Assignment

1. Let G be a connected graph. Prove that G is Eulerian if and only if, for every nonempty proper subset
$$S \subset V(G),$$
the number of edges having exactly one endpoint in S is even.

2. Let
$$d_1 \geq d_2 \geq \cdots \geq d_n$$
be a graphic sequence satisfying
$$d_1 = d_2 + d_3 + \cdots + d_n.$$
Determine the form of the sequence and characterize all its realizations.

3. Let
$$V(G) = \{A, B, C, D, E, F\}$$
and
$$E(G) = \{AB, BC, CA, AD, DE, EA, CF, FD, DC\}.$$
Determine whether G has an Euler circuit or an Euler trail. If applicable, apply Fleury's algorithm, indicating at each step which available edges are bridges in the remaining graph, and produce the final traversal.

4. Determine whether the sequence
$$(6, 5, 5, 4, 3, 3, 2, 2)$$
is graphic. Show every Havel–Hakimi step. If it is graphic, construct one realization.

5. Let G be connected. Prove that
$$e \text{ belongs to every spanning tree of } G \quad \Longleftrightarrow \quad e \text{ is a cut-edge.}$$

6. Reconstruct the tree on $[9]$ whose Prüfer sequence is
$$(4, 4, 7, 2, 7, 2, 5).$$
Then determine its degree sequence and its leaves.

7. For the labelled tree
$$E(T) = \{12, 24, 25, 36, 46, 47, 58, 59\},$$
find its Prüfer sequence and verify that each vertex v occurs exactly
$$d(v) - 1$$
times.

8. Let P, Q be fixed labelled sets with
$$|P| = p, \qquad |Q| = q.$$
Prove that the number of labelled trees whose bipartition is exactly (P, Q) is
$$p^{q-1} q^{p-1}.$$