Solution 1: If a and b are nonzero rational numbers, then ab is a nonzero rational number, and so is ab/2, showing that the operation is closed on the set G. The operation is associative since
a * (b * c) = a * (bc/2) = a (bc/2)/2 = a(bc)/4
and
(a * b) * c = (ab/2) * c = (ab/2)c/2 = (ab)c/4.
The number 2 acts as the multiplicative identity, and if a is nonzero, then 4/a is a nonzero rational number that serves as the multiplicative inverse of a, since
a * (4/a) = (4a)/(2a) = 2.

Solution 2: Assume that a and b are elements of G for which (ab)2 = a2 b2. Expanding this equation gives us
(ab)(ab) = a2 b2 .
Since G is a group, both a and b have inverses, denoted by a-1 and b-1, respectively. Multiplication in G is well-defined, so we can multiply both sides of the equation on the left by a-1 without destroying the equality.

If we are to be precise about using the associative law, we have to include the following steps.

a-1 ((ab)(ab)) = a-1 (a2 b2)
(a-1 (ab)) (ab) = (a-1 a2) b2
((a-1 a) b)) (ab) = ((a-1 a) a) b2
(e b) (ab) = (e a) b2
b (ab) = a b2

The next step is to multiply on the right by b-1. The associative law for multiplication essentially says that parentheses don't matter, so we don't really need to include all of the steps we showed before.

b (ab) b-1 = (a b2) b-1
(ba)(b b-1) = (a b)(b b-1)
ba = ab

This completes the proof, since we have shown that if (ab)2 = a2 b2, then ba = ab.

Solution 3: the generators correspond to the numbers less than 28 and relatively prime to 28. The Euler -function allows us to compute how many there are:
(28) = (1/2) · (6/7) · 28 = 12.
The list of generators is { ± 1, ± 3, ± 5, ± 9, ± 11, ± 13 }.

Solution 4 :
(a) Show that the operation * is closed on G.
Solution: If a,b in G, then a>1 and b>1, so b-1>0, and therefore a(b-1)>(b-1). It follows immediately that ab-a-b+2>1.

(b) Show that the associative law holds for *.

Solution: For a,b,c in G, we have

a * (b * c) = a * (bc-b-c+2)
= a(bc-b-c+2) -a -(bc-b-c +2) +2
= abc -ab -ac -bc +a +b +c .

On the other hand, we have

(a * b) * c = (ab-a-b+2) * c
= (ab-a-b+2)c - (ab-a-b+2) -c +2
= abc -ab -ac -bc +a +b +c .

Thus a * (b * c) = (a * b) * c.

(c) Show that 2 is the identity element for the operation *.

Solution: Since the operation is commutative, the one computation
2 * y = 2y - 2 - y + 2 = y suffices to show that 2 is the identity element.

(d) Show that for element a in G there exists an inverse a-1 in G.

Solution: Given any a in G, we need to solve a * y = 2. This gives us the equation
ay - a - y + 2 = 2, which has the solution y = a / (a-1). This solution belongs to G since
a > a - 1 implies a / (a-1) > 1. Finally,

a * (a / a-1) = a2 / (a-1) - a - a / (a-1) + 2
= (a2 -a2 + a - a) / (a-1) + 2 = 2.

Solution 5: The function μ preserves multiplication in R× since for all a,b in R× we have
μ (ab) = (ab)3 = a3 b3 = μ (a) μ (b).
The function is one-to-one and onto since for each y in R× the equation μ (x) = y has the unique solution .