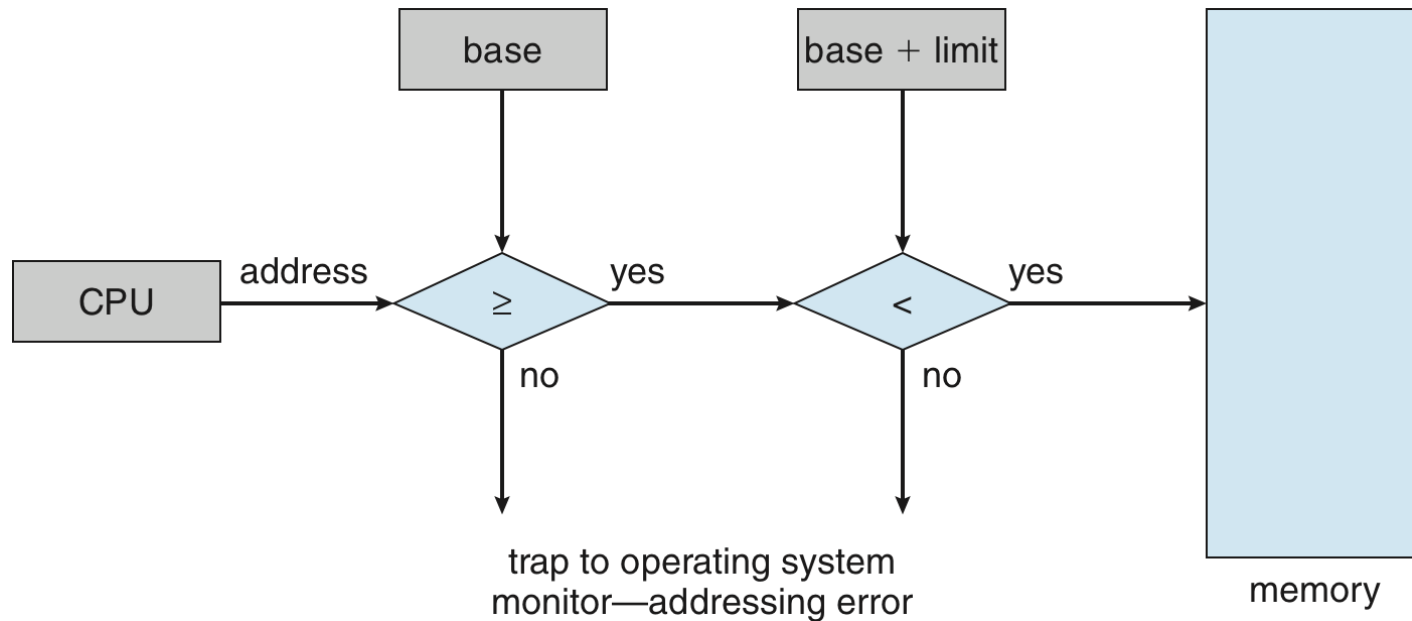


# Base and Limit Registers

- A pair of **base** and **limit** registers define the logical address space



# Hardware Address Protection with Base and Limit Registers



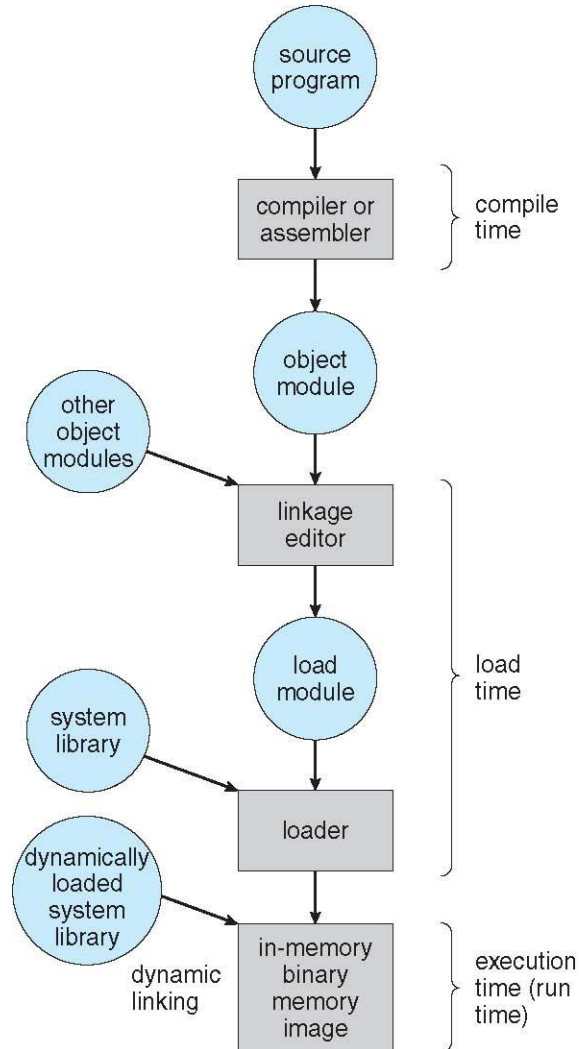
- OS loads the base & limit reg.
- Privileged instruction



# Address Binding

- Process resides in main memory
- Associate each data element with memory address
- Further, addresses represented in different ways at different stages of a program's life
  - Source code addresses usually symbolic
  - Compiled code addresses **bind** to relocatable addresses
    - i.e. “14 bytes from beginning of this module”
  - Linker or loader will bind relocatable addresses to absolute addresses
    - i.e. 74014

# Multistep Processing of a User Program

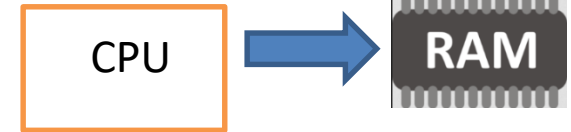


# Binding of Instructions and Data to Memory

- Address binding of instructions and data to memory addresses can happen at three different stages
  - **Compile time:** If memory location known a priori, **absolute code** can be generated; must recompile code if starting location changes
  - **Load time:** Must generate **relocatable code** if memory location is not known at compile time
  - **Execution time:** If the process can be moved during its execution from one memory segment to another
    - Binding delayed until run time
    - Need hardware support for address maps (e.g., base and limit registers)

# Logical vs. Physical Address Space

**Logical address** – generated by the CPU; also referred to as **virtual address**



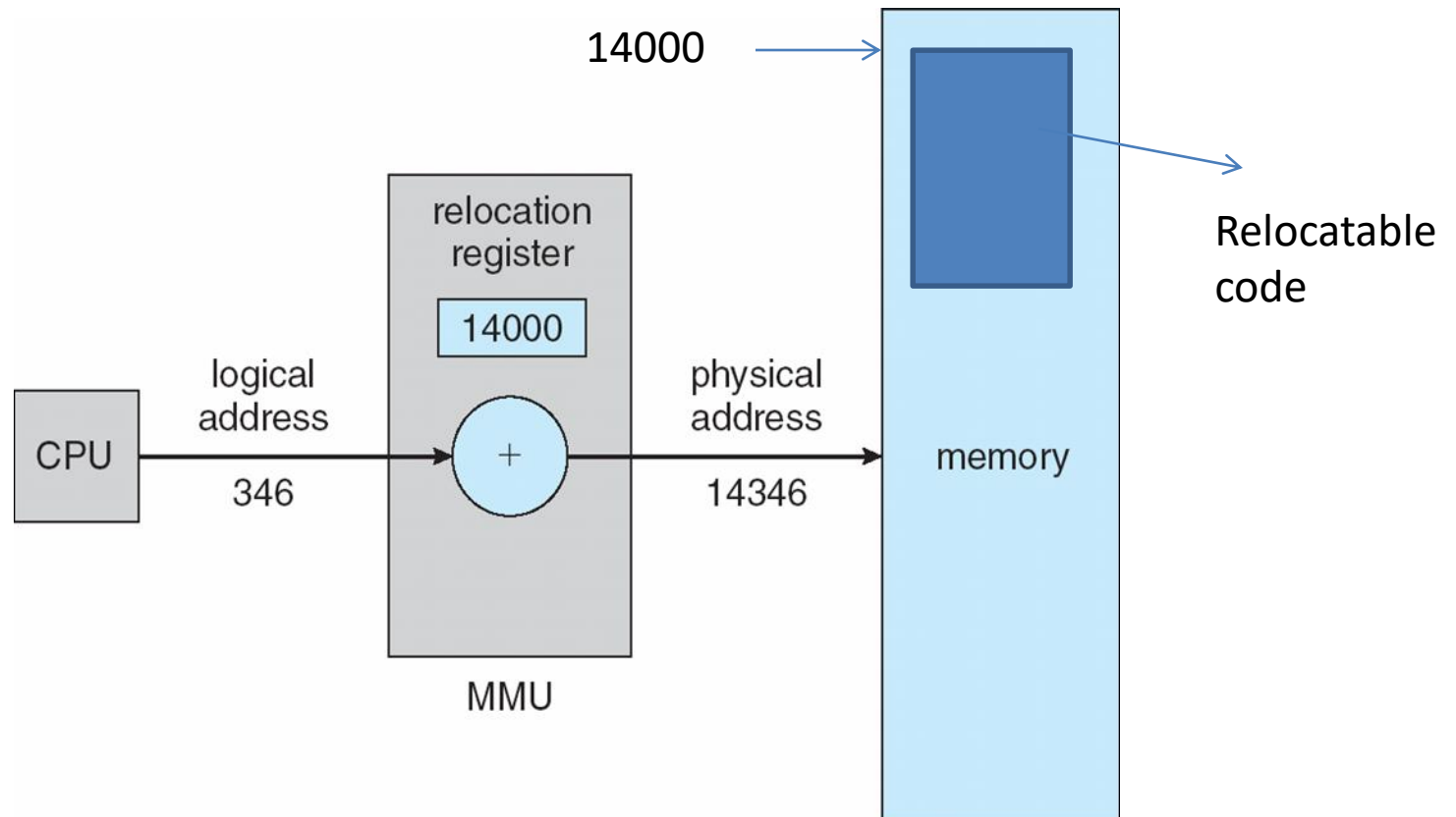
**Physical address** – address seen by the memory unit

- Logical and physical addresses are the same in compile-time and load-time address-binding schemes;
- logical (virtual) and physical addresses differ in execution-time address-binding scheme
- **Logical address space** is the set of all logical addresses generated by a program
- **Physical address space** is the set of all physical addresses generated by a program

# Memory-Management Unit (MMU)

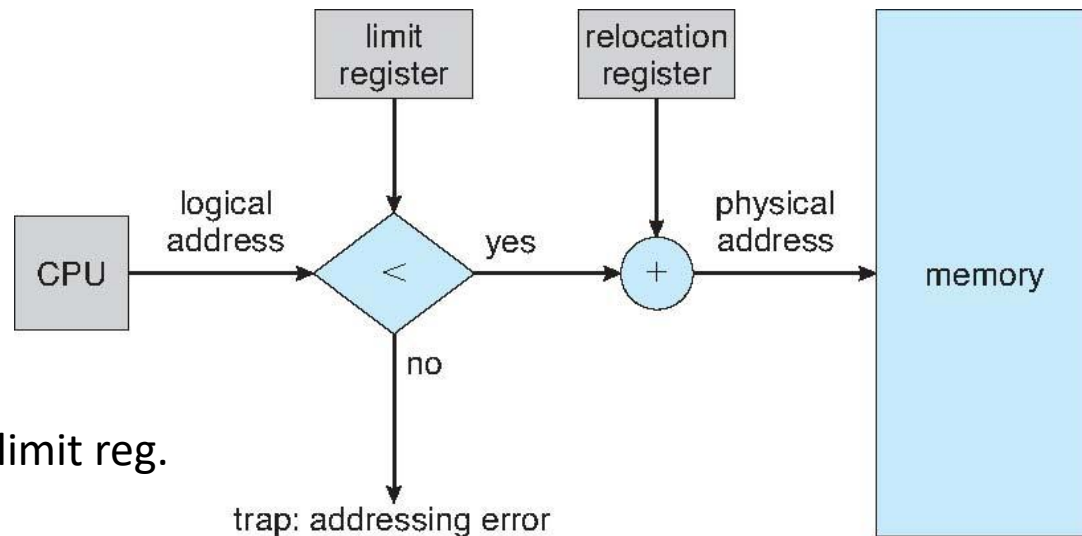
- Hardware device that at run time maps virtual to physical address
- Many methods possible
- To start, consider simple scheme where the value in the **relocation register** is added to every address generated by a user process at the time it is sent to memory
  - **relocation register**
  - MS-DOS on Intel 80x86 used 4 relocation registers
- The user program deals with *logical* addresses (0 to max); it never sees the *real* physical addresses (R to R+max)
  - Say the logical address 25
  - **Execution-time binding** occurs when reference is made to location in memory
  - Logical address bound to physical addresses

# Dynamic relocation using a relocation register



# Hardware Support for Relocation and Limit Registers

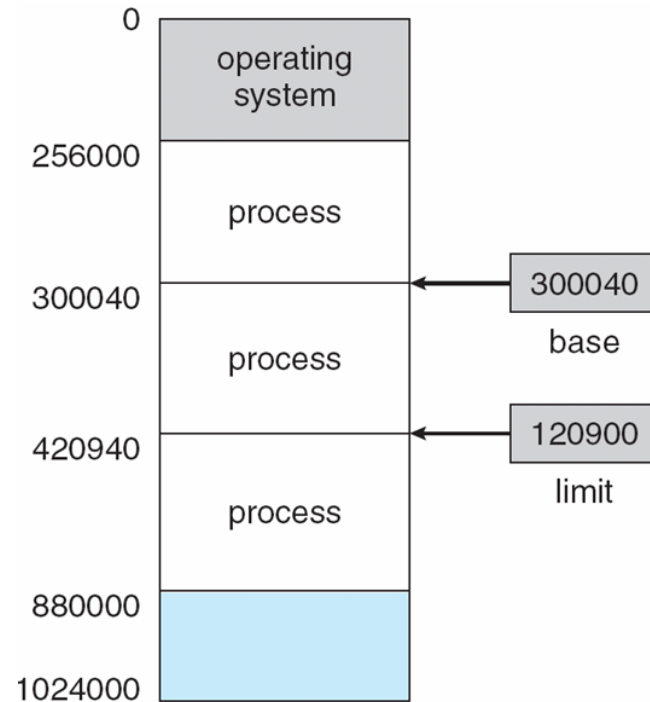
- **Relocation registers** used to protect user processes from each other, and from changing operating-system code and data
  - Relocation register contains value of smallest physical address
  - Limit register contains range of logical addresses – each logical address must be less than the limit register
  - Context switch
  - MMU maps logical address *dynamically*



- OS loads the reloc & limit reg.
- Privileged instruction

# Contiguous Allocation

Multiple processes resides in memory





# Contiguous Allocation

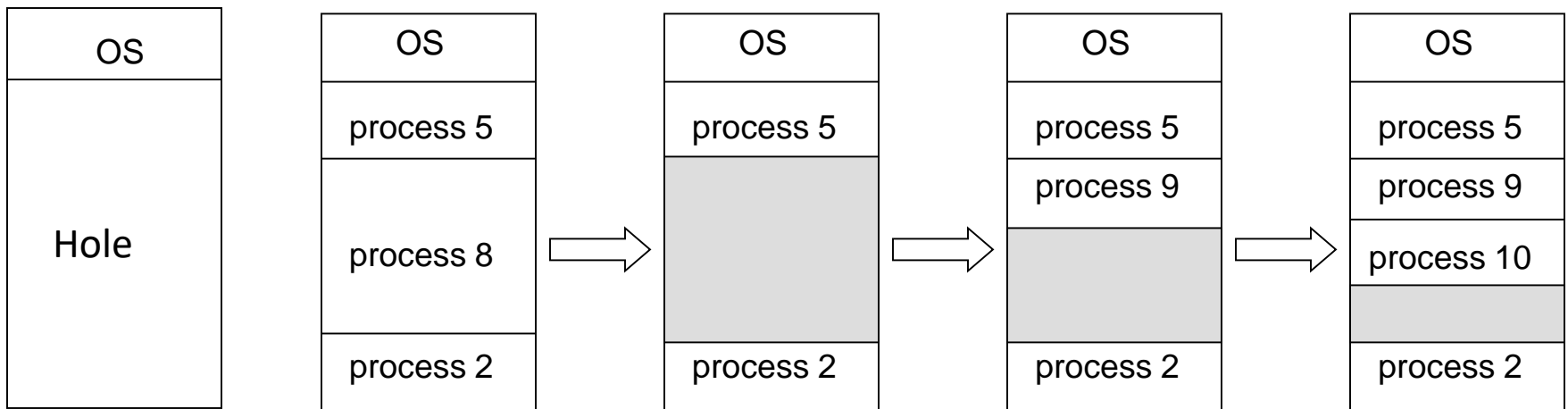
- Main memory usually divided into two partitions:
  - Resident operating system, usually held in high memory
  - User processes then held in low memory
  - Each process contained in **single contiguous section** of memory

# Contiguous Allocation (Cont.)

- Multiple-partition allocation
  - Divide memory into several **Fixed size partition**
  - Each partition stores one process
  - Degree of multiprogramming limited by number of partitions
  - If a partition is free, load process from job queue
  - MFT (IBM OS/360)

# Contiguous Allocation (Cont.)

- Multiple-partition allocation
  - **Variable partition scheme**
  - Hole – block of available memory; holes of various size are scattered throughout memory
  - Keeps a table of free memory
  - When a process arrives, it is allocated memory from a hole large enough to accommodate it
  - Process exiting frees its partition, adjacent free partitions combined
  - Operating system maintains information about:
    - a) allocated partitions
    - b) free partitions (hole)



# Dynamic Storage-Allocation Problem

How to satisfy a request of size  $n$  from a list of free holes?

Dynamic storage allocation problem

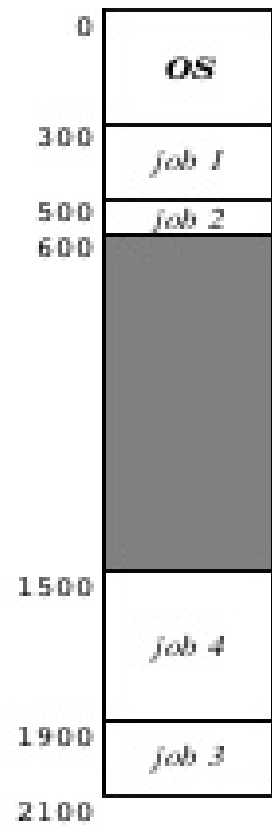
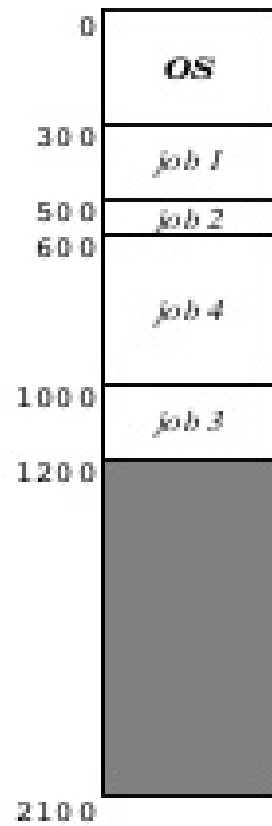
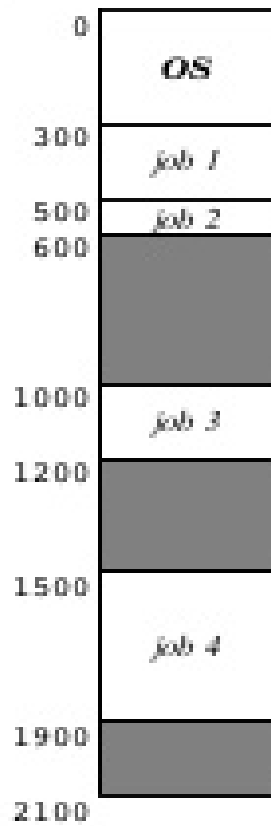
- **First-fit**: Allocate the *first* hole that is big enough
- **Best-fit**: Allocate the *smallest* hole that is big enough; must search entire list, unless ordered by size
  - Produces the smallest leftover hole
- **Worst-fit**: Allocate the *largest* hole; must also search entire list
  - Produces the largest leftover hole

# Fragmentation

- Processes loaded and removed from memory
  - Memory is broken into little pieces
- **External Fragmentation** – total memory space exists to satisfy a request, but it is not contiguous
- First fit analysis reveals that given  $N$  blocks allocated,  $0.5 N$  blocks lost to fragmentation
  - $1/3$  may be unusable -> **50-percent rule**

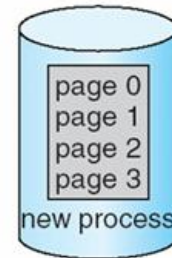
# Fragmentation (Cont.)

- Reduce external fragmentation by **compaction**
  - Shuffle memory contents to place all free memory together in one large block
  - Compaction is possible *only* if **relocation is dynamic**, and is done at execution time
    - Change relocation reg.
  - Cost
- **Internal Fragmentation** – allocated memory may be slightly larger than requested memory; this size difference is memory internal to a partition, but not being used



# Paging

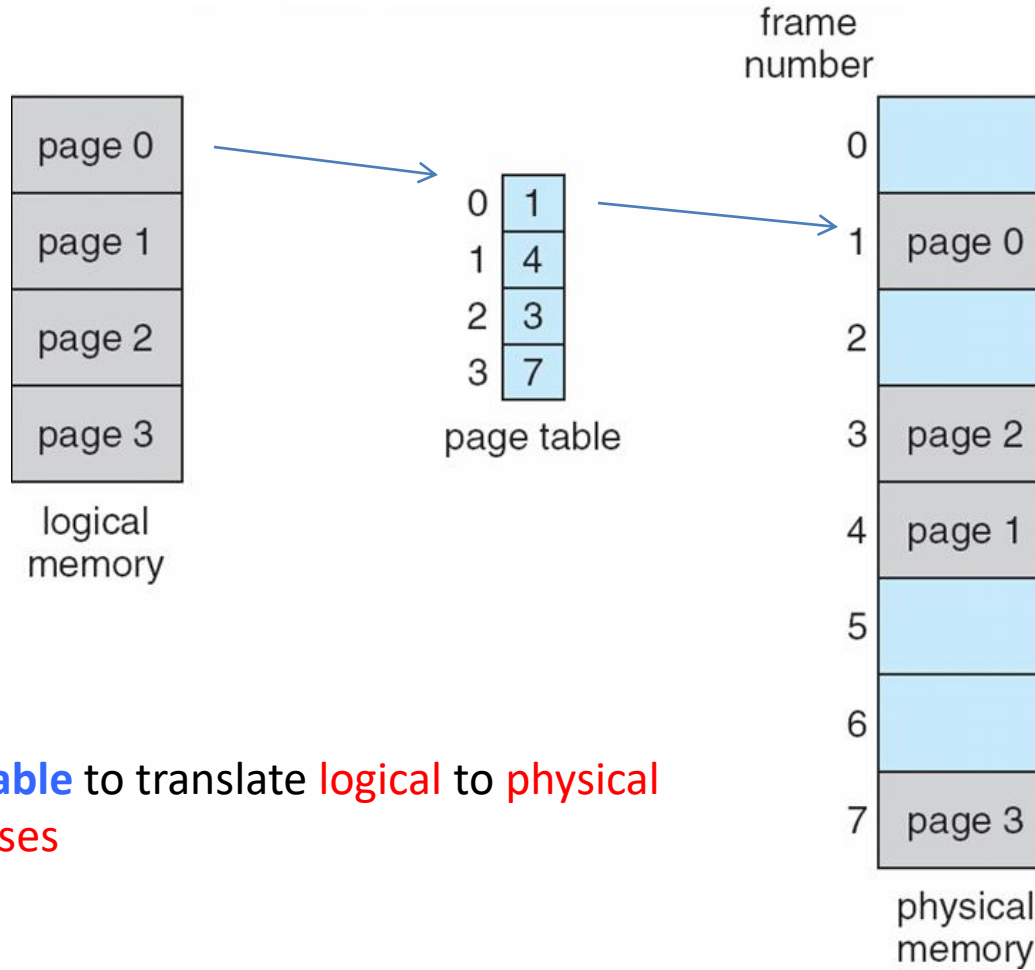
- Physical address space of a process can be **noncontiguous**;
  - process allocates physical memory whenever the latter is available
- Divide physical memory into fixed-sized blocks called **frames**
  - Size is power of 2, between 512 bytes and 16 Mbytes
- Divide logical memory into blocks of same size called **pages**
  - To run a program of size  $N$  pages, need to find  $N$  free frames and load program



- Backing store likewise split into pages
- Set up a **page table** to translate **logical** to **physical** addresses
- System keeps track of all free frames

