Tutorial - 7

1. Construct truth tables for the following. Also classify each as tautology, contingency and contradiction.
   a. $(p \wedge \sim q) \vee (\sim p \wedge q)$
   b. $\sim(p \wedge q) \vee (\sim p \wedge \sim q)$
2. Write the contrapositive, converse and inverse of the following:
   "Indian team win whenever match is played in Kolkata, home town of Sourav Ganguly."
3. Determine the truth value of the following statements, taking the set of real numbers the universal set. Also, negate each of the statement.
   a. $\forall x, |x| = x$
   b. $\exists x, x^2 = x$
   c. $\forall x, x + 1 > x$
   d. $\exists x, x + 1 = x$
4. Let $\text{NAND}(p,q) = \sim(p \wedge q)$, $\text{XOR}(p,q) = (p \wedge \sim q) \vee (\sim p \wedge q)$.
   a. Can we express XOR using only NAND.
   b. Can we express NAND using only XOR.
5. Find the DNF of the following forms:
   a. $(p \rightarrow q) \wedge (\sim p \wedge q)$
   b. $\sim(p \rightarrow (q \wedge r))$
6. Find the CNF of the following forms:
   a. $\sim(p \rightarrow r) \wedge (p \leftrightarrow q)$
   b. $(p \wedge q) \vee (\sim p \wedge q \wedge r)$
7. Prove that the following arguments are valid without using truth tables:
   a. $p \rightarrow (q \vee r)$, $(s \wedge t) \rightarrow q$, $(q \vee r) \rightarrow (s \wedge t)$ infer $p \rightarrow q$
   b. $p \vee (q \rightarrow p)$, $\sim p \wedge r$ infer $\sim q$
8. Show that *s* is a valid conclusion from the premises: $p \rightarrow q$, $p \rightarrow r$, $\sim(q \wedge r)$, $s \vee p$
9. Show that *~p* is a valid conclusion from the premises: $(p \vee q) \rightarrow s$, $s \rightarrow r$, $\sim(r \vee q)$
10. Use predicate calculus to express the following:
    a. None of my friends are perfect.
    b. Some real numbers are rational.
    c. Tigers and lions attack if they are hungry or threatened.
    d. Every teacher is liked by some student.