

Graph Theory - Questions

Tutorial

1. Suppose a simple graph G on n vertices is isomorphic to its complement $\overline{G}$. Show that

$$n \equiv 0 \text{ or } 1 \pmod{4}.$$

2. Let A be the adjacency matrix of a simple graph G . Show that $(A^k)_{ij}$ equals the number of walks of length k from v_i to v_j . Deduce that the number of triangles in G is

$$\frac{\text{tr}(A^3)}{6}.$$

3. Let G be a connected graph and let $e \in E(G)$. Prove that e is a cut-edge if and only if every closed walk in G uses e an even number of times.
4. Show that every graph G has a spanning bipartite subgraph containing at least half of the edges of G .
5. Let G have three independent vertex classes A, B, C , each of size n . Suppose every vertex has degree at least $n + 1$. Prove that G contains a triangle.

Assignment

1. Among any six people, prove that there are either three people who pairwise know one another or three people who pairwise do not know one another.
2. Show that a graph in which every vertex has even degree has no cut-edge.
3. Let G be a connected graph with exactly two odd-degree vertices u and v . Prove that every cut-edge of G separates u and v .
4. Find the maximum possible number of edges in a simple bipartite graph on n vertices.
5. Suppose G is a graph such that $G - v$ is bipartite for every vertex $v \in V(G)$. Prove that either G is bipartite or G is an odd cycle.