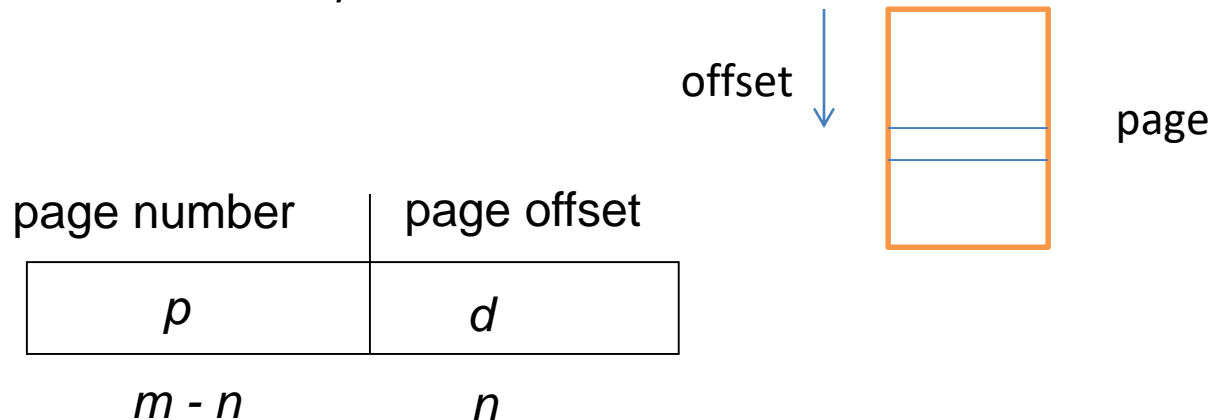


# Paging Model of Logical and Physical Memory



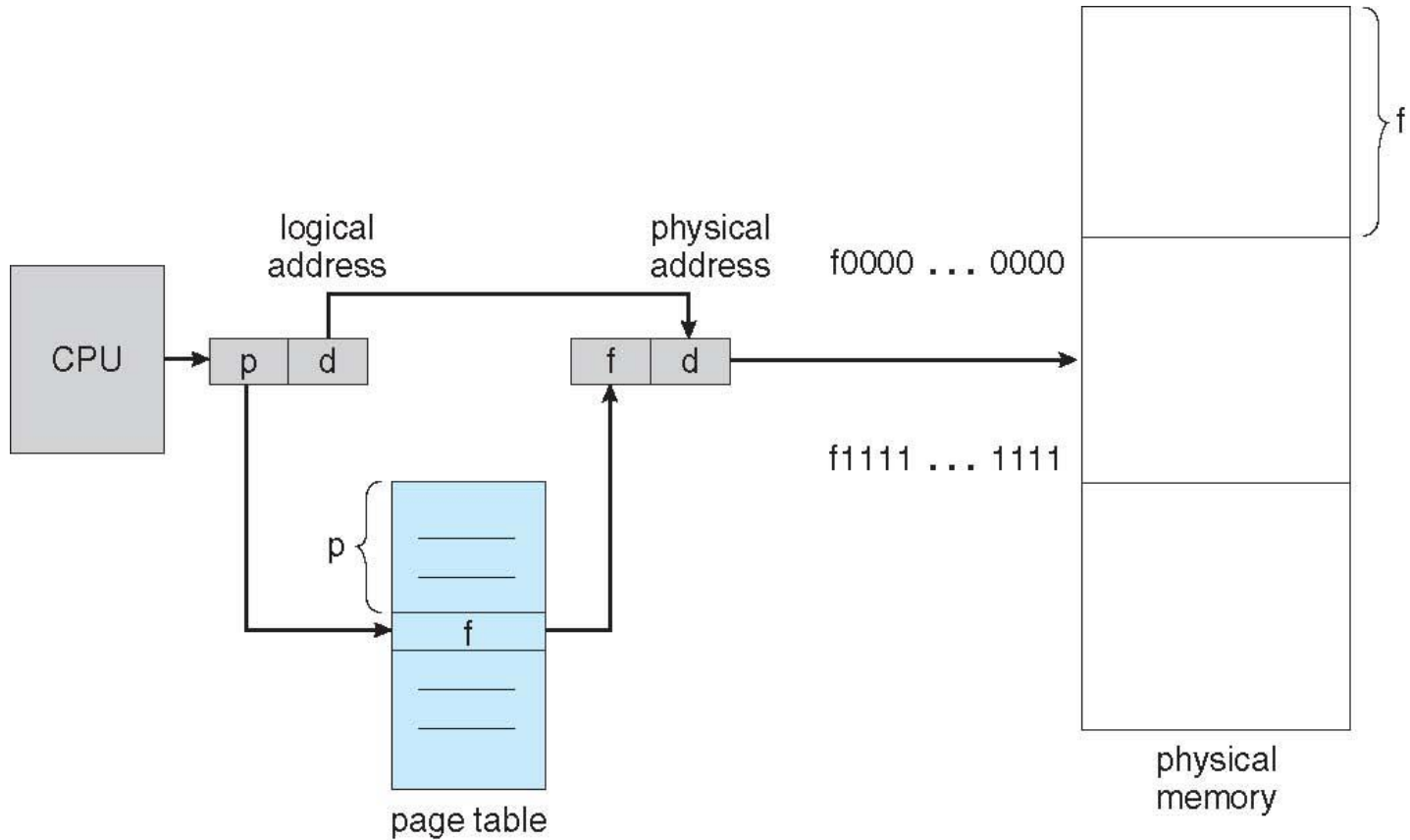
# Address Translation Scheme

- Address generated by CPU is divided into:
  - **Page number ( $p$ )** – used as an index into a **page table**
    - which contains base address of each frame in physical memory
  - **Page offset ( $d$ )** – offset within a page
    - combined with base address to define the physical memory address that is sent to the memory unit



- For given logical address space  $2^m$  and page size  $2^n$

# Paging Hardware



# Paging Example

Logical address = 16

Page size=4

Physical memory=32

0	a
1	b
2	c
3	d
4	e
5	f
6	g
7	h
8	i
9	j
10	k
11	l
12	m
13	n
14	o
15	p

logical memory

User's view

0	5
1	6
2	1
3	2

page table

0	
4	i j k l
8	m n o p
12	
16	
20	a b c d
24	e f g h
28	

physical memory

Logical address 0

$$(0*4+0)$$

Physical address:

$$(5*4+0)=20$$

Logical address 3

$$(0*4+3)$$

Physical address:

$$(5*4+0)=23$$

Logical address 4

$$(1*4+0)$$

Physical address:

$$(6*4+0)=24$$

Logical address 13

$$(3*4+1)$$

Physical address:

$$(2*4+1)$$

Run time address binding

$n=2$  and  $m=4$  32-byte  
memory and 4-byte pages

# Paging

- External fragmentation??
- Calculating internal fragmentation
  - Page size = 2,048 bytes
  - Process size = 72,766 bytes
  - 35 pages + 1,086 bytes
  - Internal fragmentation of  $2,048 - 1,086 = 962$  bytes
- So small frame sizes desirable?
  - But increases the page table size
  - Poor disk I/O
  - Page sizes growing over time
    - Solaris supports two page sizes – 8 KB and 4 MB
- User's view and physical memory now very different
  - user view=> process contains in single contiguous memory space
- By implementation process can only access its own memory
  - protection

# Page table entries

- Each page table entry 4 bytes (32 bits) long
- Each entry can point to  $2^{32}$  page frames
- If each frame is 4 KB
- The system can address  $2^{44}$  bytes (16TB) of physical memory

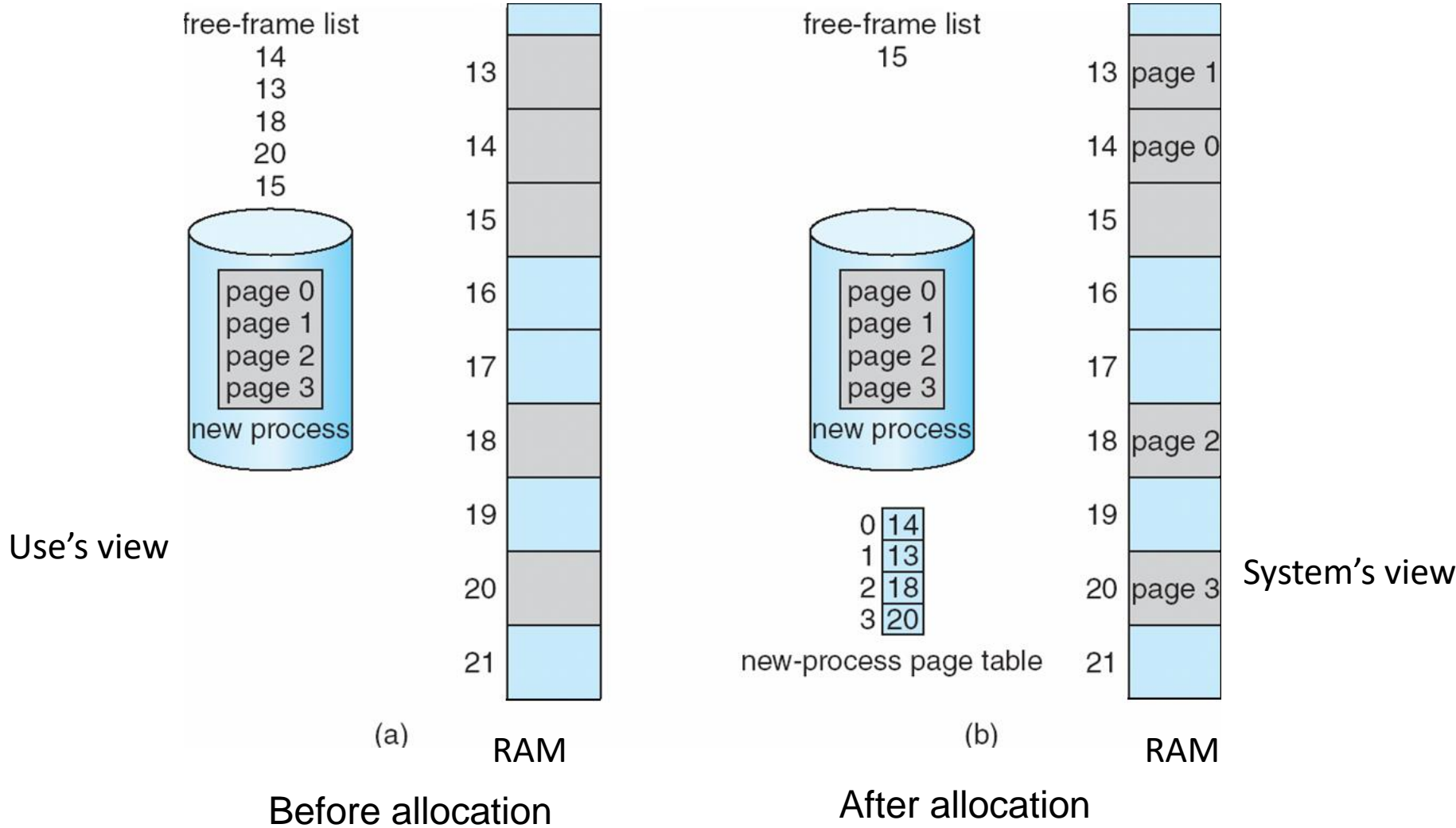
Virtual address space 16MB.

Page table size?

- Process P1 arrives
- Requires n pages => n frames must be available
- Allocate n frames to the process P1
- Create page table for P1

# Free Frames

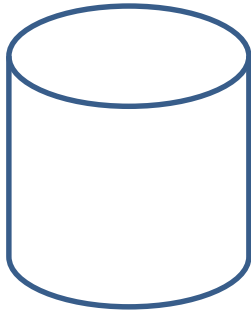
Frame table



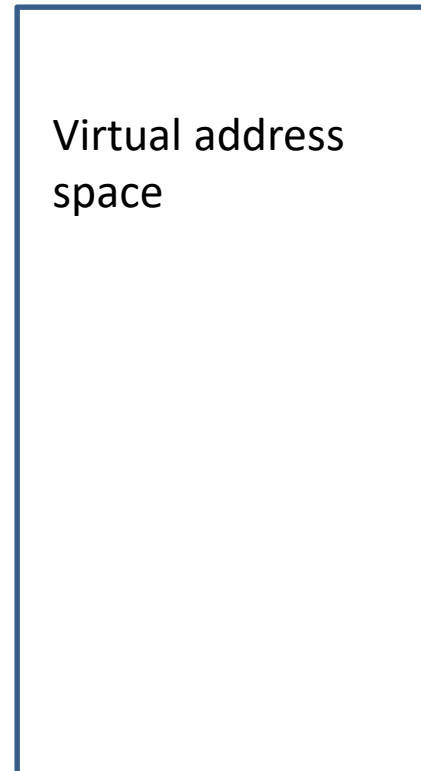


# Executable file and virtual address

a.out



Symbol table	
Name	address
SQR	0
SUM	4

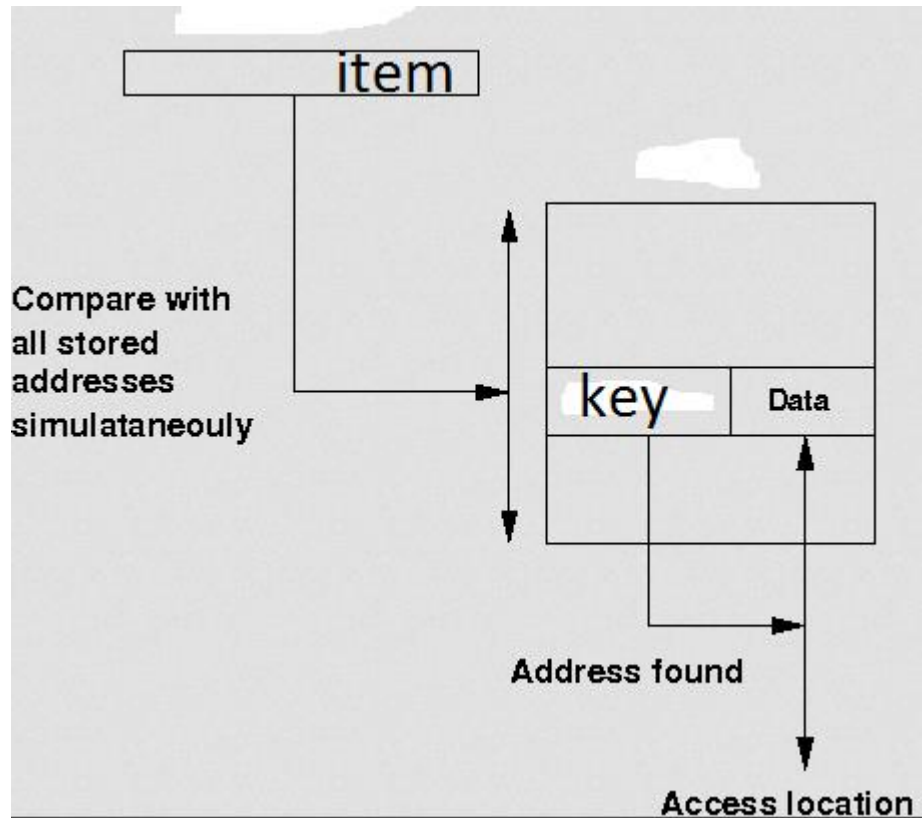


```
TEXT      code
MAIN      movf    NUM_1,w    ; Get Number 1
          call   SQR      ; Square it
          movf   SQUARE+1,w ; Get lower byte
          movwf  SUM+1    ; Is the low byte of sum
          movf   SQUARE,w  ; Get upper byte
          movwf  SUM      ; Is the high byte of sum
```

# Implementation of Page Table

- For each process, Page table is kept in main memory
- **Page-table base register (PTBR)** points to the page table
- **Page-table length register (PTLR)** indicates size of the page table
- In this scheme every data/instruction access requires two memory accesses
  - One for the page table and one for the data / instruction
- The two memory access problem can be solved by the use of a special fast-lookup hardware cache called **associative memory** or **translation look-aside buffers (TLBs)**

# Associative memory



# Associative Memory

- Associative memory – parallel search

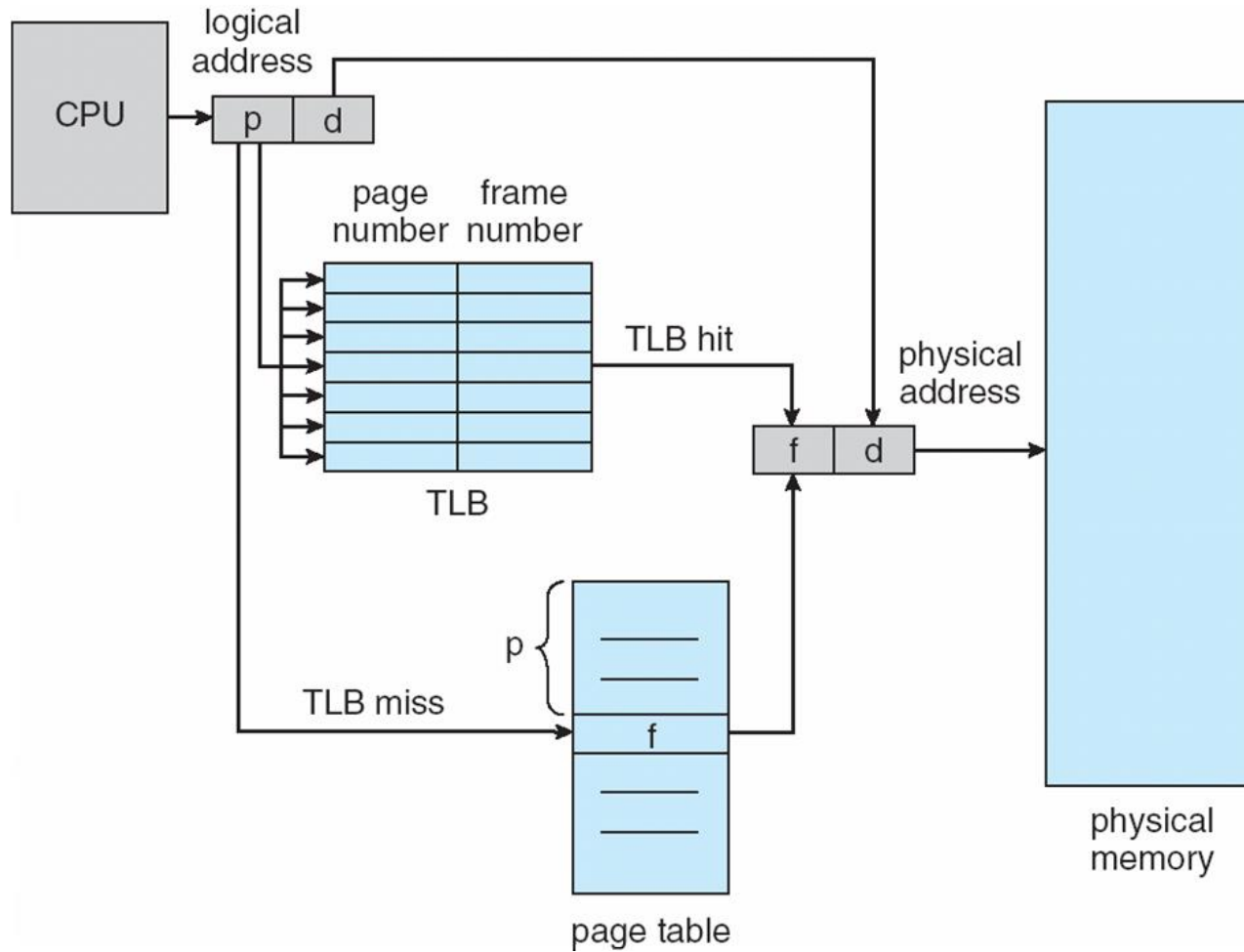
Page #	Frame #

- Address translation (p, d)
  - If p is in **associative register**, get frame # out
  - Otherwise get frame # from **page table in memory**

# Implementation of Page Table

- For each process, Page table is kept in main memory
- **Page-table base register (PTBR)** points to the page table
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- In this scheme every data/instruction access requires two memory accesses
  - One for the page table and one for the data / instruction
- The two memory access problem can be solved by the use of a special fast-lookup hardware cache called **associative memory** or **translation look-aside buffers (TLBs)**
- **TLBs typically small (64 to 1,024 entries)**
- On a **TLB miss**, value is loaded into the TLB for faster access next time
  - Replacement policies must be considered (LRU)
  - Some entries can be **wired down** for permanent fast access
- Some TLBs store **address-space identifiers (ASIDs)** in each TLB entry – uniquely identifies each process (PID) to provide address-space protection for that process
  - Otherwise need to flush at every context switch

# Paging Hardware With TLB



# Effective Access Time

- Associative Lookup =  $\varepsilon$  time unit
  - Can be < 10% of memory access time
- Hit ratio =  $\alpha$ 
  - Hit ratio – percentage of times that a page number is found in the associative registers; ratio related to size of TLB
- Consider  $\alpha = 80\%$ ,  $\varepsilon = 20\text{ns}$  for TLB search,  $100\text{ns}$  for memory access
- **Effective Access Time (EAT)**
$$\text{EAT} = (100 + \varepsilon) \alpha + (200 + \varepsilon)(1 - \alpha)$$
- Consider  $\alpha = 80\%$ ,  $\varepsilon = 20\text{ns}$  for TLB search,  $100\text{ns}$  for memory access
  - $\text{EAT} = 0.80 \times 120 + 0.20 \times 220 = 140\text{ns}$
- Consider better hit ratio  $\rightarrow \alpha = 98\%$ ,  $\varepsilon = 20\text{ns}$  for TLB search,  $100\text{ns}$  for memory access
  - $\text{EAT} = 0.98 \times 120 + 0.02 \times 220 = 122\text{ns}$

# Memory Protection

- Memory protection implemented by associating protection bit with each frame to indicate if read-only or read-write access is allowed
  - Can also add more bits to indicate page execute-only, and so on
- **Valid-invalid** bit attached to each entry in the page table:
  - “valid” indicates that the associated page is in the process’ logical address space, and is thus a legal page
  - “invalid” indicates that the page is not in the process’ logical address space
  - Or use PTLR
- Any violations result in a trap to the kernel



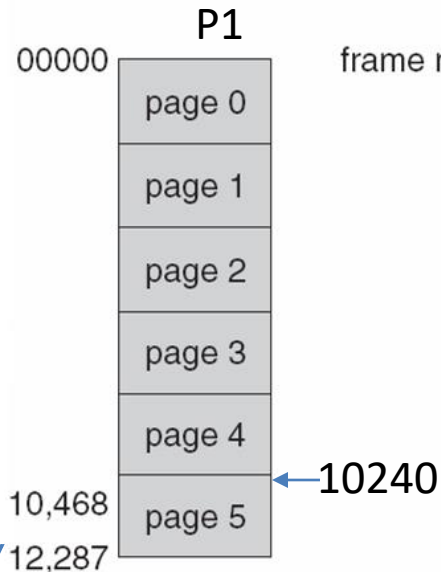
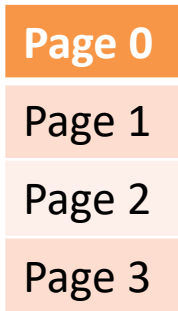
# Valid (v) or Invalid (i) Bit In A Page Table

14 bit address space (0 to 16383)

Page size 2KB

Process P1 uses only 0 to 10468

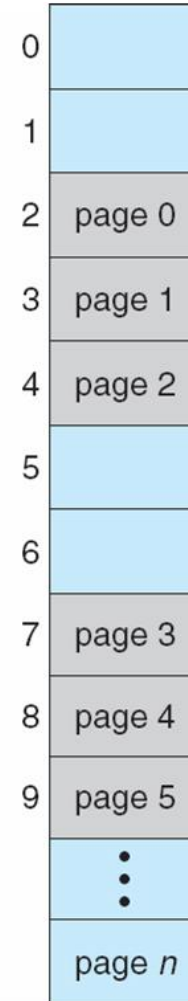
P2



frame number      valid-invalid bit

0	2	v
1	3	v
2	4	v
3	7	v
4	8	v
5	9	v
6	0	i
7	0	i

page table



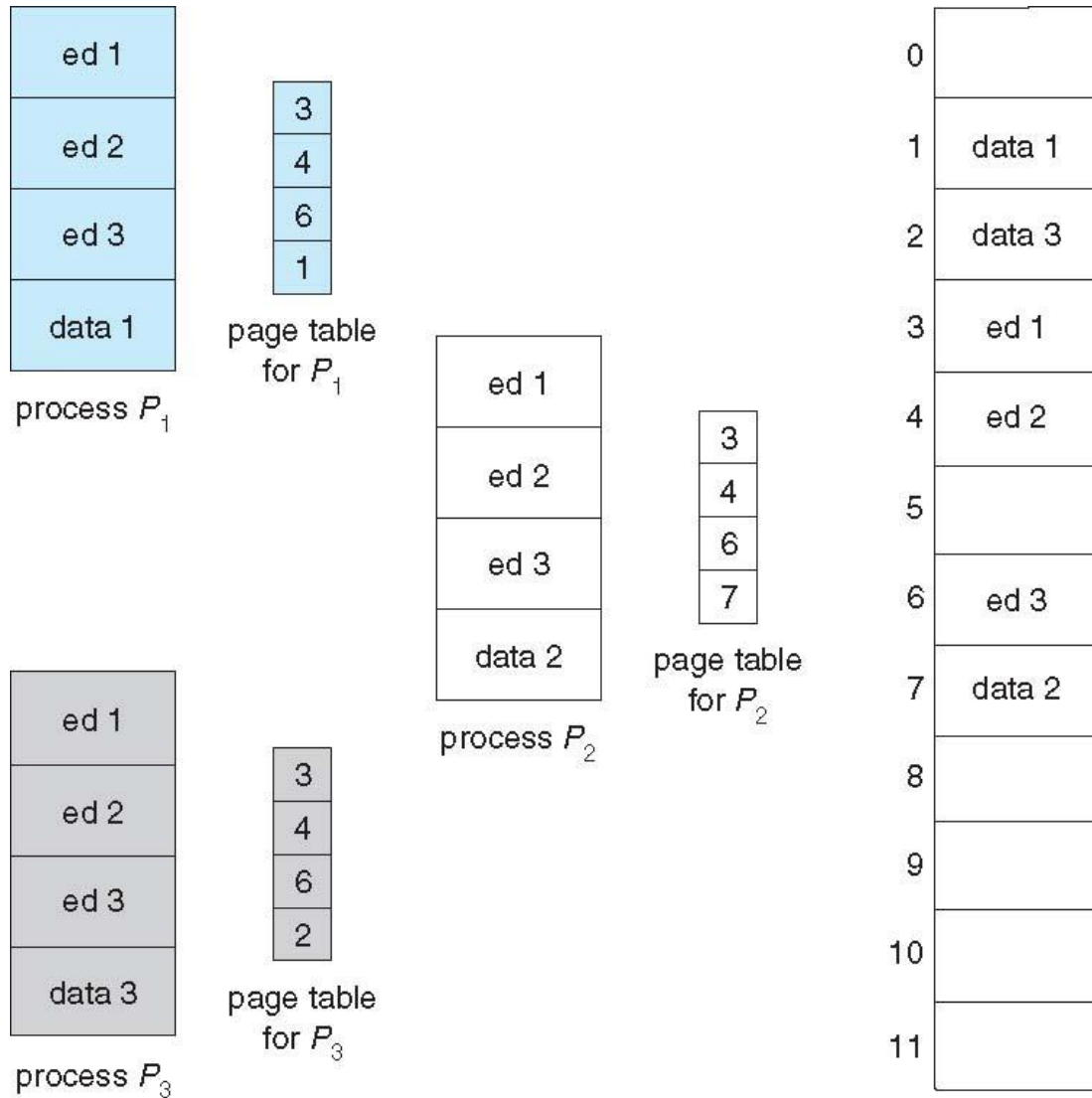
Internal fragmentation

Use of PTLR (length)

# Shared Pages Example

- System with 40 users
  - Use common text editor
- Text editor contains 150KB code 50KB data (page size 50KB)
  - 8000KB!
- **Shared code**
  - One copy of read-only (**reentrant**) code shared among processes (i.e., text editors, browsers)
    - Code never changes during execution
- Only one copy of the editor in the memory
- Total memory consumption
  - $40*50+150=2150$ KB

