Assignment 6

(1) A 2xn grid is given. In how many ways you can cover it using 1x2 dominoes ?

(2) Show that every rational number written in decimal from will either have terminating or periodic decimal digits.

(3) In a party n people were there. Some of these people did handshakes with each others. Prove that there are at least 2 people who did handshakes same number of times.

(4) Prove the binomial theorem for natural index using only probability theory. (Hint: tossing a biased coin)

(5) Using generating functions solve the following problem:
In how many ways you can distribute m coins among n persons ?

(6) Prove the following identity in the most elegant method(no calculus or algebra)
nC1*1 + nC2*2 + … nCn * n = n *2^(n-1)

(7) How many strings of [a,b,c] are there such that it contains no two consecutive pairs of same character ?

(8) If p is a prime number then prove that p divides a^p - a (*trust me It can be solved by combinatorics, try at home or check from net if unable to do*)

(9) How many n-ary trees (every node is either a leaf or has exactly n children) are there of height d ?

(10) For any given integer prove that given a sequence of n^2+1 numbers, there exist a sub sequence of monotonically increasing numbers or monotonically decreasing numbers of length n+1.

(11) Prove that among any set of 51 positive integers less than 100, there is a pair whose sum is 100

(12) A chess master who has 11 weeks to prepare for a tournament decides to play at least one game every day but, in order not to tire himself, he decides not to play more than 12 games during any calendar week. Show that there exists a succession of consecutive days during which the chess master will have played exactly 21 games.