Assignment 2 —— Discrete Structures

1. Prove that the relation defined by $R = \{(x, y)|2x + y$ is divided by $3\}$ is an equivalence relation.
2. Is the relation defined as $R = \{(x, y)|x - y$ is an odd integer $\}$ where x, y are integers an equivalence relation - give proper justification ?
3. Let G be the digraph representation of a relation R on set S where $a, b \in S$,Prove that there is a path between a, b of length $n \iff$ (a,b) $\in R^n$
4. Let A = {0, 1, 2, 3} and let R = {(0,1), (1,1), (1,2), (2,0), (2,2), (3,0)} be a relation on R. Find the Reflexive, Symmetric and Transitive Closure of R.
5. Given a directed graph, find whether it is possible to reach x from y, where x and y are nodes of the graph. Can you extend this to find the smallest cost of the path (assume every edge cost is ≥ 0)?

6. If R and S are equivalence relations on a set A , then the smallest equivalence relation containing both R and S is $(R \cup S)^{\text{inf}}$where $B^{\text{inf}} = B^1 \cup B^2 \cup ...$