# Shared Pages Example



# Data share: example

writer.c

```
int main()
{
    int shmid,f,key=3,i,pid;
    char *ptr;

    shmid=shmget((key_t)key,100,IPC_CREAT|0666);
    ptr=shmat(shmid,NULL,0);
    printf("shmid=%d ptr=%u\n",shmid, ptr);
    strcpy(ptr,"hello");
    i=shmdt((char*)ptr);
}
```

reader .c

```
int main()
{
    int shmid,f,key=3,i,pid;
    char *ptr;

    shmid=shmget((key_t)key,100,IPC_CREAT|0666);
    ptr=shmat(shmid,NULL,0);
    printf("shmid=%d ptr=%u\n",shmid, ptr);
    printf("\nstr %s\n",ptr);
}
```

ptr

Shared  
memory

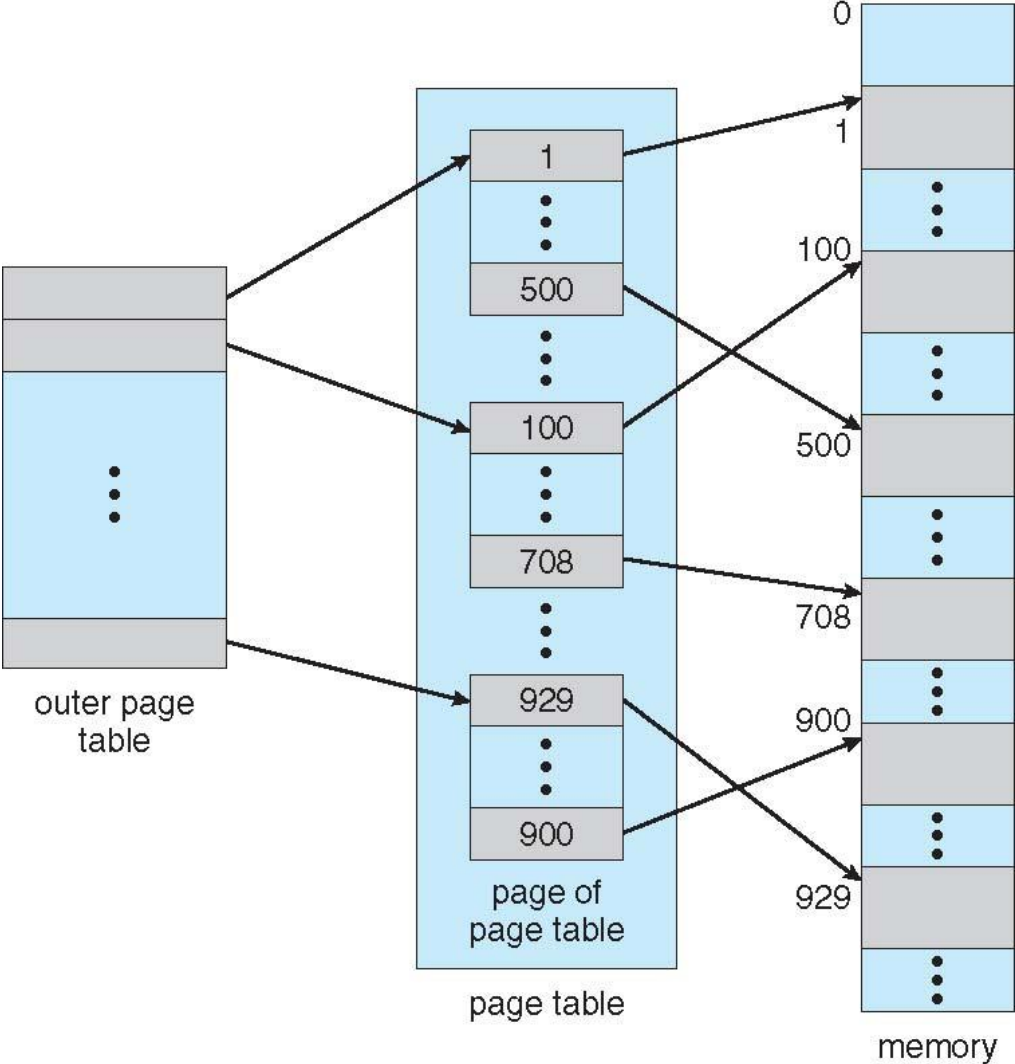
# Structure of the Page Table

- Memory requirement for page table can get huge using straightforward methods
  - Consider a 32-bit logical address space as on modern computers
  - Page size of 4 KB ( $2^{12}$ )
  - Page table would have 1 million entries  $2^{20}$  ( $2^{32} / 2^{12}$ )
  - If each entry is 4 bytes -> 4 MB of physical address space / memory for page table alone
    - That amount of memory used to cost a lot
    - Don't want to allocate that contiguously in main memory
- Hierarchical Paging
- Hashed Page Tables
- Inverted Page Tables

# Hierarchical Page Tables

- Break up the page table into multiple pages
- We then page the page table, and store it in frame
- Create outer page table
- A simple technique is a two-level page table

# Two-Level Page-Table Scheme



# Two-Level Paging Example

- A logical address (on 32-bit machine with 4KB page size) is divided into:
  - a page number consisting of 20 bits
  - a page offset consisting of 12 bits
- Since the page table is paged, the page number is further divided into:
  - a 10-bit page number
  - a 10-bit page offset
- Thus, a logical address is as follows:

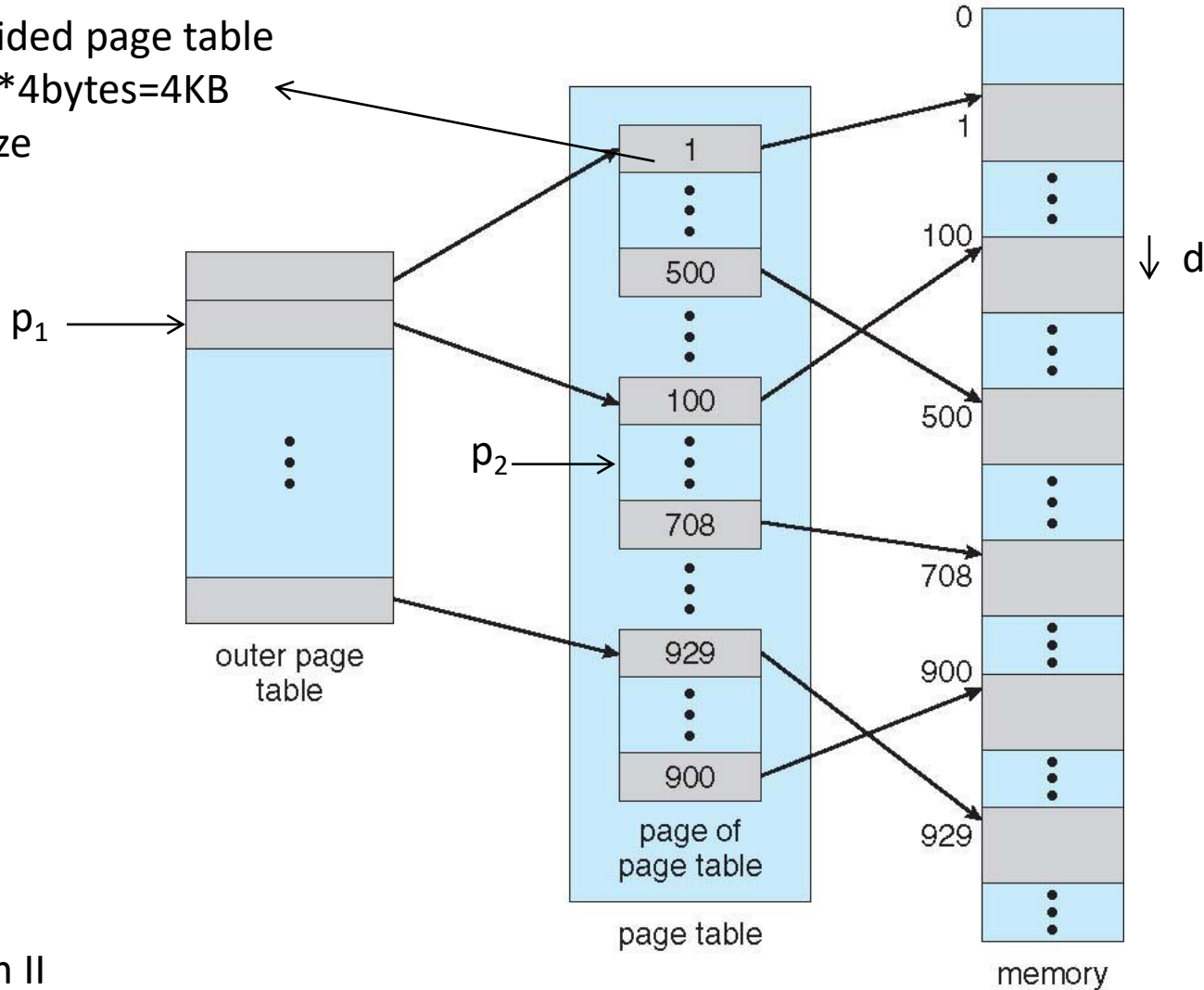
page number		page offset
$p_1$	$p_2$	$d$
10	10	12

- where  $p_1$  is an index into the outer page table, and  $p_2$  is the displacement within the page of the inner page table

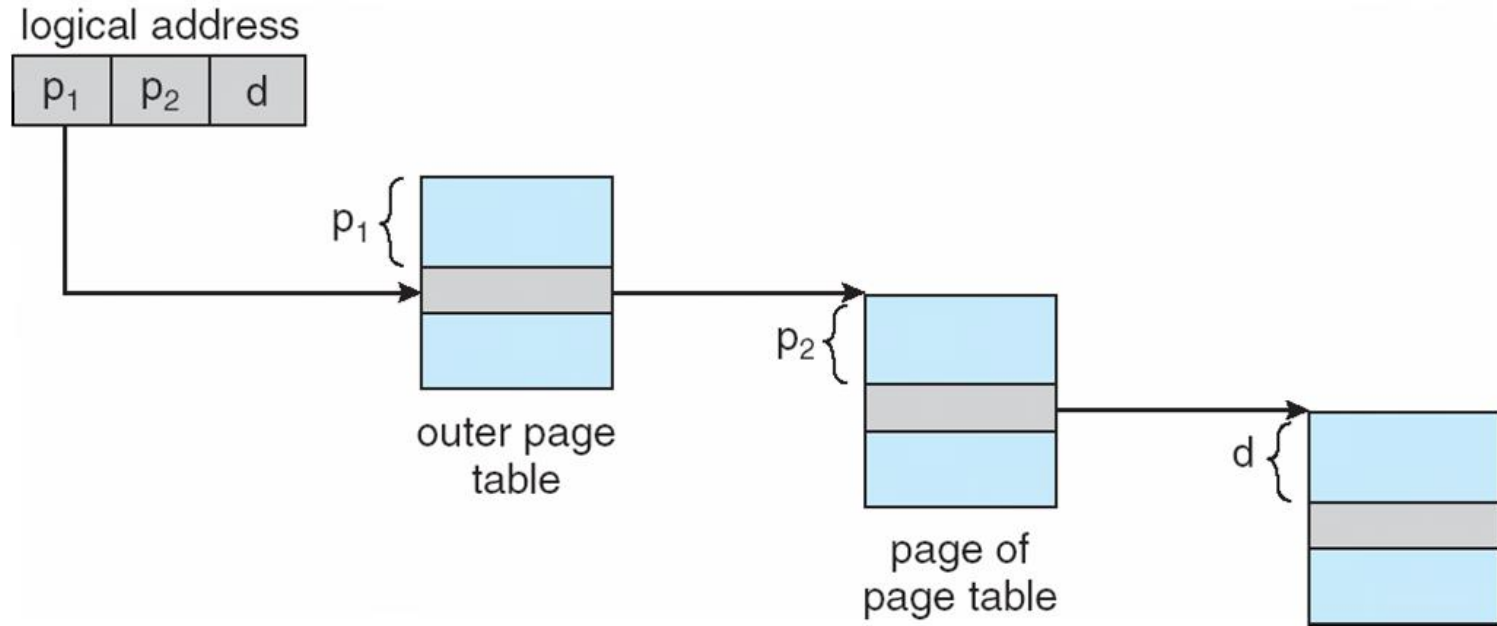


# Two-Level Page-Table Scheme

Each divided page table  
size =  $2^{10} * 4\text{bytes} = 4\text{KB}$   
= Page size

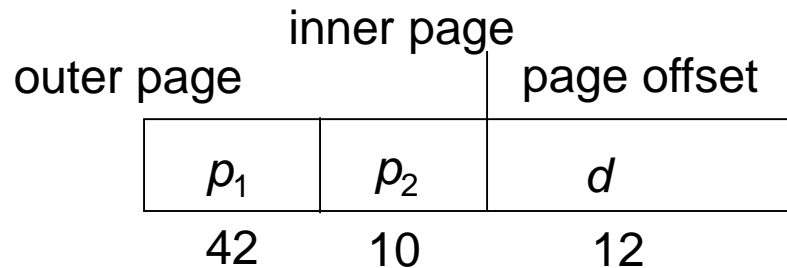


# Address-Translation Scheme



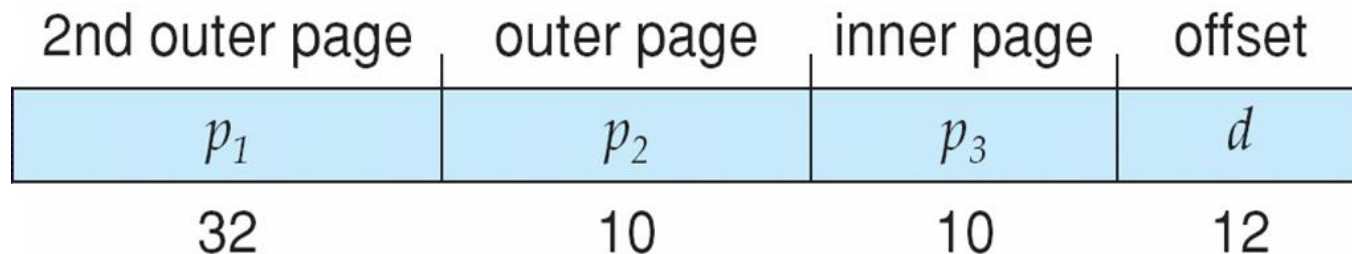
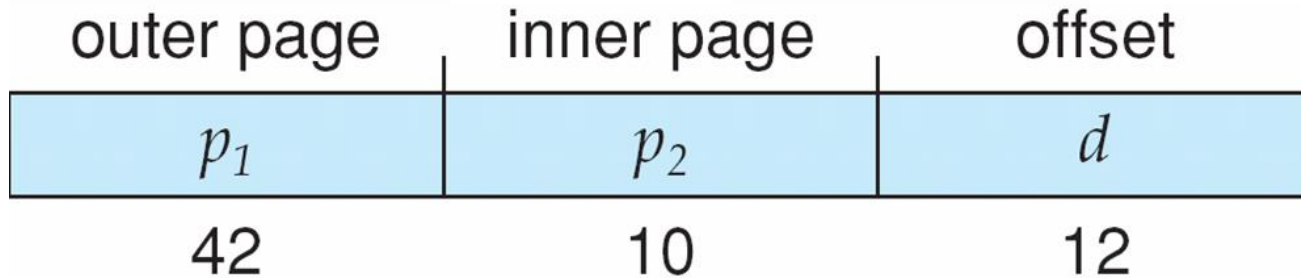
# 64-bit Logical Address Space

- Even two-level paging scheme not sufficient
- If page size is 4 KB ( $2^{12}$ )
  - Then page table has  $2^{52}$  entries
  - If two level scheme, inner page tables could be  $2^{10}$  4-byte entries
  - Address would look like



- Outer page table has  $2^{42}$  entries or  $2^{44}$  bytes
- One solution is to add a  $2^{\text{nd}}$  outer page table
- But in the following example the  $2^{\text{nd}}$  outer page table is still  $2^{34}$  bytes in size
  - And possibly 4 memory access to get to one physical memory location

# Three-level Paging Scheme

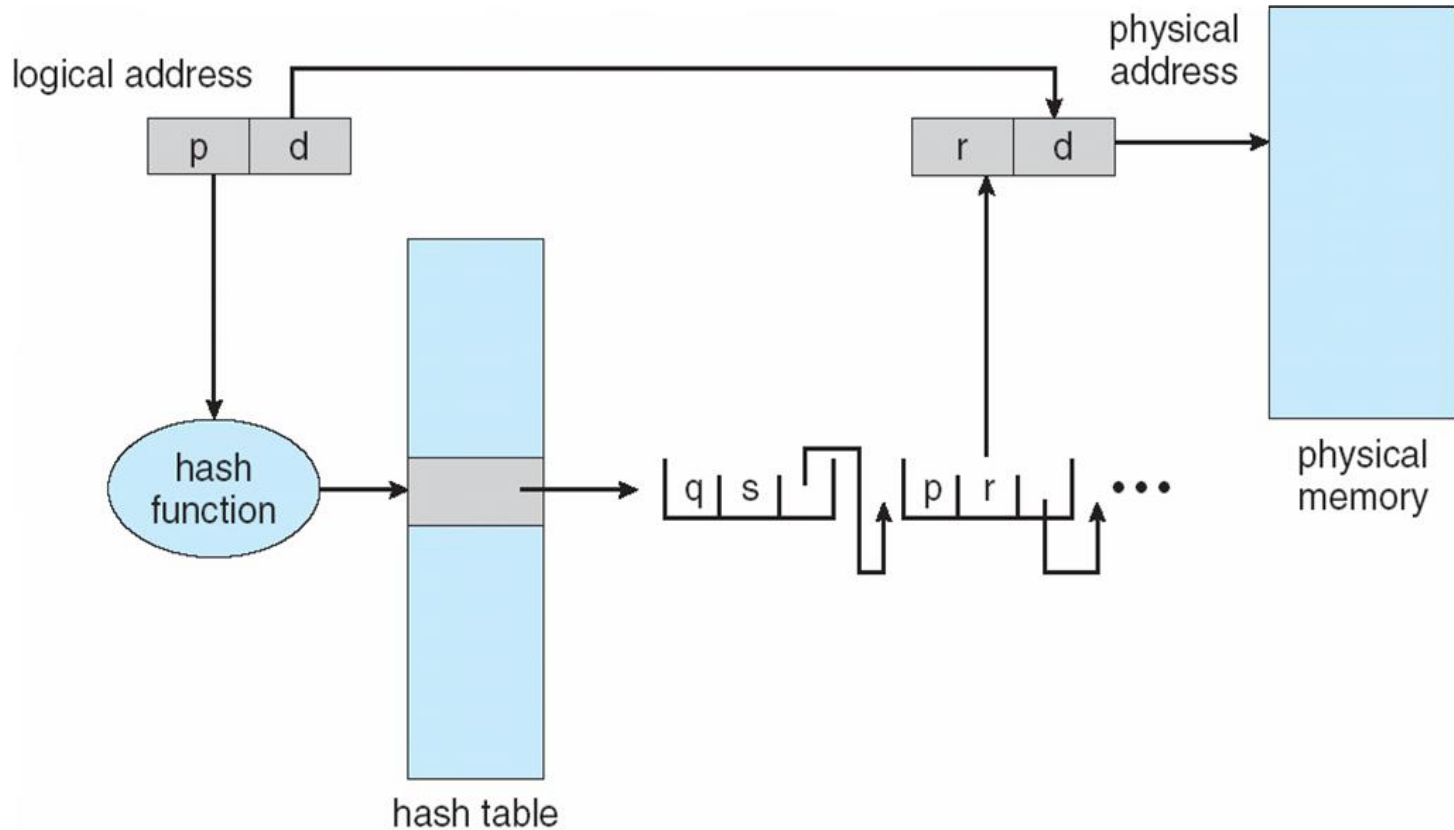


SPARC (32 bits), Motorola 68030 support three and four level paging respectively

# Hashed Page Tables

- Common in virtual address spaces  $> 32$  bits
- The page number is hashed into a page table
  - This page table contains a chain of elements hashing to the same location
- Each element contains (1) the page number (2) the value of the mapped page frame (3) a pointer to the next element
- Virtual page numbers are compared in this chain searching for a match
  - If a match is found, the corresponding physical frame is extracted

# Hashed Page Table

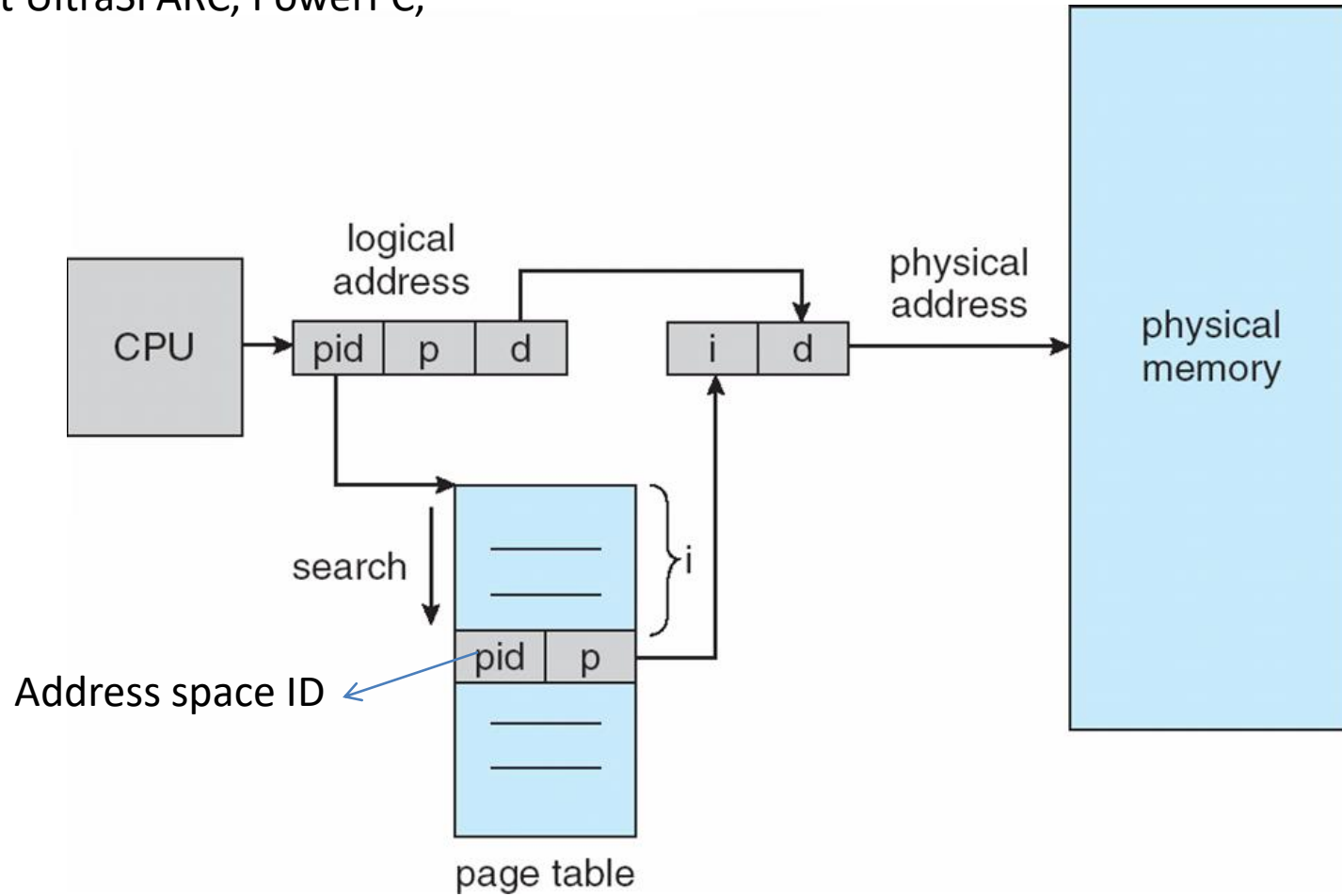


# Inverted Page Table

- Rather than each process having a page table and keeping track of all possible logical pages, **track all frames**
- One entry for each frame
- Entry consists the **page number** stored in that frame, with **information about the process** that owns that page
- Decreases memory needed to store each page table,
  - but increases time needed to search the table when a page reference occurs

# Inverted Page Table Architecture

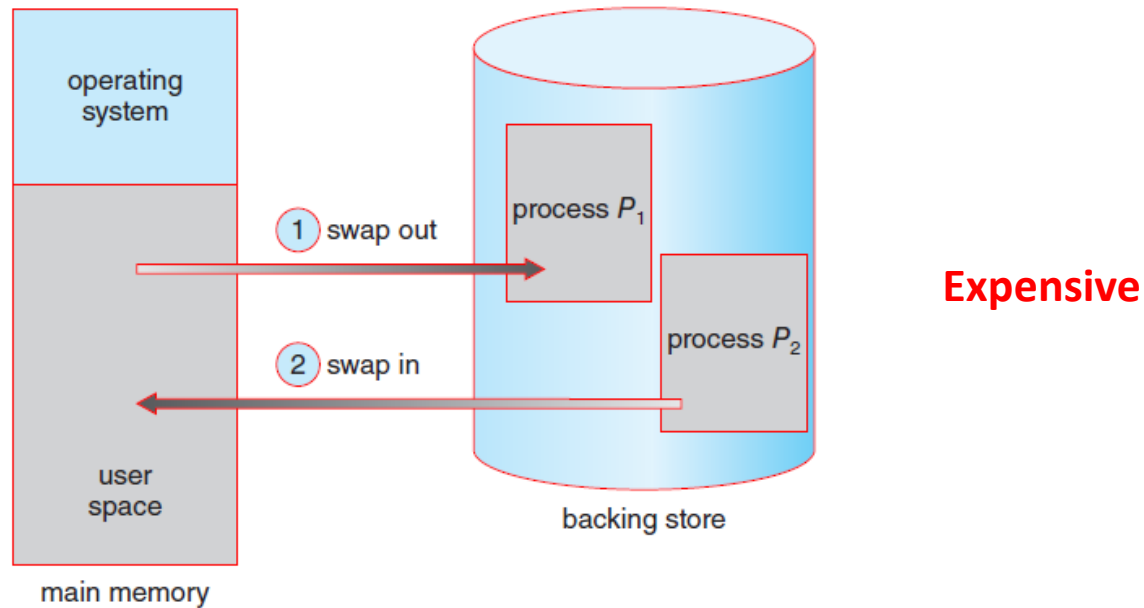
64 bit UltraSPARC, PowerPC,





# Swapping

A process, or a portion of a process, can be swapped temporarily out of memory to a backing store and then brought back into memory for continued execution

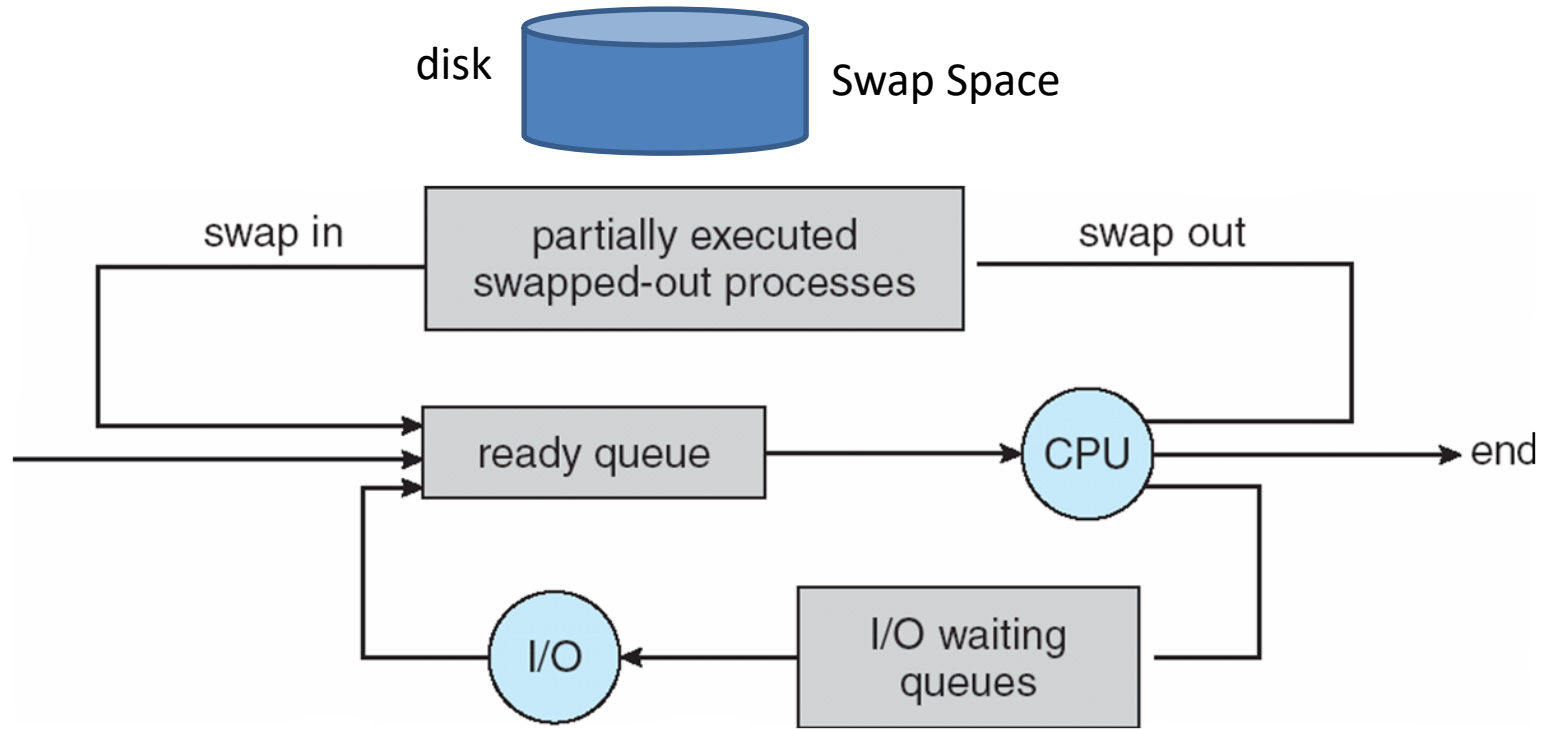


Advantages :

Allows physical memory to be oversubscribed, so that the system can accommodate more processes than there is actual physical memory to store them.

