

Ans 1)

All primes b/w 100 and 200 and powers of 2,5,11,13,17 and then $3 \cdot 61$ and $7 \cdot 23$.

Ans 2)

No. Since each domino covers one black and 1 white square and the number of white square or black square will be more.

Ans 3)

Given a run of $2n$ consecutive integers: $a + 1, a + 2, \dots, a + 2n - 1, a + 2n$, there are n pairs of numbers that differ by n : $(a+1, a+n+1), (a + 2, a + n + 2), \dots, (a + n, a + 2n)$. Therefore, by the Pigeonhole Principle, if one selects more than n numbers from the set, two are liable to belong to the same pair that differ by n .

Ans 4)

If there are N people in the room and each has a different number of acquaintances then one is bound to have $N - 1$ and one 0 acquaintances. This is a contradiction.

Ans 5)

Let a_1 be the number of games played on the first day, a_2 the total number of games played on the first and second days, a_3 the total number games played on the first, second, and third days, and so on. Since at least one game is played each day, the sequence of numbers a_1, a_2, \dots, a_{77} is strictly increasing, that is, $a_1 < a_2 < \dots < a_{77}$. Moreover, $a_1 \leq 1$; and since at most 12 games are played during any one week, $a_{77} \leq 12 \times 11 = 132$. Thus

$1 \leq a_1 < a_2 < \dots < a_{77} \leq 132$:

Note that the sequence $a_1 + 21, a_2 + 21, \dots, a_{77} + 21$ is also strictly increasing, and

$22 \leq a_1 + 21 < a_2 + 21 < \dots < a_{77} + 21 \leq 132 + 21 = 153$.

Now consider the 154 numbers

$a_1, a_2, \dots, a_{77}, a_1 + 21, a_2 + 21, \dots, a_{77} + 21$.

each of them is between 1 and 153. It follows that two of them must be equal. Since a_1, a_2, \dots, a_{77} are distinct and $a_1 + 21, a_2 + 21, \dots, a_{77} + 21$ are also distinct, then the two equal numbers must be of the forms a_i and $a_j + 21$. Since the number games played up to the i th day is $a_i = a_j + 21$, we conclude that on the days $j + 1, j + 2, \dots, i$ the chess master played a total of 21 games.

Ans 6)

Simple question on de arrangement of numbers.(Basic Permutation and combination)