Tutorial-6

1. Solve these recurrence relations together with the initial conditions given.
   I. $a_n = 5a_{n-1} - 6a_{n-2}$ for $n>=2$, $a_0 = 1$, $a_1 = 0$
   II. $a_n = -4a_{n-1} - 4a_{n-2}$ for $n>=2$, $a_0 = 0$, $a_1 = 4$
2. How many different messages can be transmitted in n microseconds using three different signal if one signal requires 1 microsecond for transmittal, the other two signals require 2 microseconds each for transmittal, and a signal in message is followed immediately by the next signal?
3. A new employee at an exciting new software company starts with a salary of $50,000 and is promised that at the end of each year her salary will be double her salary of the previous year, with an extra increment of $10,000 for each year she has been with the company.
   a) Construct a recurrence relation for her salary for her nth year of employment.
   b) Solve this recurrence relation to find her salary for her nth year of employment.
4. Let $\mathbf{a_n}$ be the sum of first n perfect squares, that is , $\sum_{k=1}^{n} k^2$. Show that the sequence {an} satisfies the linear nonhomogeneous recurrence relation $a_n = a_{n-1} + n^2$ and the initial condition $a_1 = 1$.

5. Use generating functions to solve the recurrence relation $a_k = 5a_{k-1} - 6a_{k-2}$ with initial conditions $a_0 = 6$ and $a_1 = 30$.
6. Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.
   I. Show that $a_n = -2^{n+1}$ is a solution of this recurrence relation.
   II. Find all the solutions of this recurrence relation.
   III. Find the solution with $a_0 = 1$
7. Suppose that a valid codeword is an n digit number in decimal; notation containing an even number of 0s. Let $\mathbf{a_n}$ denote the number of valid codeword of length n. Find the recurrence relation of $\mathbf{a_n}$. Use generating functions to find an explicit formula for $\mathbf{a_n}$.
8. For the recurrence given bellow answer the following question
$$T(n) = 7T\left(\frac{n}{2}\right) - 6T\left(\frac{n}{4}\right), T(1) = 2, T(2) = 7$$
   I. Does recurrence for T(n) involve previous terms which are within a fixed range of n?
   II. If not , use the domain transform such that for given terms T(f(n)), T(f(f(n)))… , select a function g(m) which hold the property f(g(m)) = g(m-1).
   III. Solve the transformed recurrence with initial conditions written above.
   IV. Find the solution of original recurrence using inverse transformation.
9. For the recurrence given bellow , answer the following question
$$T(2) = 2.\frac{T(n-1)^3}{T(n-2)^2}, T(0) = 2, T(1) = 2$$
   I. Is the recurrence linear?
   II. If not, apply the range transformations to the recurrence to make it linear. Idea is for given relation for T(n) in terms of T(n-1) , T(n-2),……., T(n-k), find a function f(x) such that f(T(n)) is a linear combination of f(T(n-1)),……, f(T(n-k)).
   III. Solve the transformed recurrence with initial condition written above.
   IV. Find the solution of original recurrence using the inverse transformation.