Idle or mostly idle processes are good candidates for swapping; any memory that has been allocated to these inactive processes can then be dedicated to active processes.

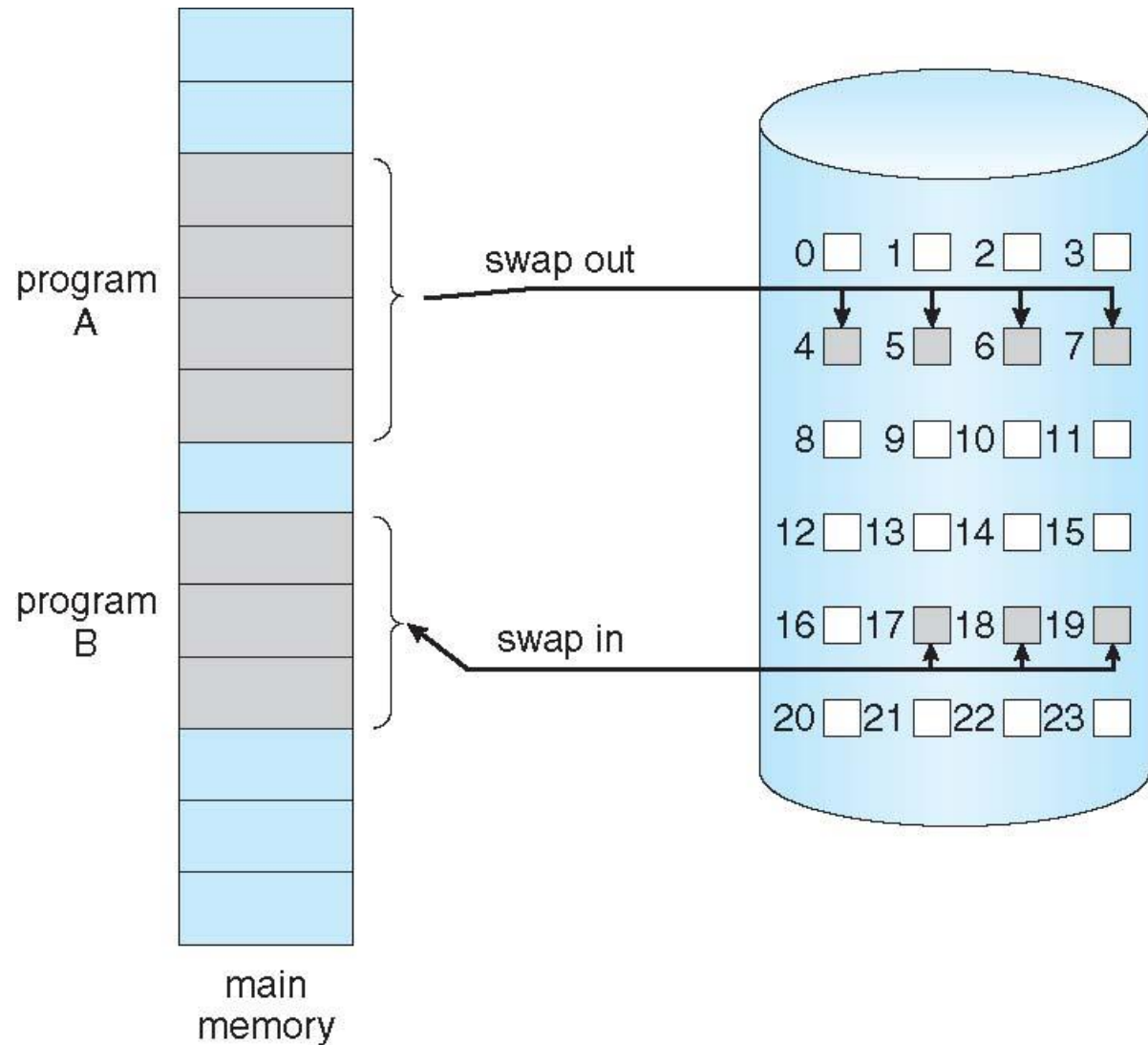
# Medium Term Scheduling



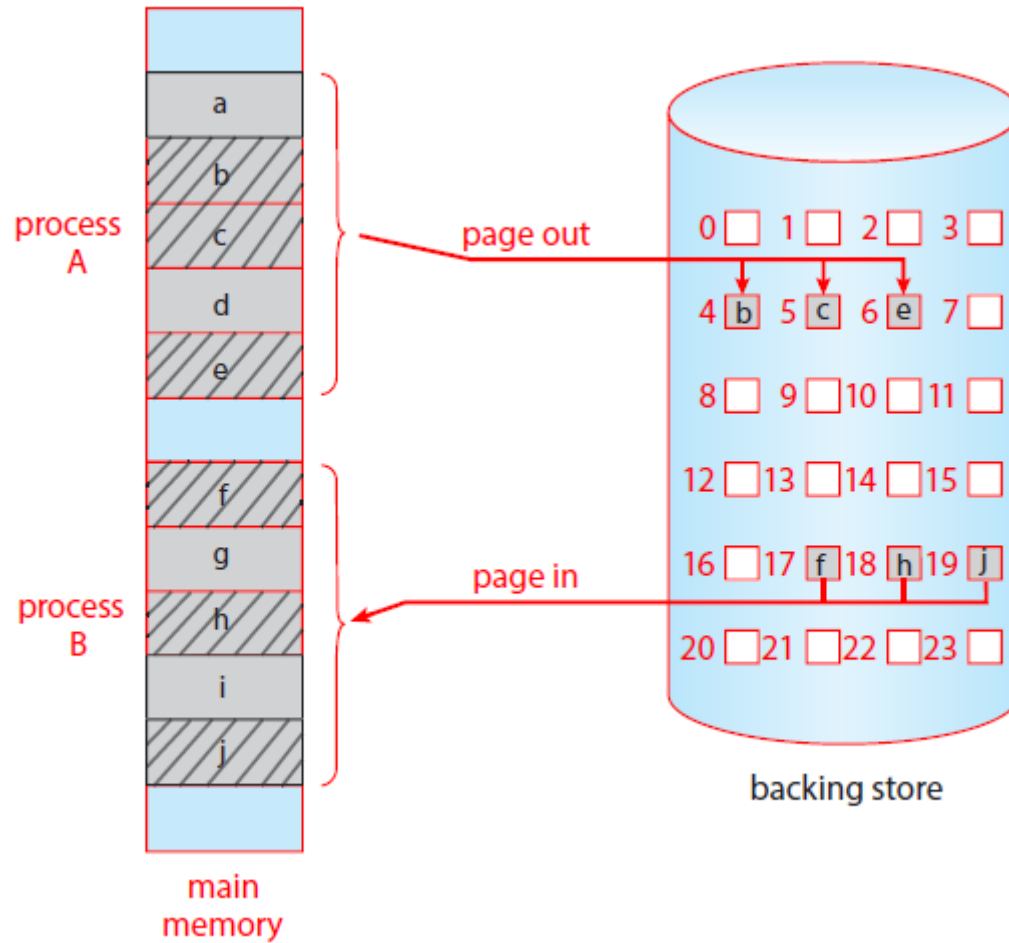
Swapper

# Swapping with Paging

- When we want to execute a process, swap in
- Instead of swap in entire process, load page
- Pager



# Swapping with Paging



# Segmentation

- Memory-management scheme that supports **user view** of memory
- A program is a collection of segments
  - A segment is a logical unit such as:

main program

procedure

function

method

object

local variables, global variables

common block

stack

symbol table

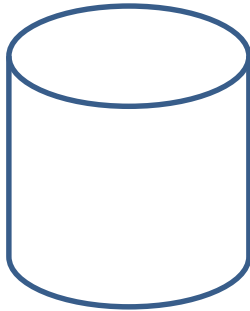
arrays

Compiler generates the segments

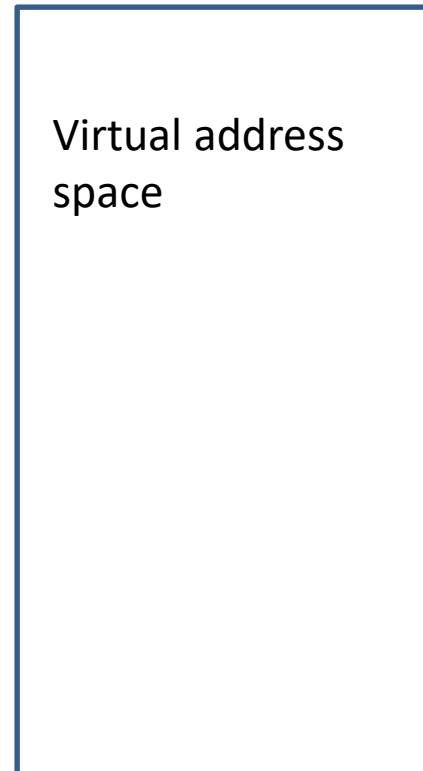
Loader assign the seg#

# Executable file and virtual address

a.out

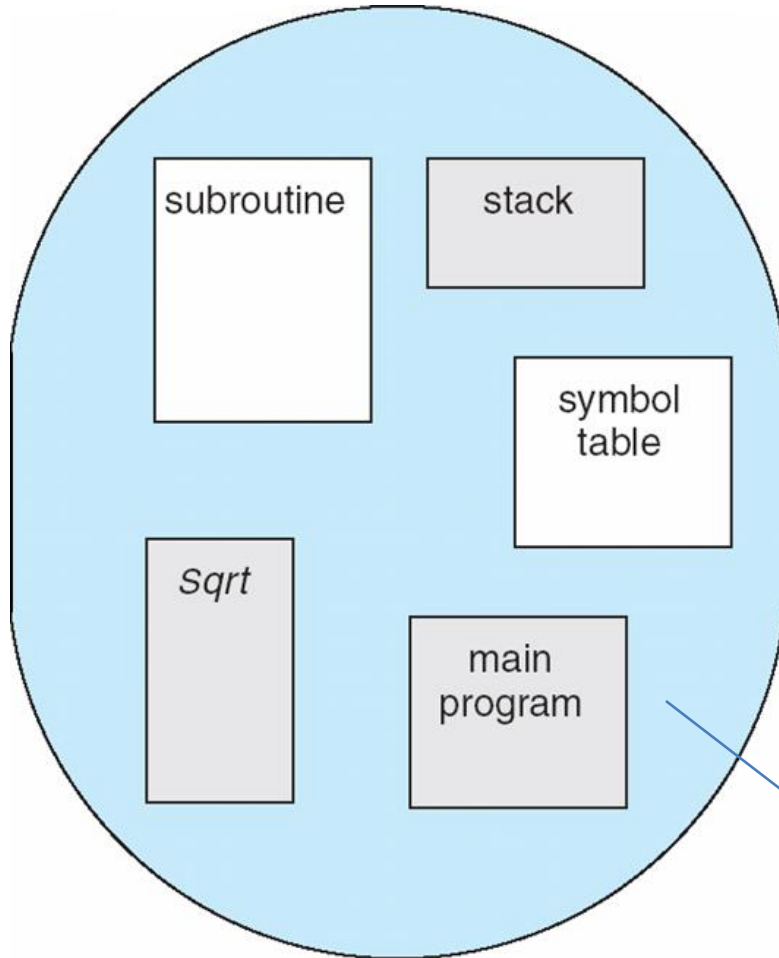


Symbol table	
Name	address
SQR	0
SUM	4



```
TEXT      code
MAIN      movf    NUM_1,w    ; Get Number 1
          call   SQR      ; Square it
          movf   SQUARE+1,w ; Get lower byte
          movwf  SUM+1    ; Is the low byte of sum
          movf   SQUARE,w  ; Get upper byte
          movwf  SUM      ; Is the high byte of sum
```

# User's View of a Program



logical address

User specifies each address by two quantities  
(a) Segment name  
(b) Segment offset

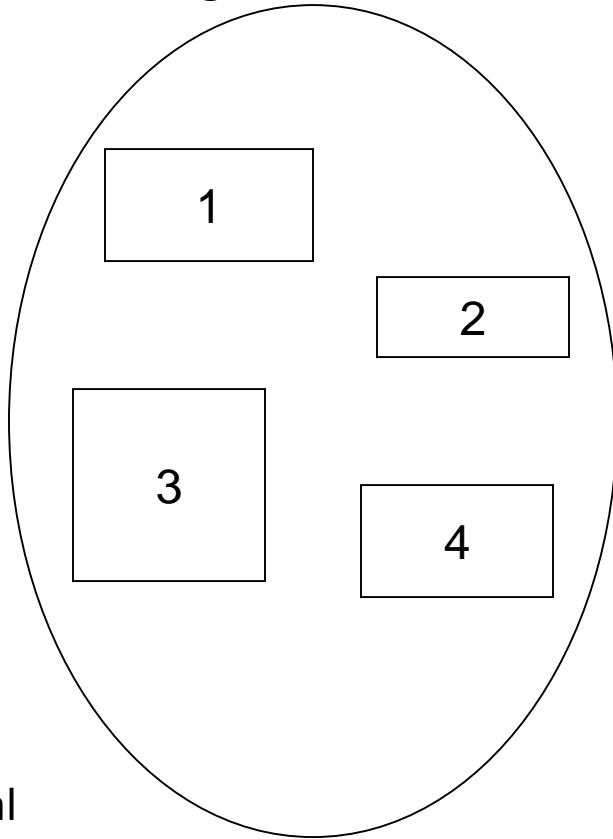
Logical address contains the tuple  
<segment#, offset>

- Variable size segments without order
- Length=> purpose of the program
- Elements are identified by offset

# Logical View of Segmentation

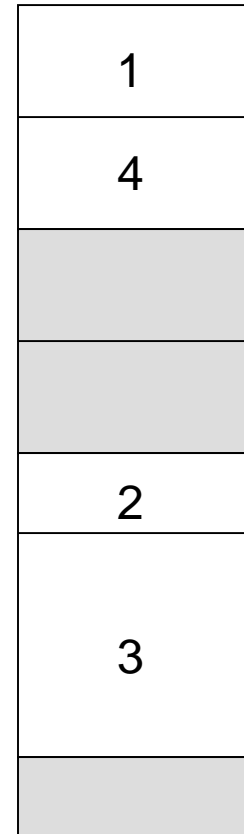
Logical address

<segment-number, offset>



Logical  
address  
space

user space



physical memory space

- Long term scheduler finds and allocates memory for all segments of a program
- Variable size partition scheme



## Windows XP Memory Usage

Segment	First Address	Last Address	Size
Code	401000x	403000x	002000x ~ 8 Kbytes
Static (Global) Data	403000x	703000x	300000x ~ 3 megabytes
Heap	760000x	3A261000x	39800000x ~ 950 megabytes
Stack	22EF00x	16EF00x	1C0000x ~ 2 megabyte

## LINUX Memory Usage

Segment	First Address	Last Address	Size
Code	8048400x	8049900x	001500x ~ 6 Kbytes
Static (Global) Data	8049A00x	8349A00	300000x ~ 3 megabytes
Heap	B7EE,B000x	01CE,4000x	B6000000x ~ 3 gigabytes
Stack	BFFB,7334x	29BA,91E0x	9640,0000x ~ 2.5 gigabyte

# Memory image

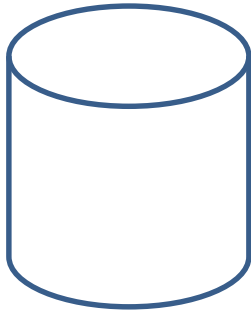
```
0x08048368 <main+0>: 55          push   %ebp
0x08048369 <main+1>: 89 e5      mov    %esp,%ebp
0x0804836b <main+3>: 83 ec 08   sub   $0x8,%esp
0x0804836e <main+6>: 83 e4 f0   and   $0xffffffff,%esp
0x08048371 <main+9>: b8 00 00 00 00 mov   $0x0,%eax
0x08048376 <main+14>: 83 c0 0f   add   $0xf,%eax
0x08048379 <main+17>: 83 c0 0f   add   $0xf,%eax
0x0804837c <main+20>: c1 e8 04   shr   $0x4,%eax
0x0804837f <main+23>: c1 e0 04   shl   $0x4,%eax
0x08048382 <main+26>: 29 c4     sub   %eax,%esp
0x08048384 <main+28>: 83 ec 0c   sub   $0xc,%esp
0x08048387 <main+31>: 68 c0 84 04 08 push  $0x80484c0
0x0804838c <main+36>: e8 1f ff ff ff call  0x80482b0
0x08048391 <main+41>: 83 c4 10   add   $0x10,%esp
0x08048394 <main+44>: e8 02 00 00 00 call  0x804839b <b>
```

```
1 void b();
2 void c();
3 int main()
4 {
5     printf( "Hello from main\n");
6     b();
7 }
8 // This routine reads the opcodes from memory and prints them out.
9 void b()
10 {
11     char *moving;
12
13     for ( moving = (char *)&main; moving < (char *)&c; moving++ )
14         printf( "Addr = 0x%x, Value = %2x\n", (int)moving, 255 & (int)*moving );
15 }
16 void c()
17 {
18 }
```

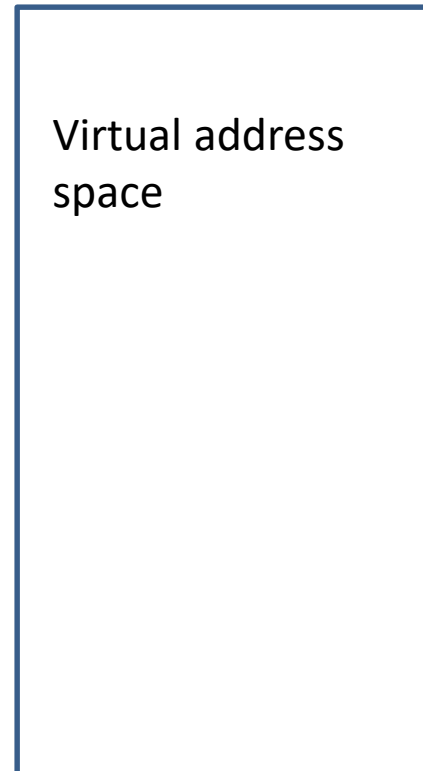
```
0x0804839b <b+0>: 55          push   %ebp
0x0804839c <b+1>: 89 e5      mov    %esp,%ebp
0x0804839e <b+3>: 83 ec 08   sub   $0x8,%esp
0x080483a1 <b+6>: c7 45 fc 68 83 04 08 movl  $0x8048368,0xffffffff(%ebp)
0x080483a8 <b+13>: 81 7d fc d9 83 04 08 cmpl  $0x80483d9,0xffffffff(%ebp)
0x080483af <b+20>: 73 26     jae   0x80483d7 <b+60>
0x080483b1 <b+22>: 83 ec 04   sub   $0x4,%esp
0x080483b4 <b+25>: 8b 45 fc   mov   0xffffffff(%ebp),%eax
0x080483b7 <b+28>: 0f be 00   movsbl (%eax),%eax
0x080483ba <b+31>: 25 ff 00 00 00 and   $0xff,%eax
```

# Executable file and virtual address

a.out



Symbol table	
Name	address
SQR	0
SUM	4



Virtual address space

Paging view

0	Load	0
4	ADD	4

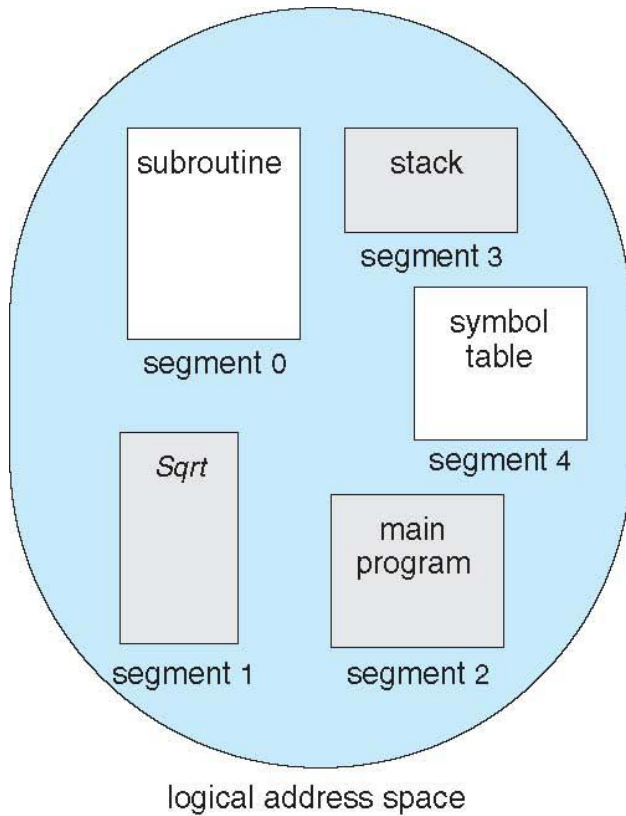
Segmentation view

<CODE, 0>	Load	<ST,0>
<CODE, 2>	ADD	<ST,4>

# Segmentation Architecture

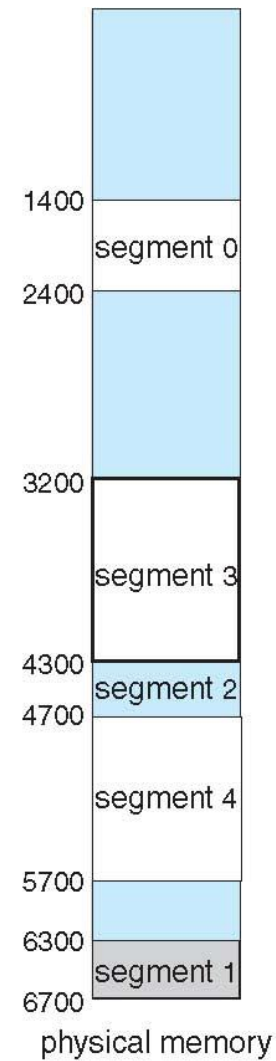
- Logical address consists of a two tuple:  
    <segment-number, offset>
- **Segment table** – maps two-dimensional logical address to physical address;
- Each table entry has:
  - **base** – contains the starting physical address where the segments reside in memory
  - **limit** – specifies the length of the segment
- **Segment-table base register (STBR)** points to the segment table's location in memory
- **Segment-table length register (STLR)** indicates number of segments used by a program;  
    segment number **s** is legal if **s** < **STLR**

