Tutorial

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1. Consider the set Q of rational numbers and let * be an operation on Q defined by a * b = a + b - ab

a> Show that Q is a commutative semigroup.
b> Find the identity element for *
c> Do any element of Q has an inverse ? What is it ?

2. Let S be a semigroup with identity e and let b and b′ be inverses of a . Show that b = b′

3. Let S = N x N and let * be an operation on S defined by
(a,b) * (a′,b′) = (a+a′,b+b′)

a> Show that S is a semigroup
b> define f : (S,*) → (Z,+) by f(a,b) = a - b . Show that f is a homomorphism
c> Find the congruence relation ~ on S determined by the homomorphism f , i.e , x~y if f(x) = f(y)

4. Prove that an infinite group has infinitely many subgroups.

5. Prove that every subgroup of a cyclic group G is cyclic

6. Suppose f : G → G′ is a group homomorphism. Prove that
a> f (e) = e ′
b> $f(a^{-1}) = f(a)^{-1}$

7. Let G be a finite group of order n. Show that $a^n = e$ *for* any element $a \in G$