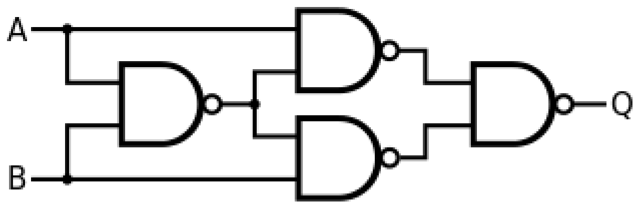
Solution Tutorial -7

1. Both *a* and *b* are contingency.
2. Converse: If the Indian team wins, then the match is played in Kolkata, hometown of Sourav Ganguly.
   Inverse: If match is not played in Kolkata, hometown of Sourav Ganguly, then the Indian team does not win.
   Contrapositive: If the Indian team does not win, then the match is not played in Kolkata, hometown of Sourav Ganguly.
3. Solutions:
   a. False. Not true for negative numbers.
      $\exists x, |x| \neq x$
   b. True. Take x = 1.
      $\forall x, x^2 \neq x$
   c. True.
      $\exists x, x + 1 \ngtr x$
   d. False. No such x exists.
      $\forall x, x + 1 \neq x$
4. Solutions:
   a. NAND is a universal gate. So we can use NAND gates alone to implement XOR gate.
      XOR(p,q) = NAND(NAND(p, NAND(p,q)) , NAND(q, NAND(p,q)))



   b. NAND gate cannot be implemented using only XOR gates.
5. Solutions:
   a. $(\sim p \land q)$
   b. $(p \land \sim q) \lor (p \land \sim r)$
6. Solutions:
   a. $(p \lor p) \land (q \lor q) \land (\sim r \lor \sim r)$
   b. $(q \lor q) \land (p \lor r)$
7. Solutions:
   a. Using Hypothetical Syllogism successively, we can infer the result.
   b. From $\sim p \land r$, we can say that p is not true. Using Disjunctive Syllogism, we can say that $(q \rightarrow p)$, is true. Now using Modus Tollens, we can infer $\sim q$.
8. Using $\sim(q \land r)$, we can say that at least one of q and r is False. Using Modus Tollens, we infer ~p. Using Disjunctive syllogism, we infer s.
9. Using $\sim(r \lor q)$, we can say that both r and q are False. Using Modus Tollens, we infer ~s. Again using Modus Tollens ,we infer $\sim(p \lor q)$. We know that q is false. Therefore, p must be false. So we infer ~p.
10. Solutions:
    a. $\forall x, F(x) \rightarrow \sim P(x)$
    b. $\exists x, Real(x) \land Rational(x)$
    c. $\forall x, (Tiger(x) \lor Lion(x)) \rightarrow [(Hungry(x) \lor Threatened(x)) \rightarrow Attack(x)]$
    d. $\forall x, Teacher(x) \rightarrow [\exists y, Student(y) \land Like(y,x)]$