# Example of Segmentation

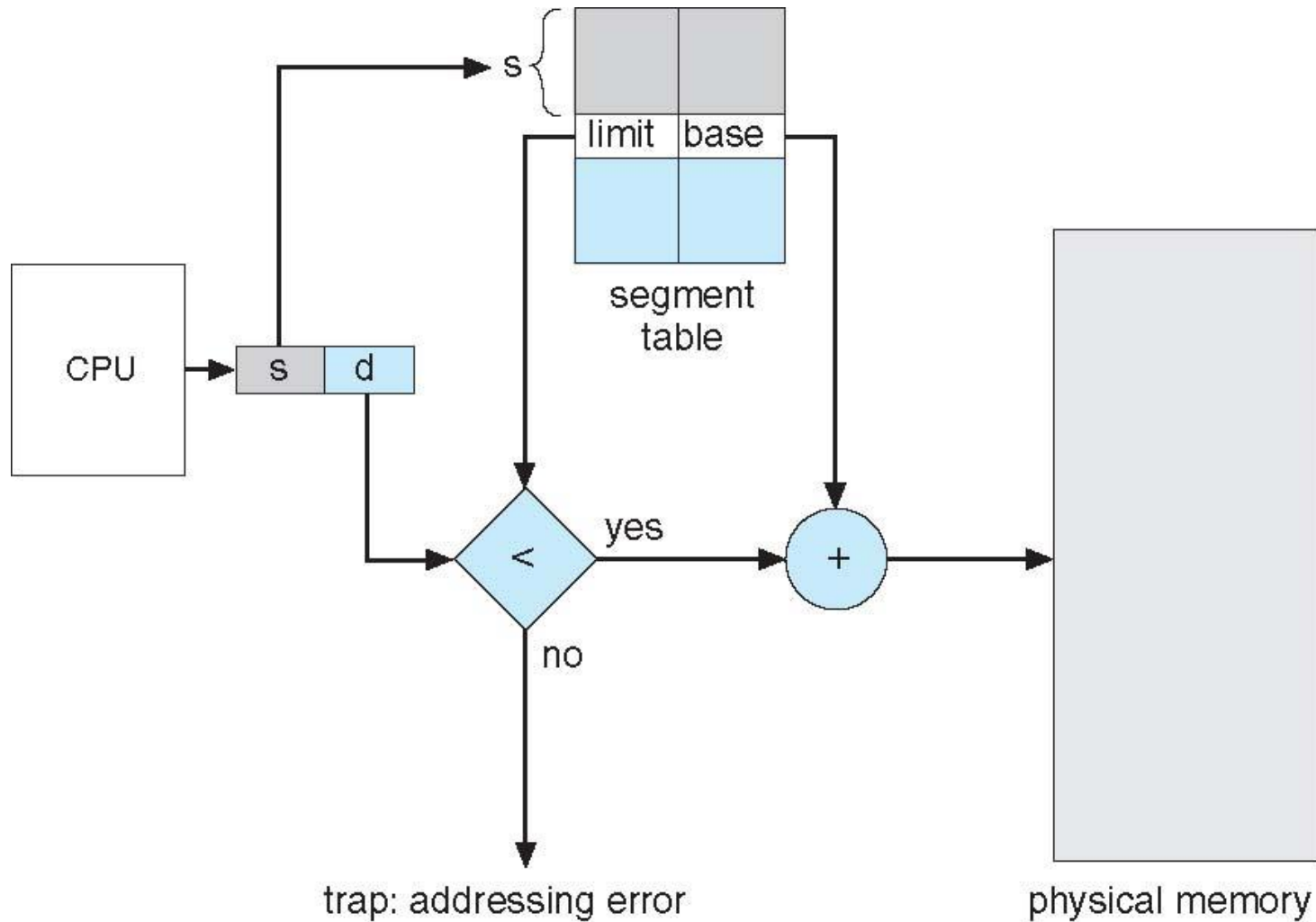


	limit	base
0	1000	1400
1	400	6300
2	400	4300
3	1100	3200
4	1000	4700

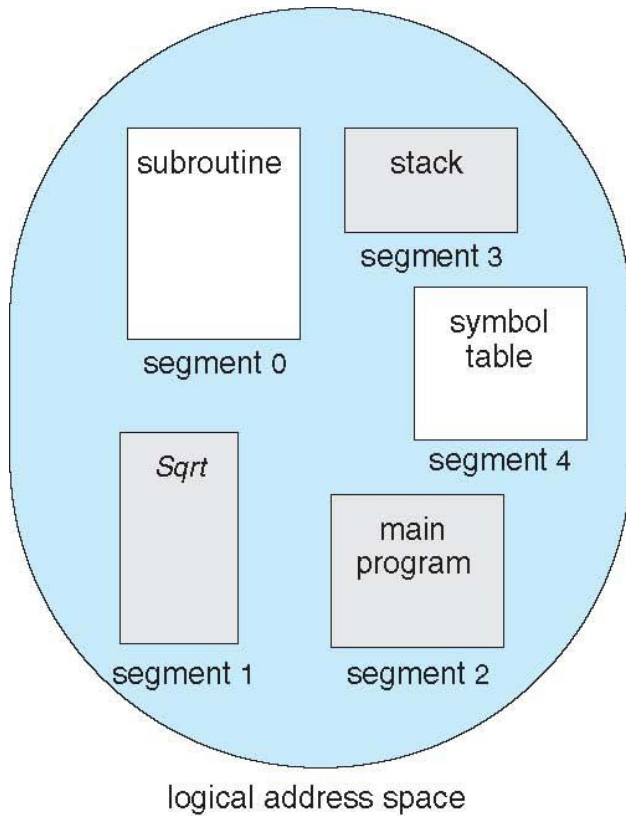
segment table



# Segmentation Hardware

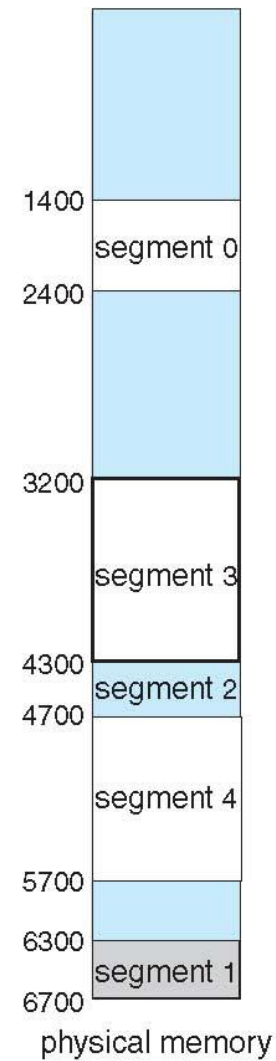


# Example of Segmentation



	limit	base
0	1000	1400
1	400	6300
2	400	4300
3	1100	3200
4	1000	4700

segment table





# Segmentation Architecture

- Protection
- Protection bits associated with segments
  - With each entry in segment table associate:
    - validation bit = 0  $\Rightarrow$  illegal segment
    - read/write/execute privileges
- Code sharing occurs at segment level
- Since segments vary in length, memory allocation is a dynamic storage-allocation problem
  - Long term scheduler
  - First fit, best fit etc
- Fragmentation

# Segmentation with Paging

Key idea:

Segments are splitted into multiple pages

Each page is loaded into frames in the memory

# Segmentation with Paging

- Supports both segmentation and segmentation with paging
  - Each segment can be 4 GB
  - Up to 16 K segments per process
  - <selector(16), offset (32)>
  - Divided into two partitions
    - First partition of up to 8 K segments are private to process (kept in **local descriptor table LDT**)
    - Second partition of up to 8K segments shared among all processes (kept in **global descriptor table GDT**)
- CPU generates logical address (six Segment Reg.)
  - Given to segmentation unit
    - Which produces linear (**logical**) addresses
  - Physical address 32 bits
  - Linear address given to paging unit
    - Which generates physical address in main memory
    - Paging units form equivalent of MMU
    - Pages sizes can be 4 KB

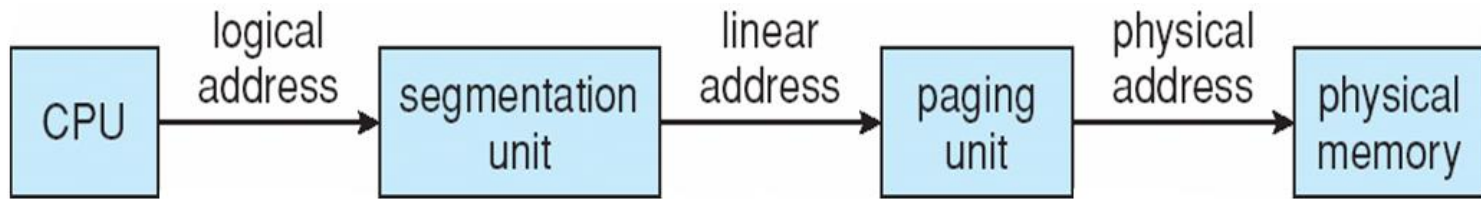
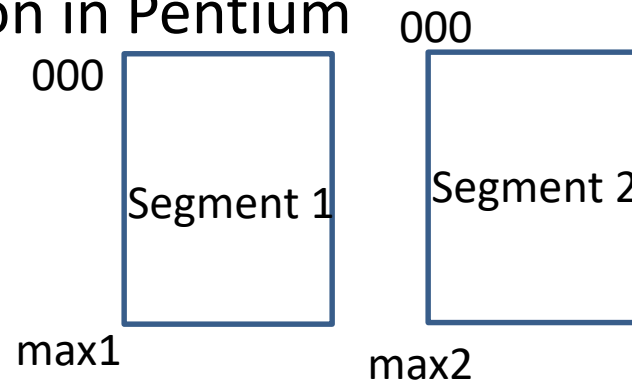


Intel 80386

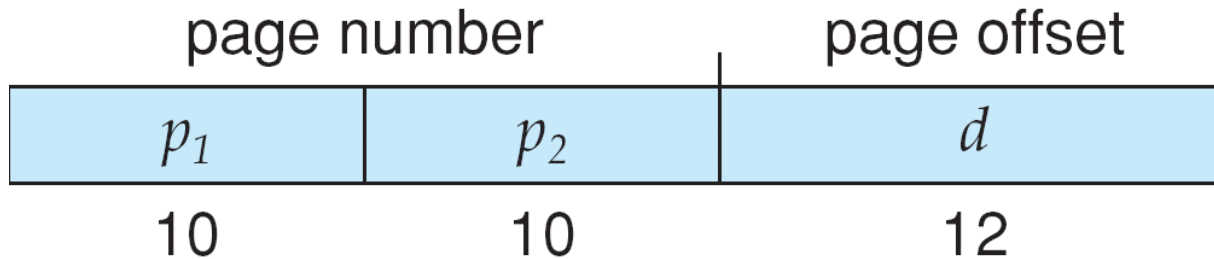
IBM OS/2

# Logical to Physical Address Translation in Pentium

Each segment works as a separate logical address space

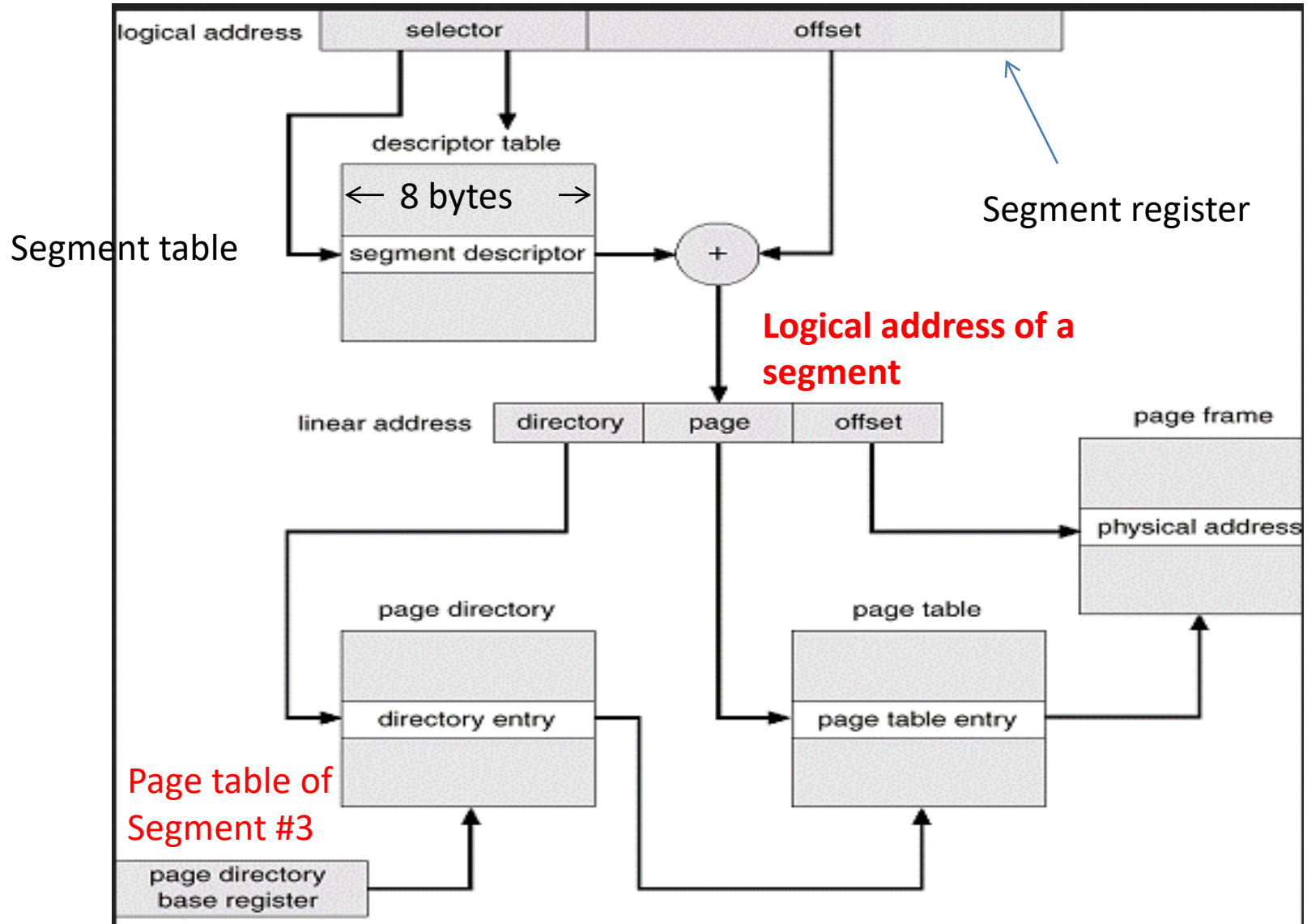


Page table =  $2^{20}$  entries

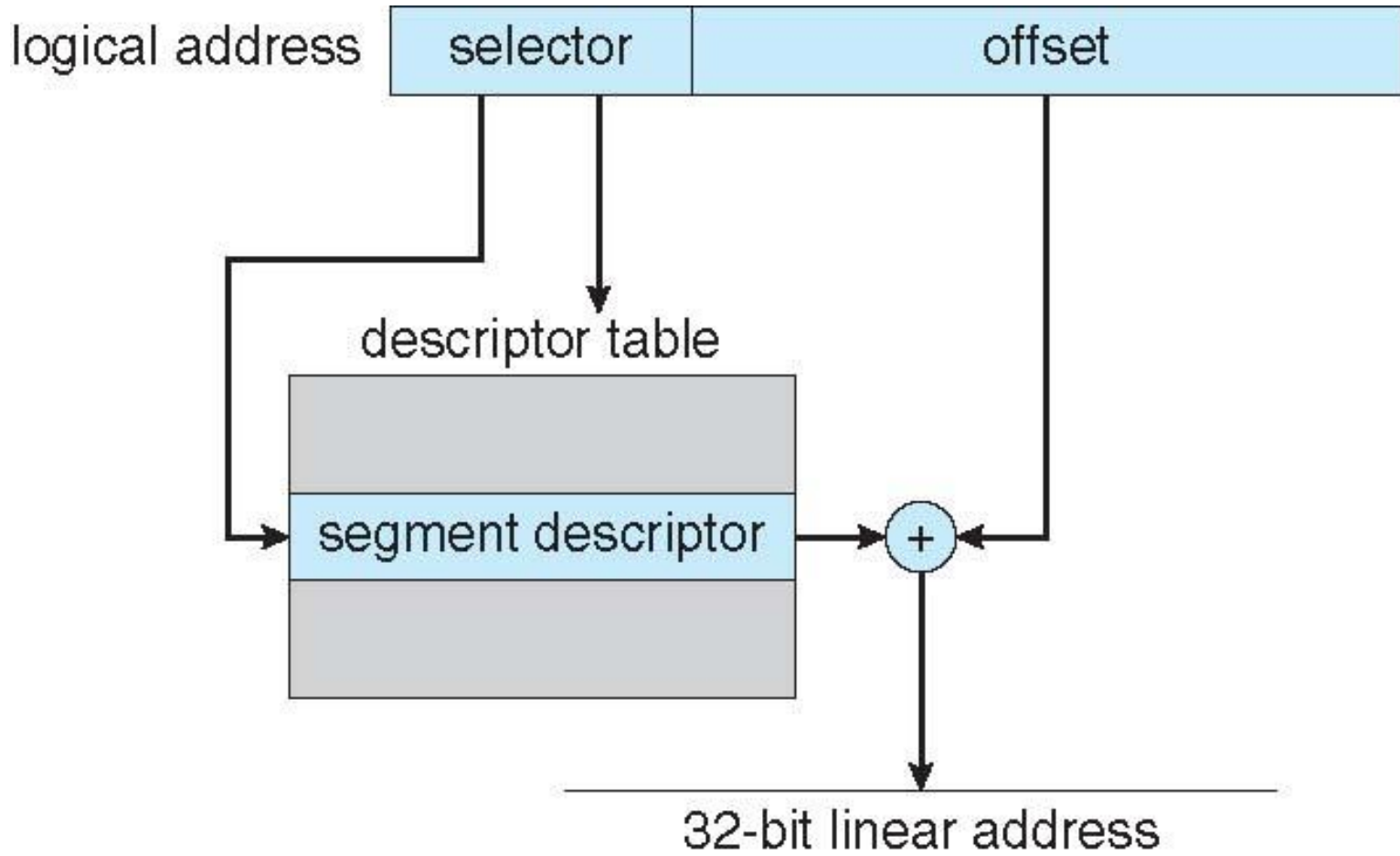


# Example: The Intel Pentium

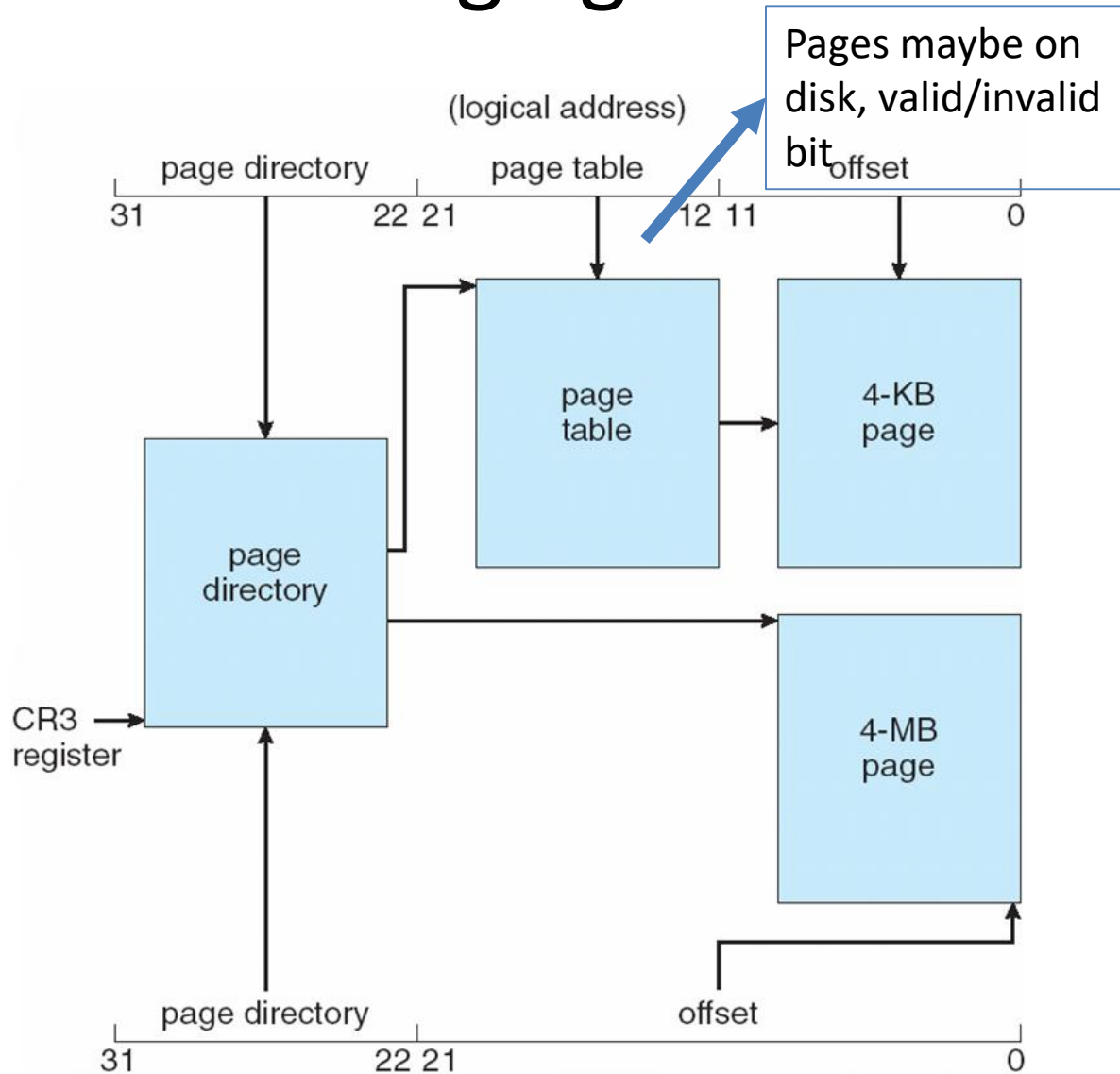
## Segment #3



# Intel Pentium Segmentation



# Pentium Paging Architecture



# Problem 1

Consider a system with byte-addressable memory, 32 bit logical addresses, 4 kilobyte page size and page table entries of 4 bytes each. Compute the size of the page table in the system in megabytes

Consider a machine with 64 MB physical memory and a 32-bit virtual address space. If the page size is 4 KB, what is the approximate size of the page table?



# Problem 2

In a virtual memory system, size of virtual address is 32-bit, size of physical address is 30-bit, page size is 4 Kbyte and size of each page table entry is 32-bit. The main memory is byte addressable. Compute the maximum number of bits that can be used for storing protection and other information in each page table entry.

# Problem 3

Consider a system with 2 level paging, where the page table is divided into 8k pages of size 16kB. The memory is byte addressable

The physical address space is 128MB which is divided into 4KB frames.

The page table entry of outer page table is 32 bits and page table entry size of inner page table is 64bits.

Show the logical and physical address format.

What will be the page table size of inner and outer page table?

Logical Address :

Outer Page ( $P_1$ ) {13-bits}	Inner Page( $P_2$ ) {14-bits}	Page offset(d) {12-bits}
--------------------------------	-------------------------------	--------------------------

Physical Address :

Frame Number (f) {27-12=15-bits}	Frame offset(d) {12-bits}
----------------------------------	---------------------------

Inner page table Size =  $2^{P_2} * \text{Inner page table entry size} = 2^{14} * 64 \text{ bits} = 2^{14} * 8 \text{ bytes} = 128 \text{ KB}$ .

Outer page table Size =  $2^{P_1} * \text{Outer page table entry size} = 2^{13} * 32 \text{ bits} = 2^{13} * 4 \text{ bytes} = 32 \text{ KB}$

So, The inner Page table size is 128 KB and outer page table size is 32 KB.

# Problem 4

Consider the segment table of a process

Segment No.	Base	Length
0	1219	700
1	2300	14
2	90	100
3	1327	580
4	1952	96

Process generates the following virtual addresses. Translate to physical address

1. 0, 430
2. 1, 11
3. 2, 100
4. 3, 425
5. 4, 95

# Virtual Memory

- Process P1 arrives
- Requires n pages => n frames must be available
- Allocate n frames to the process P1
- Create page table for P1

# Background

- Code needs to be in memory to execute, but entire program rarely used
  - Error code, unusual routines, large data structures
- Entire program code not needed at same time
- Consider ability to execute partially-loaded program
  - Program no longer constrained by limits of physical memory
  - programs could be larger than physical memory
  - More processes can be accommodated

# Background

- Example
  - 40 frames in memory
  - 8 processes each needs 10 pages
  - 5 of them rarely been used
- Two options
  - Run 4 processes (10 pages)
  - Run 8 processes (5 pages)
- Increase the degree of multiprogramming
  - Over allocating memory



# Classical paging

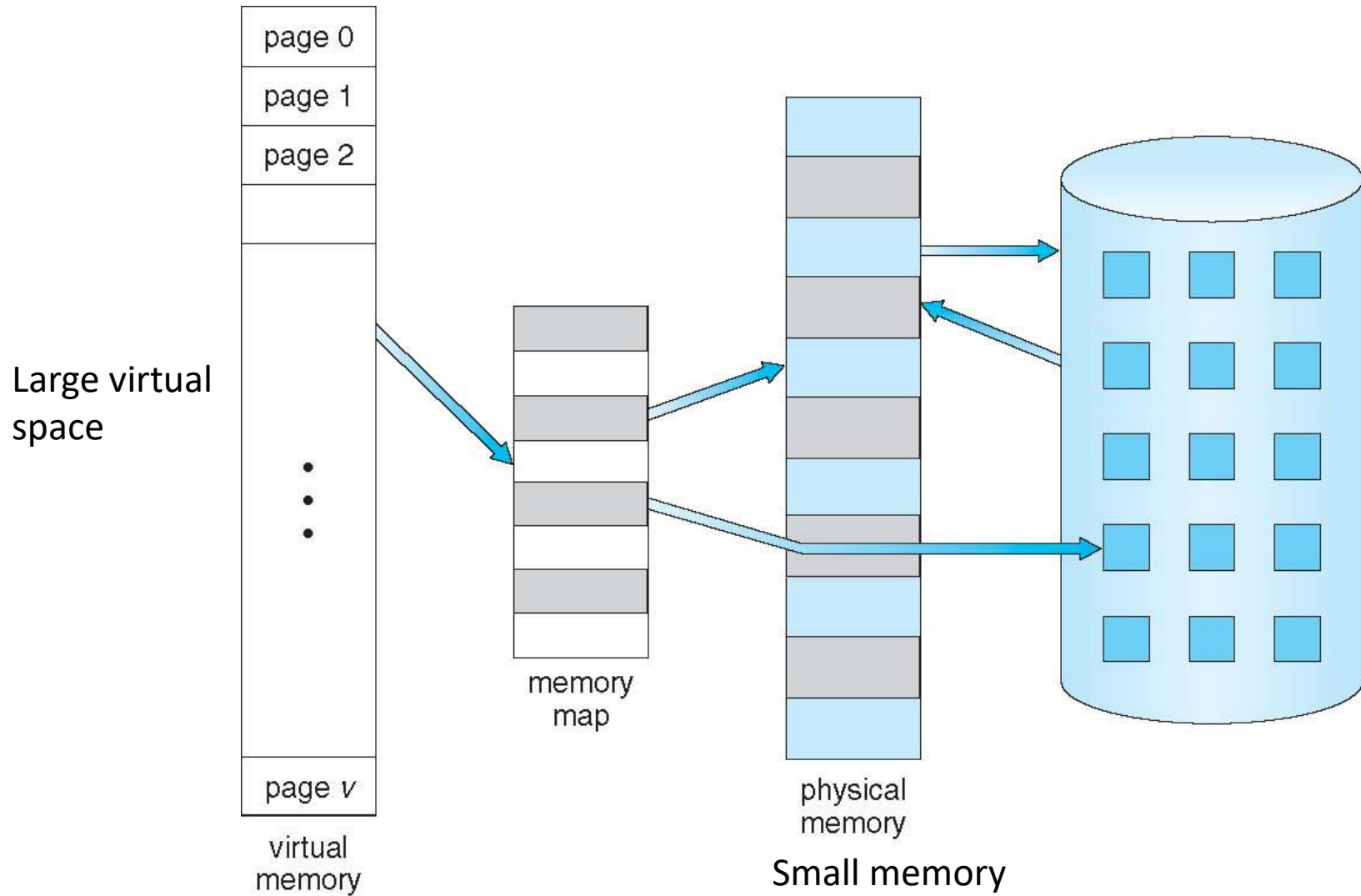
- Process P1 arrives
- Requires  $n$  pages  $\Rightarrow$   $n$  frames must be available
- Allocate  $n$  frames to the process P1
- Create page table for P1

Allocate  $< n$  frames

# Virtual memory

- **Virtual memory** – separation of user logical memory from physical memory
  - Extremely large logical space is available to programmer
  - Concentrate on the problem
- Only part of the program needs to be in memory for execution
  - Logical address space can therefore be much larger than physical address space
  - Starts with address 0, allocates contiguous logical memory
  - Physical memory
    - Collection of frame
- Virtual memory can be implemented via:
  - Demand paging
  - Demand segmentation

