1) Let G be a group, and let $H_1 \leq G$ and $H_2 \leq G$ be subgroups of G.

(a) Prove that the intersection $H1 \cap H2$ is a subgroup of G.

(b) Give an example to show that the union $H1 \cup H2$ of two subgroups need not be a subgroup of G.

2) For each of the following statements, either provide an example with the given property, or explain why no such example exists.

(a) A one-to-one function from the set {1, 2, 3, 4} to itself which is not onto.

(b) A non-cyclic group in which every proper subgroup is cyclic.

(c) An equivalence relation on $R-\{0\}$ for which every equivalence class contains exactly $\sim$ two elements.

3) Let $\sigma \in S_9$ be the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 7 & 9 & 4 & 8 & 1 & 6 & 2 \end{pmatrix}$$

a) Write σ as a disjoint union of cycles.

b) Write σ as a product of transpositions.

c) Is the element σ odd or even?

4) A group G is called divisible if given any $x \in G$ and any $n \in Z^+$ there exists a $y \in G$ so that $y^n = x$. Show that a cyclic group is never divisible.

5) Prove that every subgroup of a cyclic group is cyclic.