Answers Tutorial 1

a) $x \in (A \cup B)' \Leftrightarrow x \notin A \cup B \Leftrightarrow x \notin A\ and\ x \notin B \Leftrightarrow A \in A'\ and\ x \in B' \Leftrightarrow x \in A' \cap B'$

b) Let $S = \{a, b, c, d\}$ Let $R_1 = \{(a, b), (b, c), (a, c)\}$, $R_2 = \{(b, c), (c, d), (b, d)\}$ be two transitive relations on S Consider $R_1 \cup R_2$, $(a, b) \in R_1 \cup R_2\ and\ (b, d) \in R_1 \cup R_2$ however $(a, d) \notin R_1 \cup R_2$ hence it is not transitive

c) Let R , S be a relation on a set of natural numbers $\mathbb{N}$ a R b $\Rightarrow a < b$ and a S b $\Rightarrow a > b$, Both R and S are antisymmetric relations . Then $R \cup S = \{(a, b) : a \neq b\}$ This relation is cannot be antisymmetric

d) According to definition of R , $x \equiv (b\, mod\, n)\ if\ (a-b)/n$, $(x-x)/n\ \forall x \in \mathbb{Z}$ hence R is reflexive ; if $(x-y)$ is divisible by n then $(y-x)$ is also divisible by n , hence R is symmetric . Since $(x, y) \wedge (y, x) \in R \mid x \neq y$ the relation is not antisymmetric . If $(x, y) \in R \wedge (y, z) \in R \Rightarrow x - y + y - z = x - z$ is also divisible by n $\rightarrow (x, z) \in R$ hence R is transitive .Thus R is a equivalence relation.

e) $(A \cup B) - (C - A)$

$\Rightarrow \quad (A \cup B) \cap (C - A)'$

$\Rightarrow \quad (A \cup B) \cap (C \cap A')'$

$\Rightarrow \quad (A \cup B) \cap (C' \cup A)$

$\Rightarrow \quad (A \cup B) \cap (A \cup C')$

$\Rightarrow \quad A \cup (B \cap C')$

$\Rightarrow \quad A \cup (B - C)$

f) The distance of any point in a set from itself is zero i.e. less than 1 hence R is reflexive. If $a_1 R a_2$ exist then $a_2 R a_1$ also exist because they both are less than 1 , hence R is symmetric.If $a_1 R a_2$ exist and $a_2 R a_3$ exist $\not\Rightarrow a_1 R a_3$ because distance between $a_1\ and\ a_3$ need not be less than 1 , hence R is not transitive .