Q1) On the set $G = Q^{\times}$ of nonzero rational numbers, define a new multiplication by

$$a * b = ab/2, \qquad \text{for all } a,b \text{ in } G.$$

Show that G is a group under this multiplication.

Q2)Let G be a group, and suppose that a and b are any elements of G. Show that if (ab)2 = a2 b2, then ba = ab.

Q3) Find all generators of the cyclic group Z28.

Note:

Let a and n > 0 be integers. The set of all integers which have the same remainder as a when divided by n is called the congruence class of a modulo n, and is denoted by [a]n, where

[a]n = { x in Z | x is congruent to a (mod n) }.

The collection of all congruence classes modulo n is called the set of integers modulo n, denoted by Zn.

Q4) Let G = { x in R | x > 1 } be the set of all real numbers greater than 1. Define

x * y = xy - x - y + 2,   for x, y in G.

(a) Show that the operation * is closed on G.
(b) Show that the associative law holds for *.
(c) Show that 2 is the identity element for the operation *.
(d) Show that for element a in G there exists an inverse a-1 in G.

Q5) Let μ : $R^{\times}$ -> $R^{\times}$ be defined by μ (x) = x3, for all x in R. Show that μ is a group isomorphism.