

Tutorial 10

1. Prove that the sum of the degrees of the vertices of any finite graph is even.
2. Prove that a complete graph with n vertices contains $n(n - 1)/2$ edges.
3. Prove that a finite graph is bipartite if and only if it contains no cycles of odd length.
4. Prove that if u is a vertex of odd degree in a graph, then there exists a path from u to another vertex v of the graph where v also has odd degree.
5. Show that any graph where the degree of every vertex is even has an Eulerian cycle. Show that if there are exactly two vertices a and b of odd degree, there is an Eulerian path from a to b . Show that if there are more than two vertices of odd degree, it is impossible to construct an Eulerian path.
6. Show that in a directed graph where every vertex has the same number of incoming as outgoing paths there exists an Eulerian path for the graph.
7. An n -cube is a cube in n dimensions. A cube in one dimension is a line segment; in two dimensions, it's a square, in three, a normal cube, and in general, to go to the next dimension, a copy of the cube is made and all corresponding vertices are connected. If we consider the cube to be composed of the vertices and edges only, show that every n -cube has a Hamiltonian circuit.
8. Show that any tree with at least two vertices is bipartite.
9. Every tree with $n \geq 2$ vertices is 2-chromatic.
10. A graph is bicolourable (2-chromatic) if and only if it has no odd cycles.