

Discrete Structures Tutorial 4

1) The harmonic numbers $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{j}$

For example $H_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$

Use mathematical induction to show that

$$H_{2^n} \geq 1 + \frac{n}{2} \quad \forall n \in \mathbb{N}$$

2) Let n be a positive integer. Show that every $2^n \times 2^n$ checkerboard with one square removed can be tiled using right triominoes, where these pieces cover three squares at a time as shown in figure.



3) Prove that $\sqrt{5}$ is not a rational number.

4) Prove that if n is a positive integer with $n^2 > 100$ then $n > 10$

5) Prove that if n is a positive integer then $n^2 + 3n + 2$ is even

6) Prove that a set with n elements has 2^n subsets