# Virtual Memory That is Larger Than Physical Memory

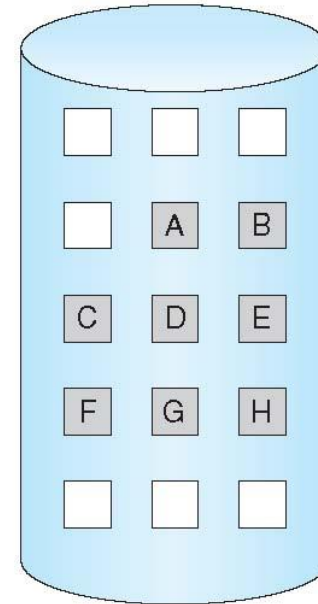
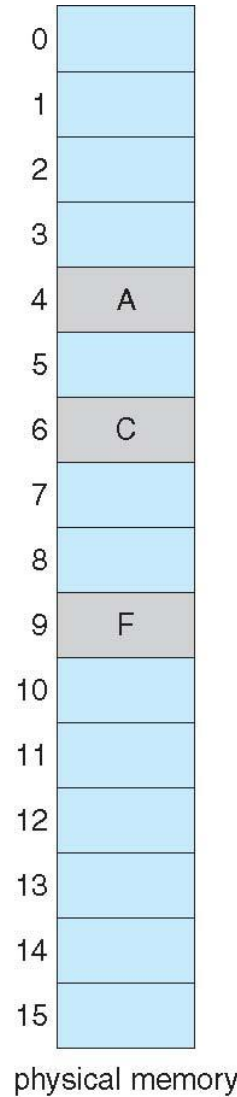
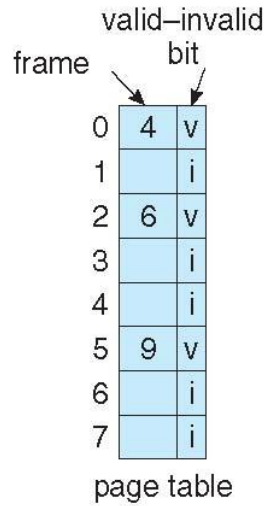
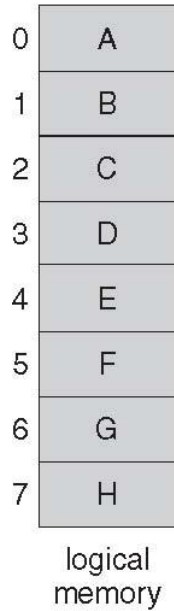


# Demand Paging

- Bring a page into memory only when it is needed
- **Lazy swapper** – never swaps a page into memory unless page will be needed
  - Swapper that deals with pages is a **pager**
- Less I/O needed, no unnecessary I/O
  - Less memory needed
  - More users
- Page is needed  $\Rightarrow$  reference to it
  - invalid reference  $\Rightarrow$  abort
  - not-in-memory  $\Rightarrow$  bring to memory

Valid address  
information is available  
in PCB

# Page Table When Some Pages Are Not in Main Memory



Pager loads few necessary pages in memory

# Valid-Invalid Bit

- With each page table entry a valid–invalid bit is associated (**v**  $\Rightarrow$  in-memory – **memory resident**, **i**  $\Rightarrow$  not-in-memory)
- Initially valid–invalid bit is set to **i** on all entries
- Example of a page table snapshot:

Frame #	valid-invalid bit
	<b>v</b>
	<b>v</b>
	<b>v</b>
	<b>v</b>
	<b>i</b>
....	
	<b>i</b>

page table

Disk address  $\rightarrow$

- During address translation, if valid–invalid bit in page table entry is **i**  $\Rightarrow$  page fault

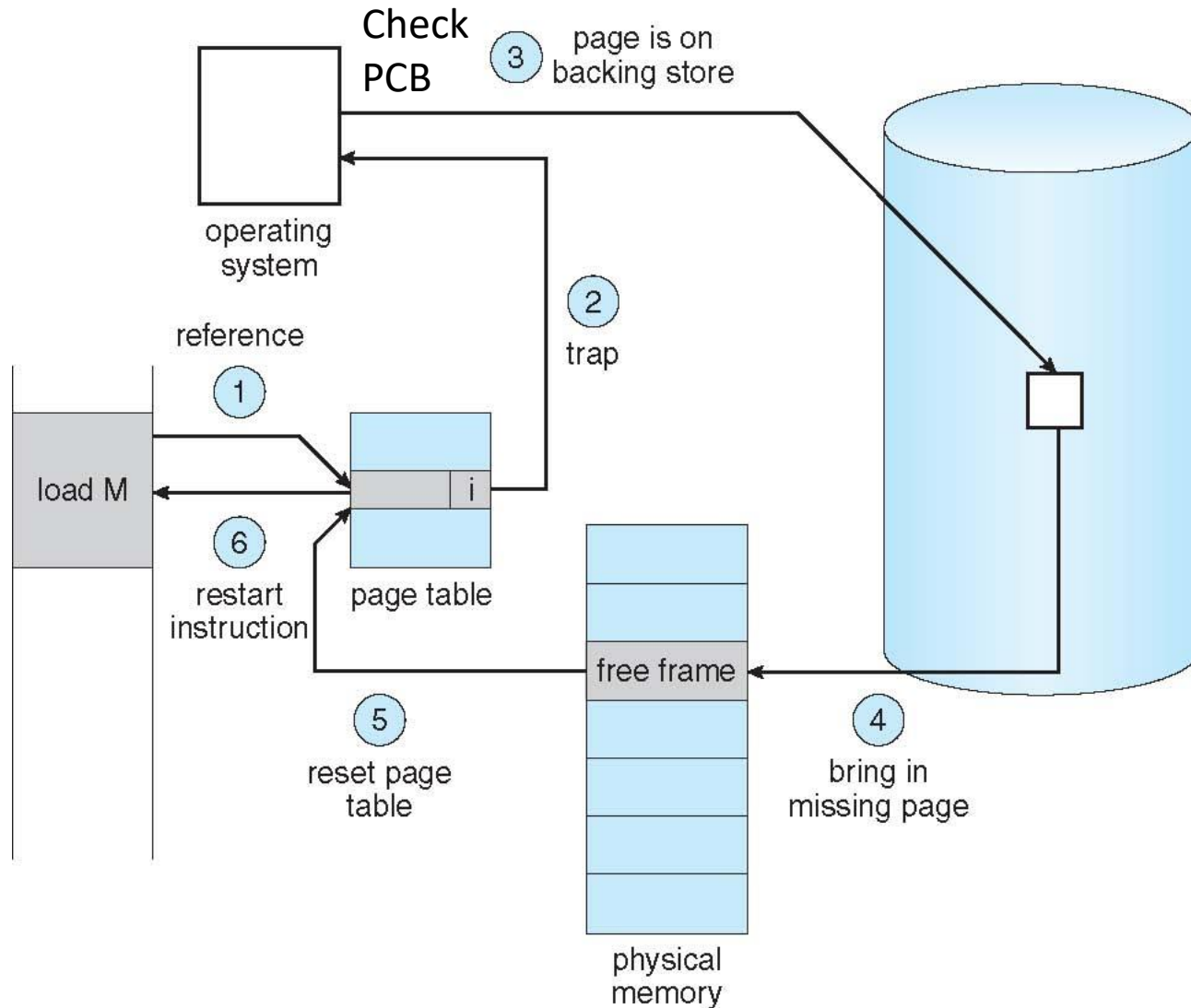
# Page Fault

- If the page is not in memory, first reference to that page will trap to operating system:

## page fault

1. Operating system looks at **PCB** to decide:
  - Invalid reference  $\Rightarrow$  abort
  - Just not in memory (load the page)
2. Get empty frame
3. Swap page into frame via scheduled disk operation
4. Reset page table to indicate page now in memory  
Set validation bit = **v**
5. Restart the instruction that caused the page fault

# Steps in Handling a Page Fault

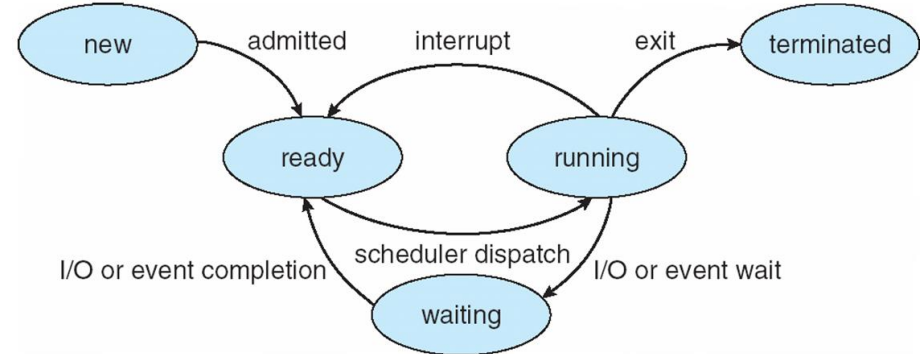




# Steps in the ISR

- In Demand Paging

1. Trap to the operating system
2. Save the user registers and process state
3. Determine that the interrupt was a page fault
4. Check that the page reference was legal and determine the location of the page on the disk
5. Get a free frame
6. Issue a read from the disk to a free frame:
  1. Wait in a queue for this device until the read request is serviced
  2. Wait for the device seek and/or latency time
  3. Begin the transfer of the page to a free frame
7. While waiting, allocate the CPU to some other user
8. Receive an interrupt from the disk I/O subsystem (I/O completed)
9. Save the registers and process state of the running process
10. Determine that the interrupt was from the disk
11. Correct the page table and other tables to show page is now in memory
12. Wait for the CPU to be allocated to this process again
13. Restore the user registers, process state, and new page table, and then resume the interrupted instruction



# Pure Demand Paging

- Extreme case – start process with *no* pages in memory
  - OS sets instruction pointer to first instruction of process, non-memory-resident -> page fault
  - Swap in that page
  - **Pure demand paging**
- Actually, a given instruction could access multiple pages (instruction + data) -> multiple page faults
  - Pain decreased because of **locality of reference**
- **Hardware support** needed for demand paging
  - Page table with valid / invalid bit
  - Secondary memory (swap device with **swap space**)
  - Instruction restart after page fault

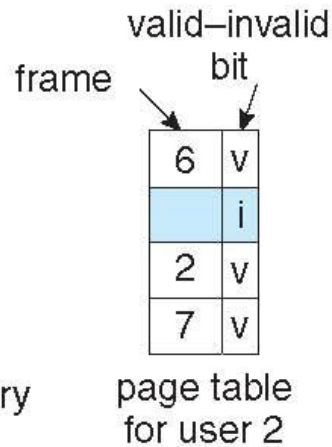
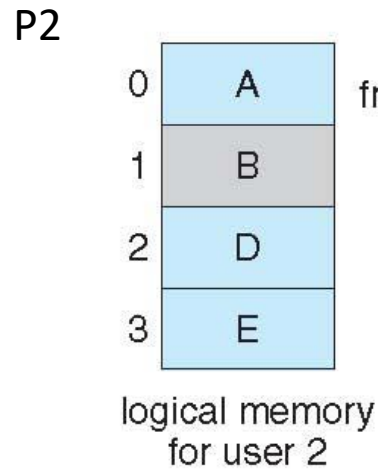
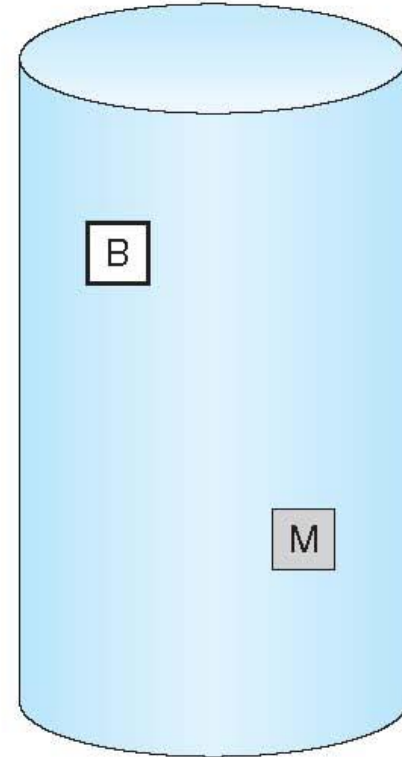
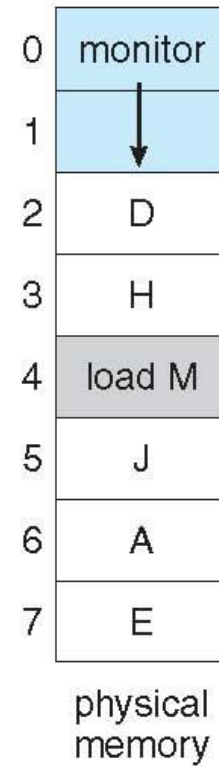
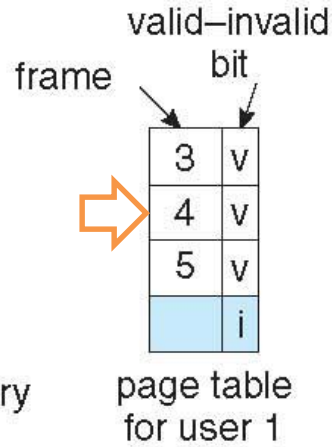
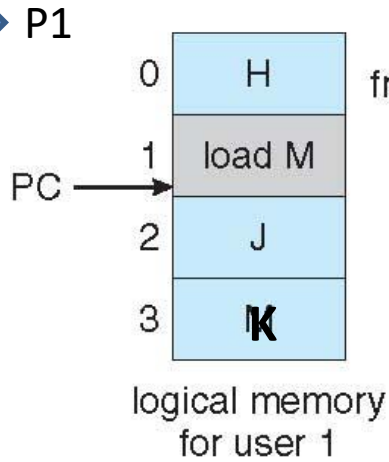
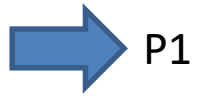
# Page and Frame Related Algorithms

- **Page-replacement algorithm**
  - Which frames to replace
  - Want lowest page-fault rate
- **Frame-allocation algorithm** determines
  - How many frames to give each process

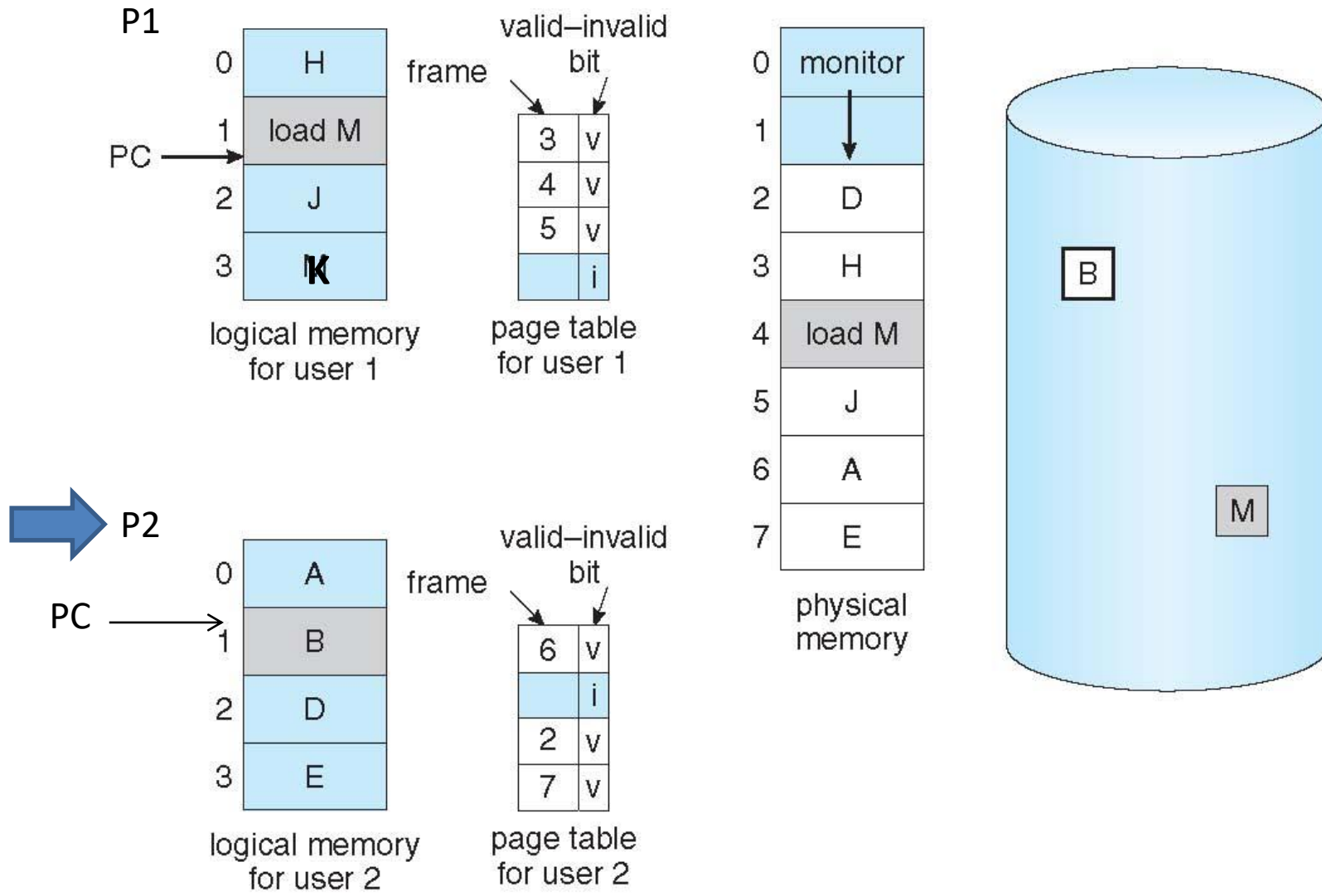
# What Happens if There is no Free Frame?

- Example
  - 40 frames in memory
  - 8 processes each needs 10 pages
  - 5 of them never used
- Two options
  - Run 4 processes (10 pages)
  - Run 8 processes (5 pages)
- Increase the degree of multiprogramming
  - Over allocating memory
- **Page fault**
  - **No free frame**
  - Terminate? swap out? replace the page?
- **Page replacement** – find some page in memory, not really in use, page it out
  - Performance – want an algorithm which will result in minimum number of page faults
- Same page may be brought into memory several times

# Need For Page Replacement



# Need For Page Replacement

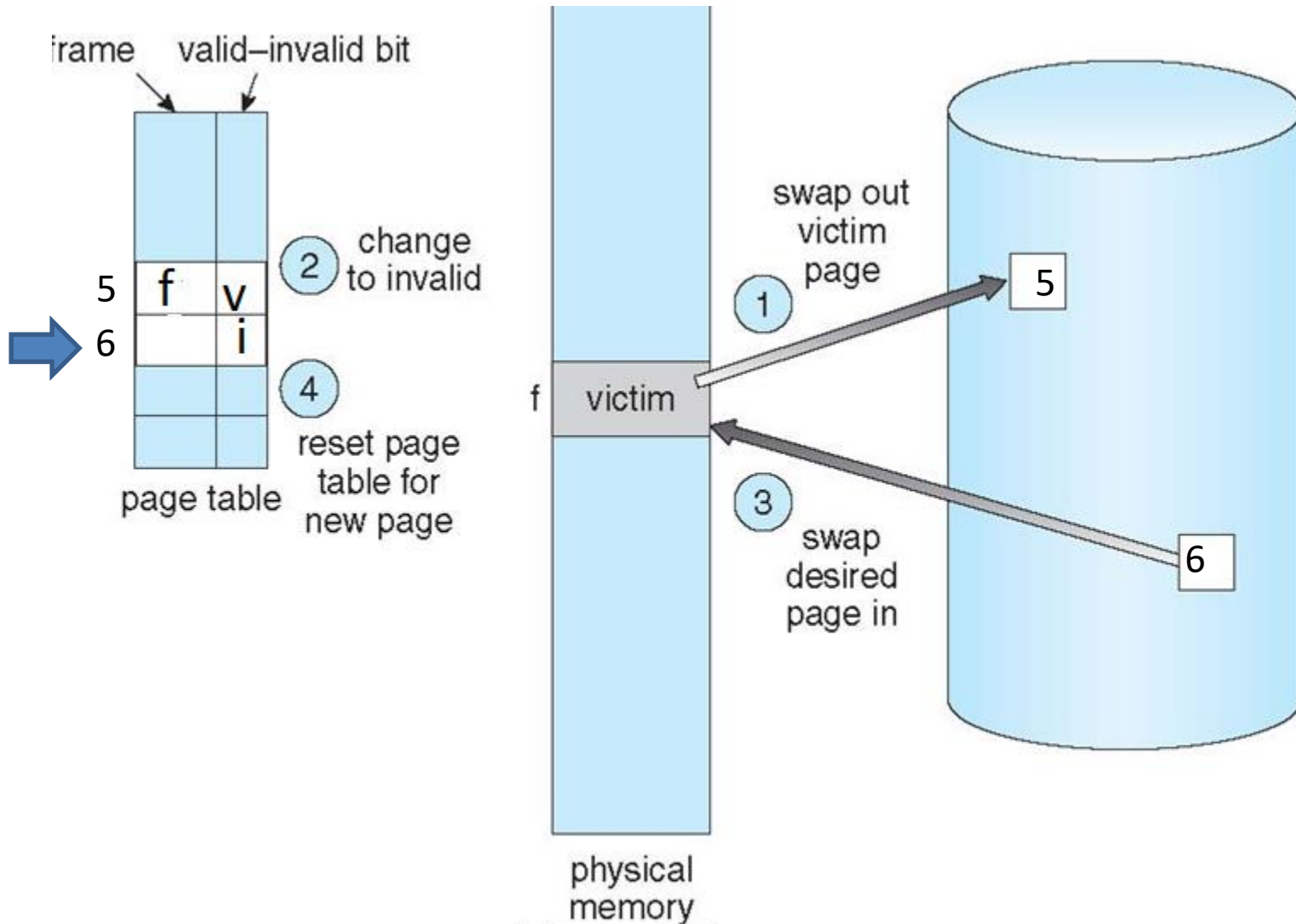


# Basic Page Replacement

1. Find the location of the desired page on disk
2. Find a free frame:
  - If there is a free frame, use it
  - If there is no free frame, use a **page replacement algorithm** to select a **victim frame (of that process)**
    - Write victim frame to disk
3. Bring the desired page into the (newly) free frame; update the page and frame tables
4. Continue the process by restarting the instruction that caused the trap

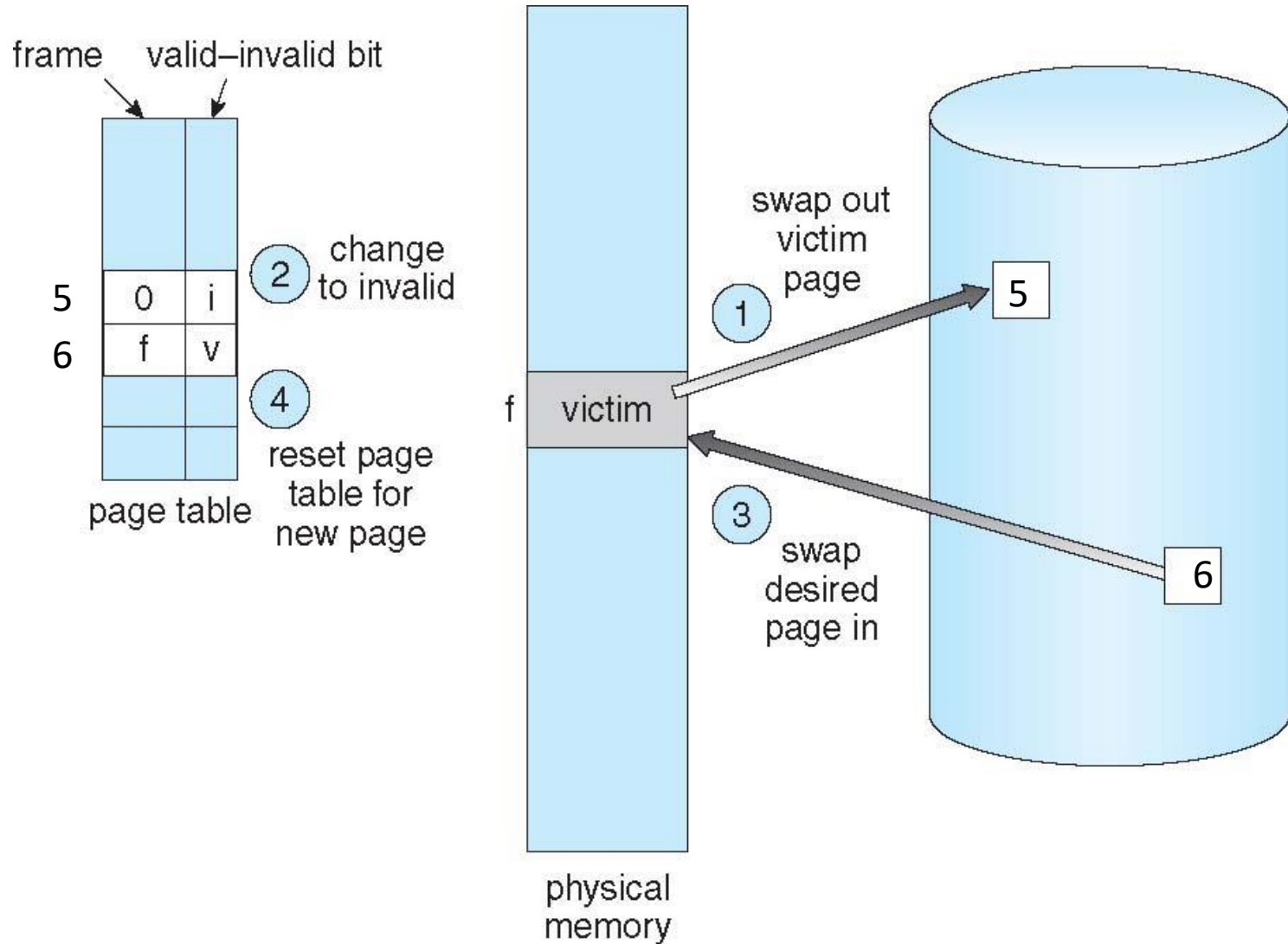
**Note now potentially 2 page transfers for page fault – increasing Effective memory access time**

# Page Replacement





# Page Replacement



# Page Replacement

- **Modify (dirty) bit** to reduce overhead of page transfers
  - only modified pages are written to disk

