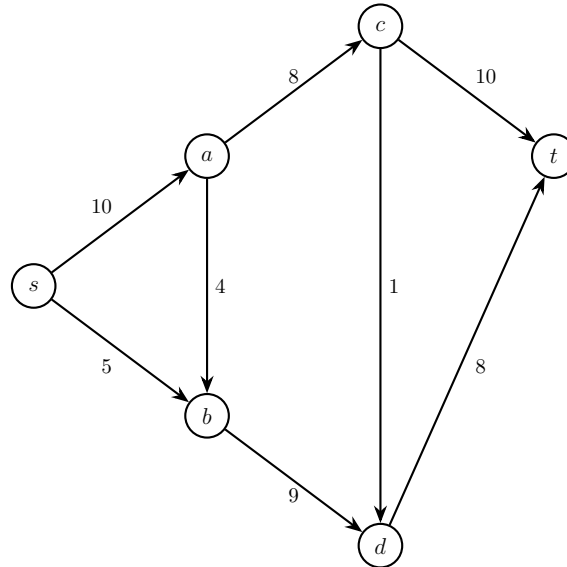


Tutorial: Flows and Cuts

1. **Compute Max Flow.** Given a network with source s , sink t , and capacities: $s \rightarrow a$: 10, $s \rightarrow b$: 5, $a \rightarrow b$: 4, $a \rightarrow c$: 8, $b \rightarrow d$: 9, $c \rightarrow t$: 10, $c \rightarrow d$: 1, $d \rightarrow t$: 8. Find the maximum flow from s to t , showing at least two augmenting paths.



2. **Compute a Cut's Capacity.** Using the network from Problem 1, let $S = \{s, a, b\}$, $T = \{c, d, t\}$. Compute $c(S, T)$. Then pick a different partition $S' \ni s, t \notin S'$, compute $c(S', T')$, and compare both to the max flow found above.
3. **No Augmenting Path $\Rightarrow$ Max Flow.** Let f be a flow with no augmenting path from s to t in the residual graph G_f . Prove f is a maximum flow.
4. **Flow Value $\leq$ Cut Capacity.** Let f be any flow and (S, T) any s - t cut. Prove $|f| \leq c(S, T)$.
5. **Max-Flow Min-Cut Theorem.** Using Problems 3 and 4, prove: the maximum flow value equals the minimum cut capacity.
6. **Verify Feasibility & Find Residual Capacities.** Consider a network with edges and capacities: $s \rightarrow a$ (7), $s \rightarrow b$ (6), $a \rightarrow t$ (5), $a \rightarrow b$ (3), $b \rightarrow t$ (8). A proposed flow assigns: $f(s, a) = 5$, $f(s, b) = 6$, $f(a, t) = 4$, $f(a, b) = 1$, $f(b, t) = 7$. Check whether this is a feasible flow (capacity constraints and flow conservation at a and b). Then compute the residual capacity of every edge (forward and backward) in G_f .