# Performance of Demand Paging

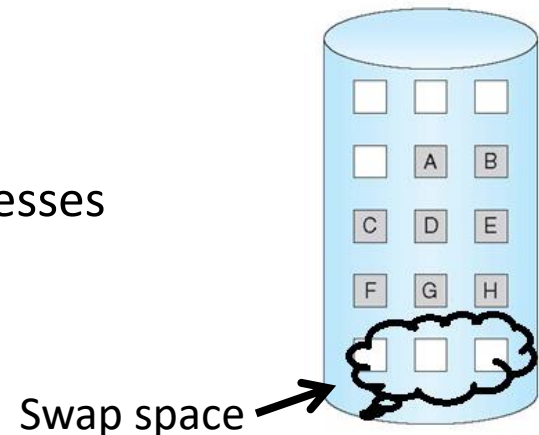
Demand paging affects the performance of the computer systems

- Page Fault Rate  $0 \leq p \leq 1$ 
  - if  $p = 0$  no page faults
  - if  $p = 1$ , every reference is a fault
- Effective Access Time (EAT)
  - EAT =  $(1 - p)$  x memory access
  - +  $p$  (page fault overhead
  - + swap page out
  - + swap page in
  - + restart overhead
  - )

# Demand Paging Example

- Memory access time = 200 nanoseconds
- Average page-fault service time = 8 milliseconds
- $EAT = (1 - p) \times 200 + p (8 \text{ milliseconds})$   
 $= (1 - p) \times 200 + p \times 8,000,000$   
 $= 200 + p \times 7,999,800$
- If one access out of 1,000 causes a page fault, then  
EAT = 8.2 microseconds.  
This is a slowdown by a factor of 40!!
- If want performance degradation < 10 percent
  - $220 > 200 + 7,999,800 \times p$   
 $20 > 7,999,800 \times p$
  - $p < .0000025$
  - < one page fault in every 400,000 memory accesses

**Better utilization of swap space**



# Page replacement algorithm

## Evaluation

- Evaluate algorithm by running it on a particular string of memory references (reference string) and computing the number of page faults on that string
  - String is just page numbers, not full addresses
  - Repeated access to the same page does not cause a page fault

- Trace the memory reference of a process

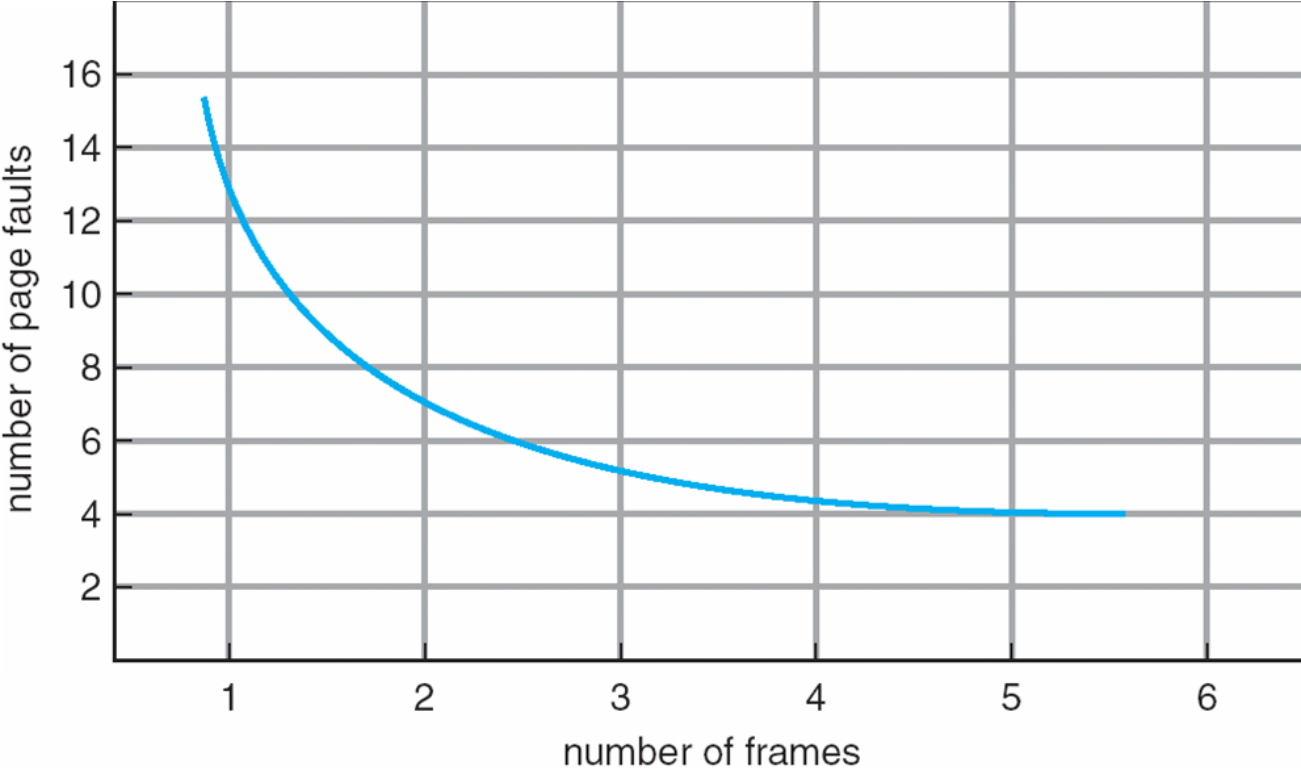
0100, 0432, 0101, 0612, 0102, 0104, 0101, 0611, 0102

- Page size 100B
- Reference string 1, 4, 1, 6, 1, 6

- In all our examples, the reference string is

**7,0,1,2,0,3,0,4,2,3,0,3,0,3,2,1,2,0,1,7,0,1**

# Graph of Page Faults Versus The Number of Frames



# First-In-First-Out (FIFO) Algorithm

Associates a time with each frame when the page was brought into memory

When a page must be replaced, the oldest one is chosen

Reference string: **7,0,1,2,0,3,0,4,2,3,0,3,0,3,2,1,2,0,1,7,0,1**

## **Limitation:**

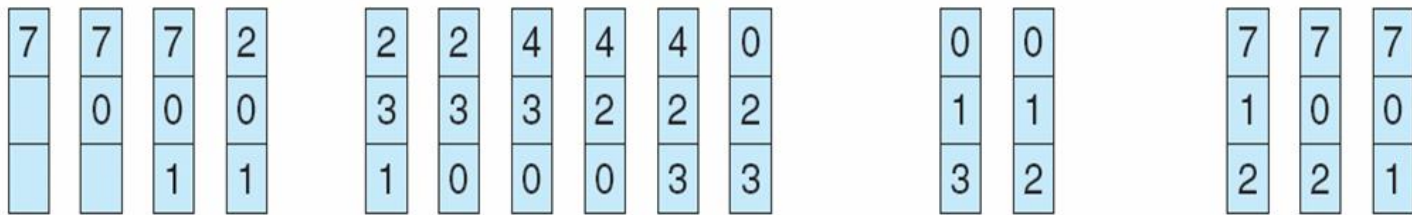
A variable is initialized early and constantly used

# FIFO Page Replacement

3 frames (3 pages can be in memory at a time)

reference string

7 0 1 2 0 3 0 4 2 3 0 3 2 1 2 0 1 7 0 1



page frames

15 page faults

- How to track ages of pages?
  - Just use a FIFO queue to hold all the pages in memory
  - Replace the page at the head
  - Insert at tail



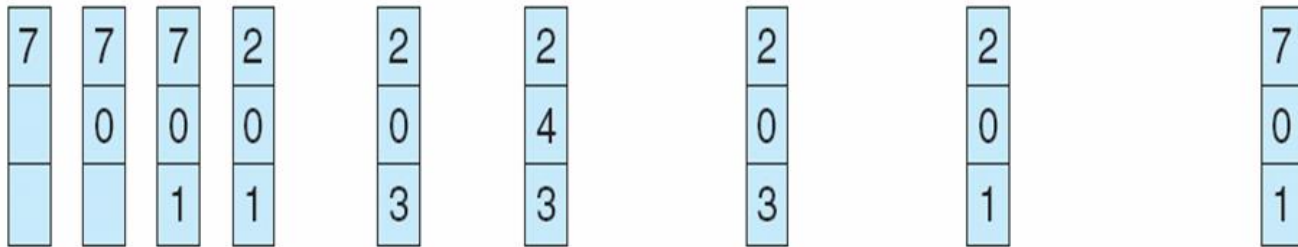
# Optimal Algorithm

- Replace page that will not be used for longest period of time
- How do you know this?
  - Can't read the future
- Used for measuring how well your algorithm performs

# Optimal Page Replacement

reference string

7 0 1 2 0 3 0 4 2 3 0 3 2 1 2 0 1 7 0 1



page frames

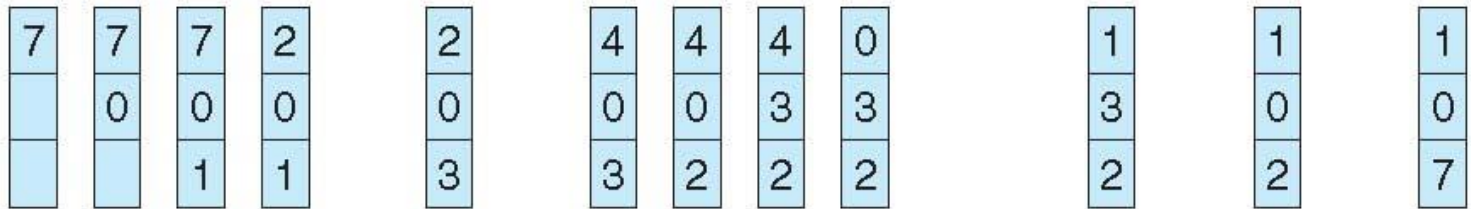
9 page faults

# Least Recently Used (LRU) Algorithm

- **Use past knowledge rather than future**
  - Past is the proxy of future
- Replace page that has not been used in the most of the time
- Associate time of last use with each page

reference string

7 0 1 2 0 3 0 4 2 3 0 3 2 1 2 0 1 7 0 1



page frames

- 12 faults – better than FIFO but worse than OPT
- Generally good algorithm and frequently used
- But how to implement?

# LRU Algorithm-Implementation

- **Counter implementation**

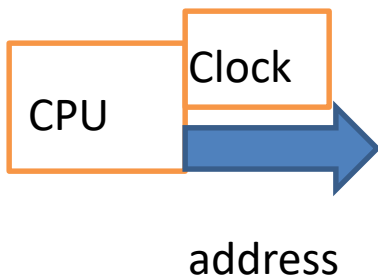
- CPU maintains a clock

- Every page entry has a **Time of use**;

- every time page is referenced, copy the clock into the **time of use**

- When a page needs to be replaced, look at the **“Time of use”** to find smallest value

- Search through table needed



**Page table**

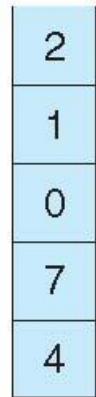
		Time of Use
0		
1		
2		
3		

# LRU Algorithm-Implementation

- **Stack implementation**
  - Keep a stack of page numbers in a double linked list form:
  - Page referenced:
    - move it to the top
  - Victim page is the bottom page

reference string

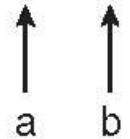
4 7 0 7 1 0 1 2 1 2 7 1 2



stack  
before  
a



stack  
after  
b

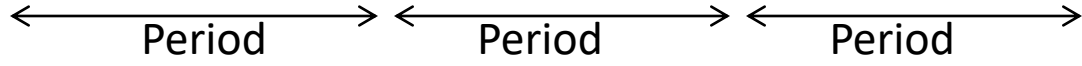


# LRU Approximation Algorithms

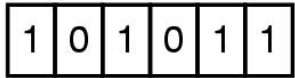
- **LRU needs special hardware and still slow**
- **Reference bit**
  - With each page associate a bit, initially = 0
  - When page is referenced bit set to 1
- **Additional reference bit algorithm**
  - Record the reference bits in regular interval
  - Keep a 8 bit string for each page in memory
  - At regular interval, timer copies the reference bit to the high order bit (MSB) of the string.
  - Shift the other bits right side by one bit

# LRU Approximation Algorithms

Reset  
reference



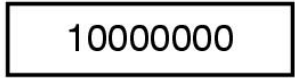
R bits for  
pages 0-5,  
clock tick 0



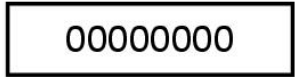
0 1 2 3 4 5

Page

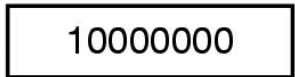
0



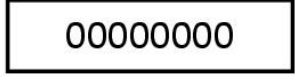
1



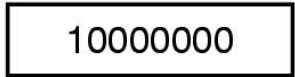
2



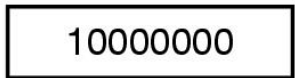
3



4

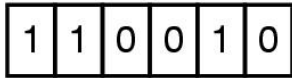


5



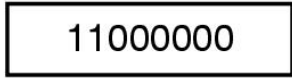
(a)

R bits for  
pages 0-5,  
clock tick 1

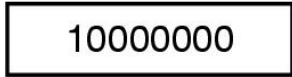


0 1 2 3 4 5

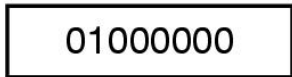
0



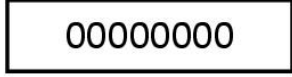
1



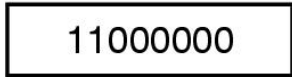
2



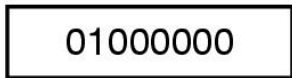
3



4

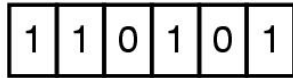


5



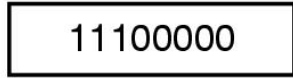
(b)

R bits for  
pages 0-5,  
clock tick 2

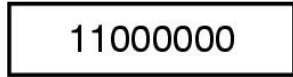


0 1 2 3 4 5

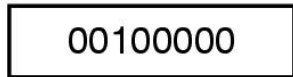
0



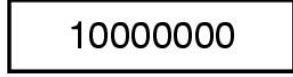
1



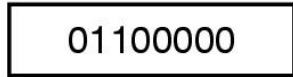
2



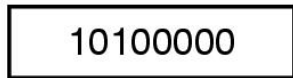
3



4

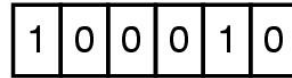


5



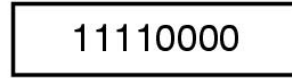
(c)

R bits for  
pages 0-5,  
clock tick 3

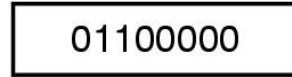


0 1 2 3 4 5

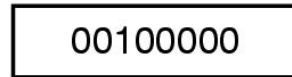
0



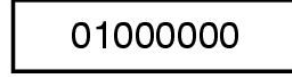
1



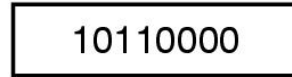
2



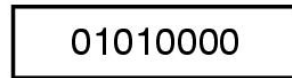
3



4

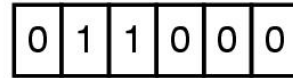


5



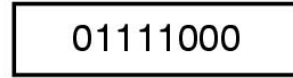
(d)

R bits for  
pages 0-5,  
clock tick 4



0 1 2 3 4 5

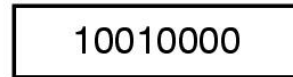
0



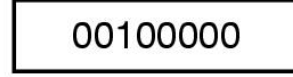
1



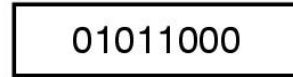
2



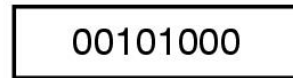
3



4



5



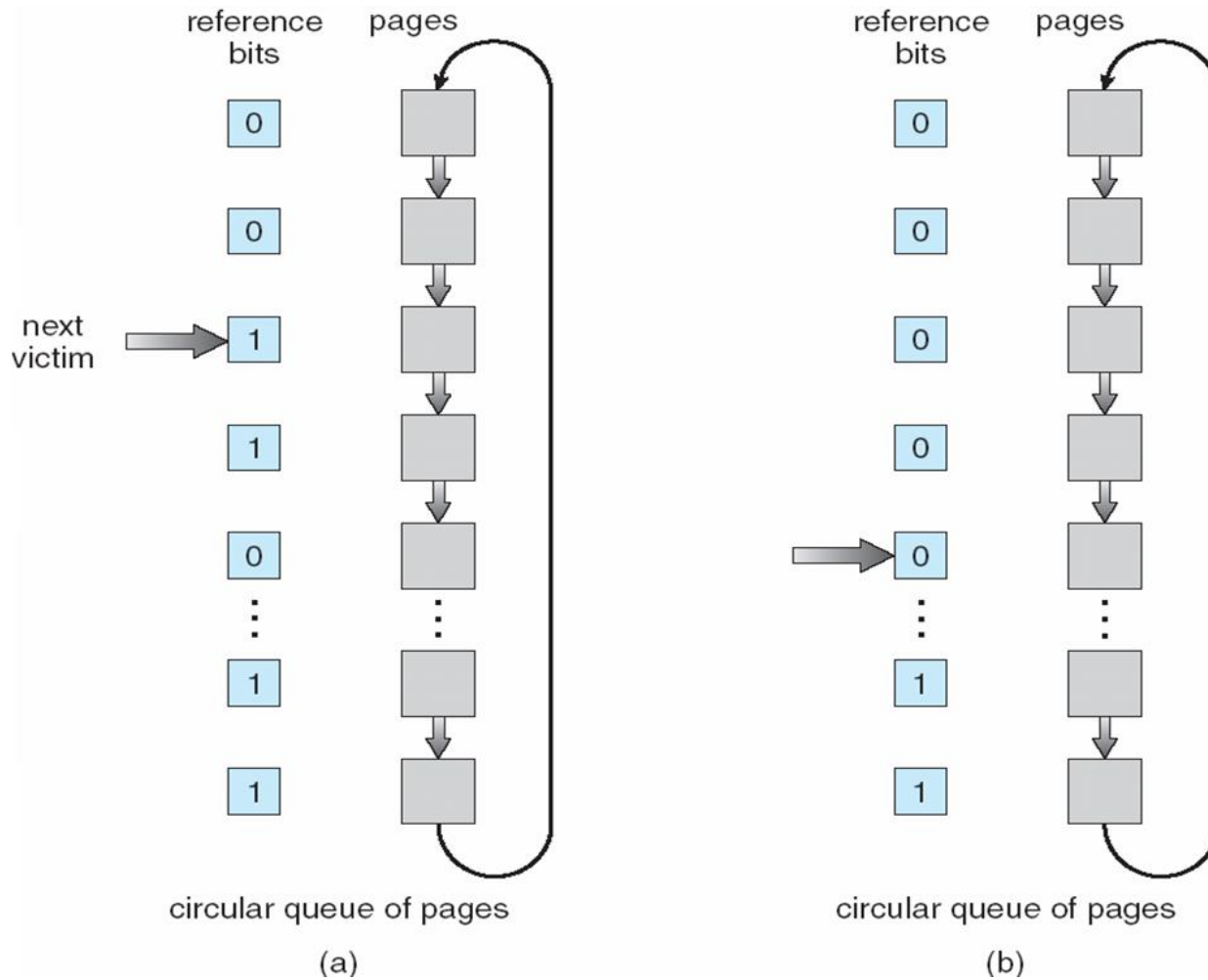
(e)

# LRU Approximation Algorithms

- LRU needs special hardware and still slow
- **Reference bit**
  - With each page associate a bit, initially = 0
  - When page is referenced bit set to 1
- Additional reference bit algorithm
- **Second-chance algorithm**
  - Generally FIFO, plus hardware-provided reference bit
  - Clock replacement
  - If page to be replaced has
    - Reference bit = 0 -> replace it
    - reference bit = 1 then:
      - set reference bit 0, leave page in memory (reset the time)
      - replace next page, subject to same rules



# Second-Chance (clock) Page-Replacement Algorithm



**{Reference bit, Dirty bit} combination**

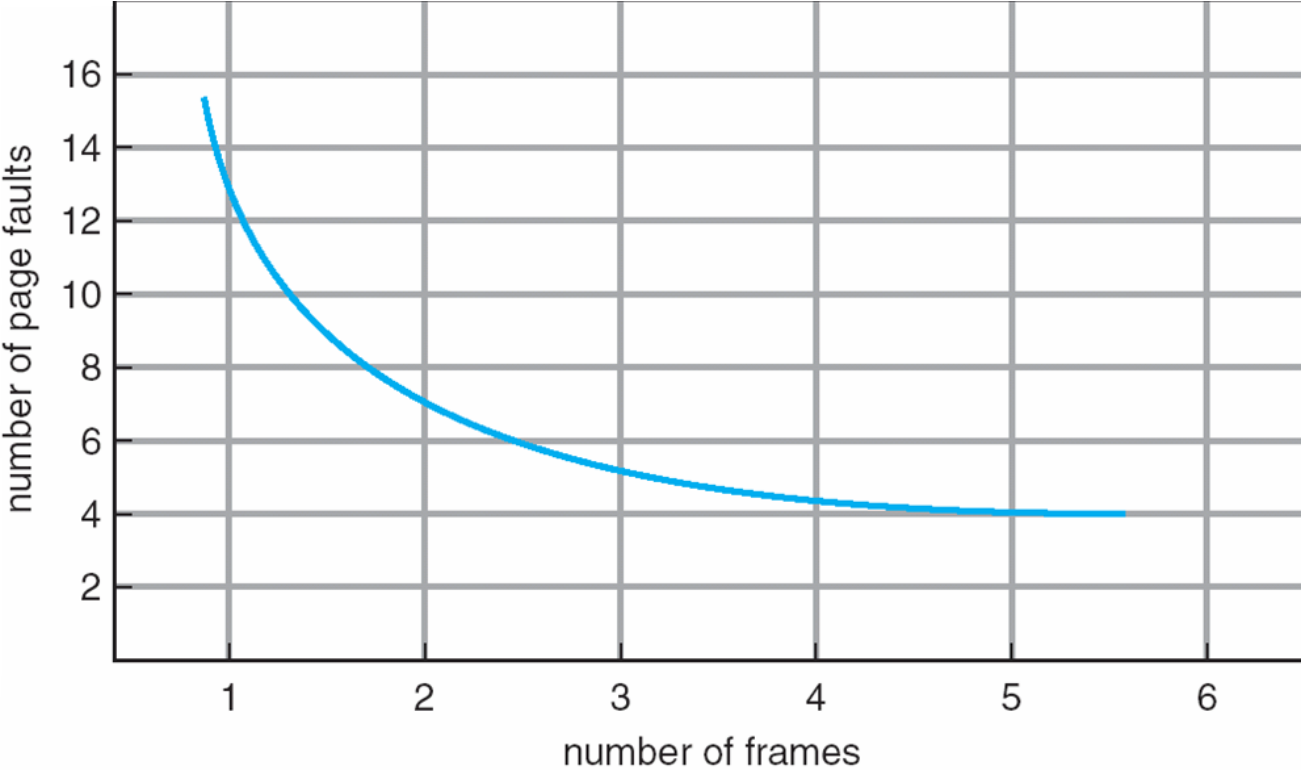
# Counting Algorithms

- Keep a counter of the number of references that have been made to each page

Least and most frequently used

- **LFU Algorithm**: replaces page with smallest count
- **MFU Algorithm**: based on the argument that the page with the smallest count was probably just brought in and has yet to be used

# Graph of Page Faults Versus The Number of Frames

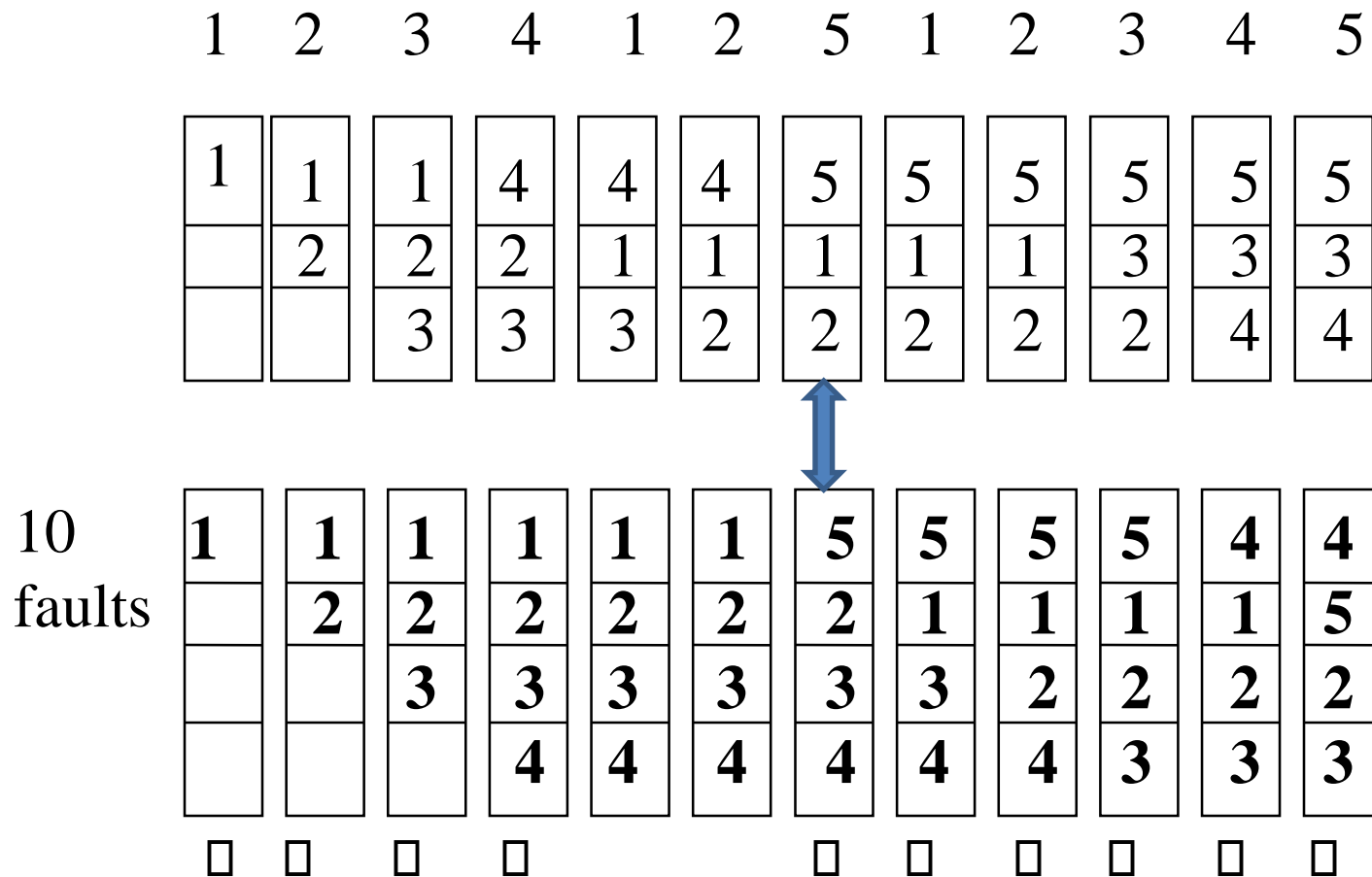


Increase in page frame decreases page fault rate?

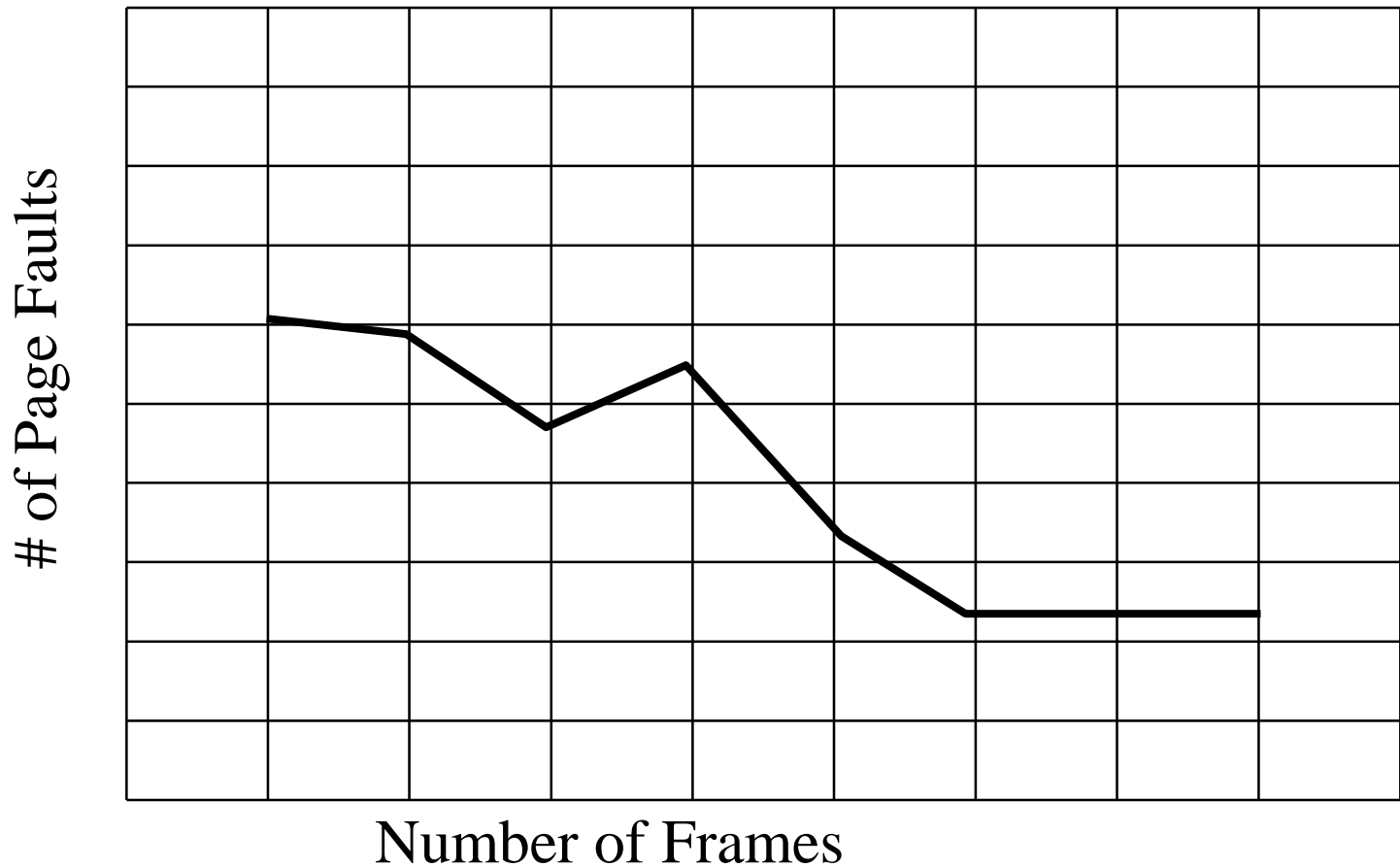
FIFO Example –  
Program - 5 pages,  
3 frames of Memory allocated

	1	2	3	4	1	2	5	1	2	3	4	5
9 faults	1	1	1	4	4	4	5	5	5	5	5	5
		2	2	2	1	1	1	1	1	3	3	3
			3	3	3	2	2	2	2	2	4	4
	□	□	□	□	□	□				□	□	

# FIFO Example – Program - 5 pages, 4 frames of Memory allocated



# Belady's Anomaly



# Belady's Anomaly

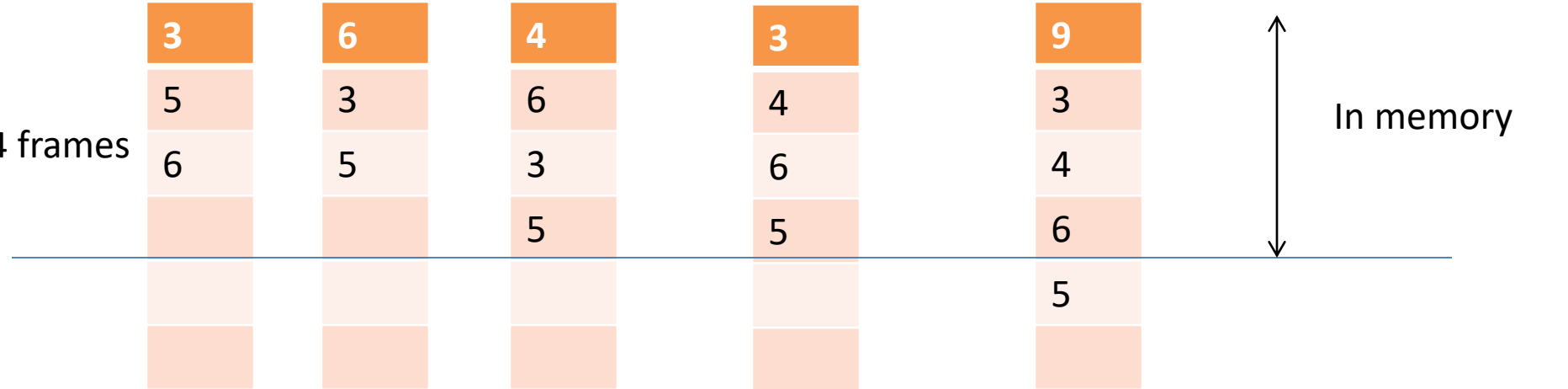
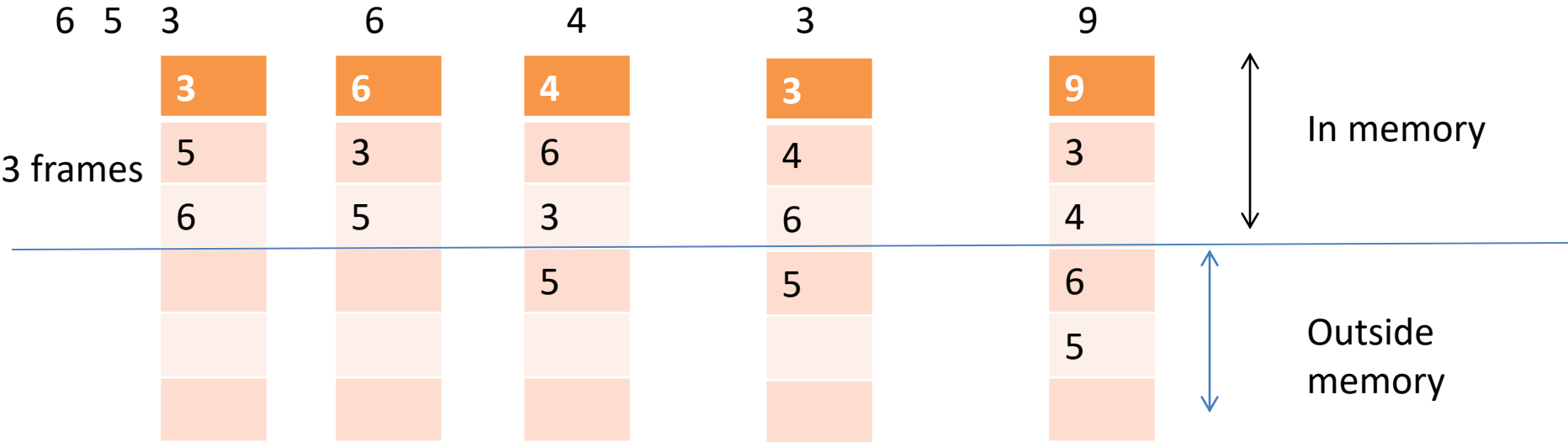
- This most unexpected result is known as **Belady's anomaly** – for some page-replacement algorithms, the page fault rate may **increase** as the number of allocated frames increases
- Is there a characterization of algorithms susceptible to **Belady's anomaly**?

# Stack Algorithms

- Certain page replacement algorithms are more “well behaved” than others
- There are a set of paging algorithms whereby the set of pages loaded with an allocation of  $m$  frames is always a subset of the set of pages loaded with an allocation of  $m + 1$  frames. This property is called the **inclusion property**
- Algorithms that satisfy the inclusion property are not subject to **Belady’s anomaly**. FIFO does not satisfy the inclusion property and is not a **stack algorithm**
- (In the FIFO example), the problem arises because the set of pages loaded with a memory allocation of 3 frames is not necessarily also loaded with a memory allocation of 4 frames



# LRU algorithm



# Page-Buffering

- Systems commonly keep a pool of free frames.
- When a page fault occurs, a victim frame is chosen as before.
- However, the desired page is read into a free frame from the pool before the victim is swapped out.
- This procedure allows the process to restart as soon as possible, without waiting
- When the victim is later swapped out, its frame is added to the free-frame pool.

# Page-Buffering

- Whenever the paging device is idle, a modified page is selected and is written to secondary storage.
- Its modify bit is then reset.
- This scheme increases the probability that a page will be clean when it is selected for replacement and will not need to be swapped out.

# Page-Buffering

- Another modification is to keep a pool of free frames
  - but to remember which page was in each frame.
- Since the frame contents are not modified when a frame is swapped to secondary storage,
  - the old page can be reused directly from the free-frame pool if it is needed before that frame is reused.
  - No I/O is needed
- When a page fault occurs, we first check whether the desired page is in the free-frame pool.

# Allocation of Frames

- How do we allocate the fixed amount of memory among various processes?
- Single user system
  - Trivial

# Allocation of Frames

- Each process needs *minimum* number of frames
- Minimum number is defined by the instruction set
- Page fault forces to restart the instruction
  - Enough frames to hold all the pages for that instruction
- Example:
  - Single address instruction (2 frames)
  - Two address instruction (3 frames)
  - Indirect Addressing mode
- ***Maximum*** of course is total frames in the system
- Two major allocation schemes
  - fixed allocation
  - proportional allocation

# Fixed and proportional Allocation

- Equal allocation –  $m$  frames and  $n$  processes
  - Each process gets  $m/n$
- For example, if there are 100 frames (after allocating frames for the OS) and 5 processes, give each process 20 frames
  - Keep some as free frame buffer pool
- Unfair for small and large sized processes
- Proportional allocation – Allocate according to the size of process –

– Dynamic as degree of multiprogramming, process sizes change

$s_i$  = size of process  $p_i$

$$S = \sum s_i$$

$m$  = total number of frames

$$a_i = \text{allocation for } p_i = \frac{s_i}{S} \times m$$

$$m = 64$$

$$s_1 = 10$$

$$s_2 = 127$$

$$a_1 = \frac{10}{137} \times 64 \approx 5$$

$$a_2 = \frac{127}{137} \times 64 \approx 59$$

# Priority Allocation

## Allocation of frames

- Depends on multiprogramming level
- Use a proportional allocation scheme using priorities along with size



# Global vs. Local Allocation

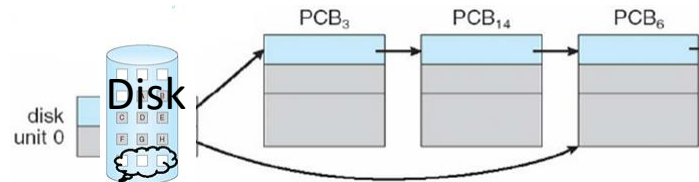
- Frames are allocated to various processes
- If process  $P_i$  generates a page fault
  - select for replacement one of its frames
  - select for replacement a frame from another process
- **Local replacement** – each process selects from only its own set of allocated frames
  - More consistent per-process performance
  - But possibly under-utilized memory
- **Global replacement** – process selects a replacement frame from the set of all frames; one process can take a frame from another
  - But then process execution time can vary greatly
  - But greater throughput ----- so more common
- Processes can not control its own page fault rate
  - Depends on the paging behavior of other processes

# Thrashing

- If a process uses a set of “active pages”
  - Number of allocated frames is less than that
- Page-fault
  - Replace some “active” page
  - But quickly need replaced “active” frame back
  - Quickly a page fault, again and again
  - **Thrashing**  $\equiv$  a process is busy swapping pages in and out
  
- OS monitors CPU utilization
  - If low? Increase the degree of multiprogramming

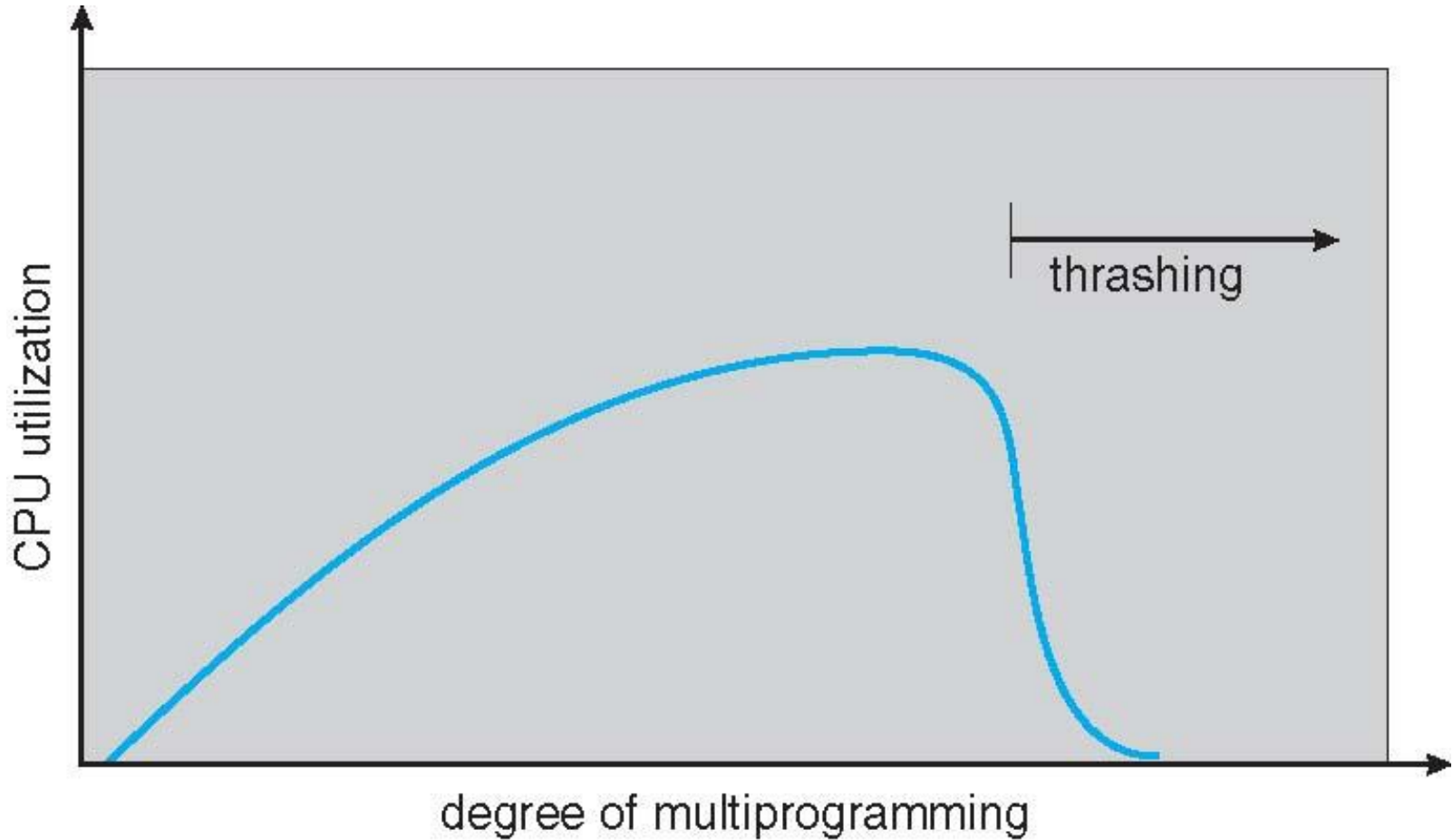
# Thrashing

- Global page replacement
  - Process enters new phase (subroutine call) execution
  - Page fault
  - Taking frames from other processes
    - Replace “active” frames of other processes
  - These processes start page fault
  - These faulting processes wait on the device queue for disk
    - Ready queue empty
  - CPU utilization decreases



- CPU scheduler increases the degree of multiprogramming
  - More page faults
  - Drop in CPU utilization
- Page fault increases tremendously

# Thrashing (Cont.)



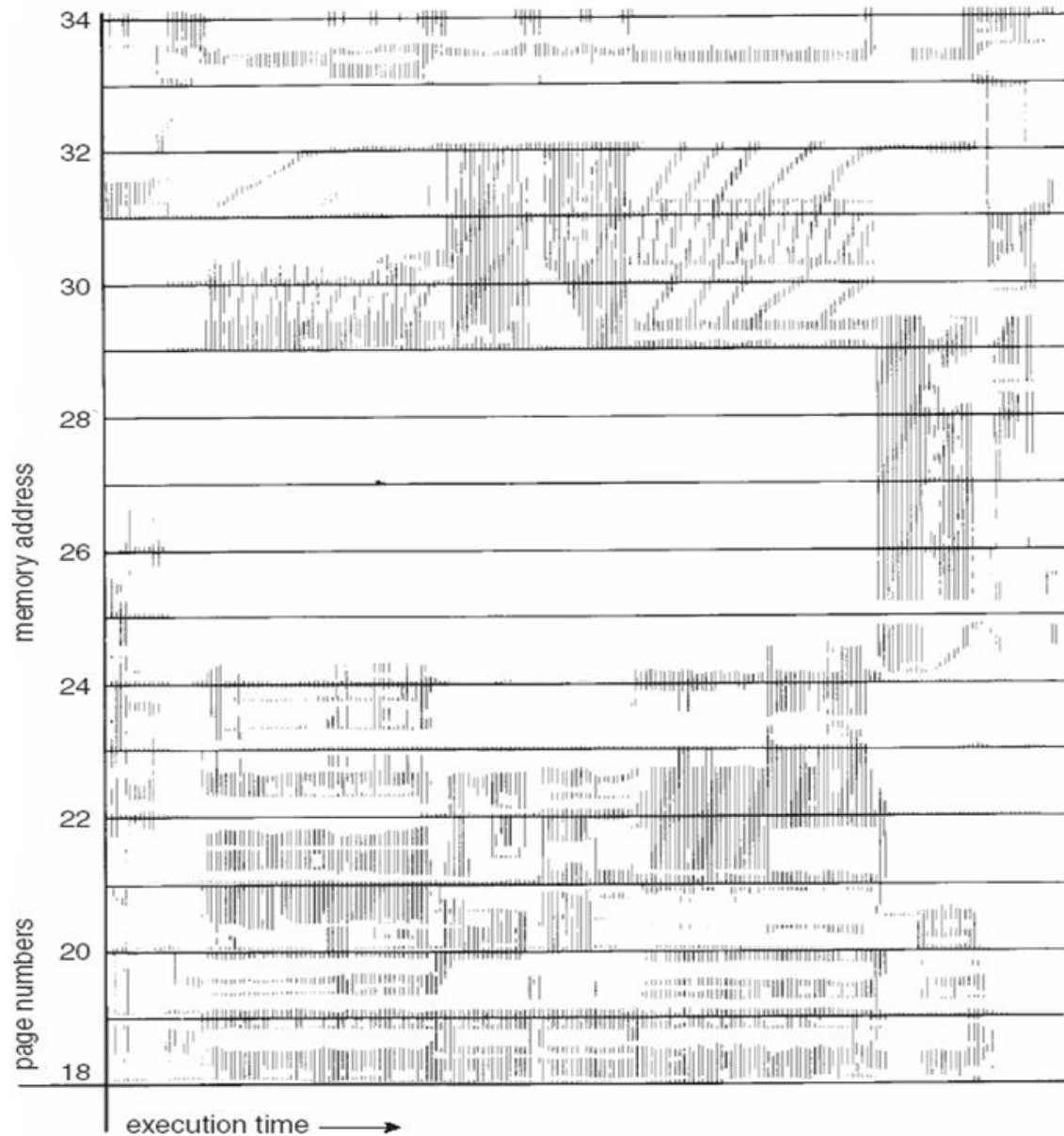
# Thrashing

- Solution
  - Local replacement
  - One process cannot steal frames from other processes
- Provide a process as many frames as needed
  - Able to load all active pages
  - How do we know?
  - Locality model

# Demand Paging and Thrashing

- Why does demand paging work?  
**Locality model**
  - Process migrates from one locality to another
  - Localities may overlap
- Allocate enough frames to a process to accommodate its locality
- Why does thrashing occur?  
 $\Sigma$  size of locality > total allocated frames
  - Limit effects by using local or priority page replacement

# Locality In A Memory-Reference Pattern



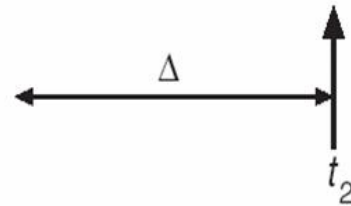
# Working-set model

page reference table

... 2 6 1 5 7 7 7 7 5 1 6 2 3 4 1 2 3 4 4 4 3 4 3 4 4 4 4 1 3 2 3 4 4 4 3 4 4 4 ...



$$WS(t_1) = \{1, 2, 5, 6, 7\}$$



$$WS(t_2) = \{3, 4\}$$



# Working-Set Model

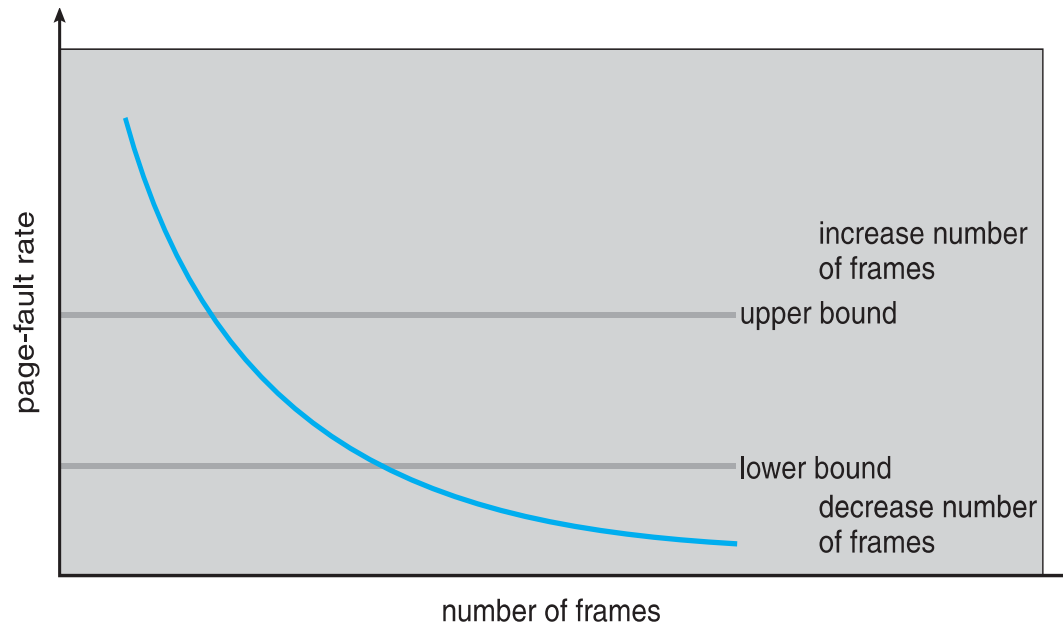
- $\Delta \equiv$  working-set window  $\equiv$  a fixed number of page references  
Example: 10,000 instructions
- $WSS_i$  (working set of Process  $P_i$ ) =  
total number of pages referenced in the most recent  $\Delta$  (varies in time)
  - if  $\Delta$  too small will not encompass entire locality
  - if  $\Delta$  too large will encompass several localities
  - if  $\Delta = \infty \Rightarrow$  will encompass entire program
- $D = \sum WSS_i \equiv$  total demand frames
  - Approximation of locality
- if  $D > m \Rightarrow$  Thrashing
- Policy if  $D > m$ , then suspend or swap out one of the processes

# Implementation of WS model

We can approximate the working-set model with a fixed-interval timer interrupt and a reference bit. For example, assume that  $\Delta$  equals 10,000 references and that we can cause a timer interrupt every 5,000 references. When we get a timer interrupt, we copy and clear the reference-bit values for each page. Thus, if a page fault occurs, we can examine the current reference bit and two in-memory bits to determine whether a page was used within the last 10,000 to 15,000 references. If it was used, at least one of these bits will be on. If it has not been used, these bits will be off. Pages with at least one bit on will be considered to be in the working set.

# Page-Fault Frequency

- More direct approach than WSS
- Establish “acceptable” **page-fault frequency (PFF)** rate and use local replacement policy
  - If actual rate too low, process loses frame
  - If actual rate too high, process gains frame



# Problem

A certain computer system has the segmented paging architecture for virtual memory. The memory is byte addressable. Both virtual and physical address spaces contain  $2^{16}$  bytes each. The virtual address space is divided into 8 non-overlapping equal size segments. The memory management unit (MMU) has a hardware segment table, each entry of which contains the physical address of the page table for the segment. Page tables are stored in the main memory and consists of 2 byte page table entries. What is the minimum page size in bytes so that the page table for a segment requires at most one page to store it?

- Let page size = n bytes.
- Now, since page table has to be stored into a single page, so we must have-
- Size of page table  $\leq$  Page size
- **Size of Each Segment-**
- Size of each segment  
 = Process size / Number of segments  
 =  $2^{16}$  bytes / 8  
 =  $2^{16}$  bytes /  $2^3$   
 =  $2^{13}$  bytes  
 = 8 KB

**Number of Pages of Each Segment-**

- Number of pages each segment is divided
- = Size of segment / Page size
- = 8 KB / n bytes
- = (8K / n) pages

## Size of Each Page Table-

Size of each page table

- = Number of entries in page table x Page table entry size
- = Number of pages the segment is divided x 2 bytes
- =  $(8K / n) \times 2$  bytes
- =  $(16K / n)$  bytes

## Page Size

- Substituting values in the above condition, we get-
- $(16K / n)$  bytes  $\leq n$  bytes
- $(16K / n) \leq n$
- $n^2 \geq 16K$
- $n^2 \geq 2^{14}$
- $n \geq 2^7$
  
- Thus, minimum page size possible =  $2^7$  bytes = 128 bytes.