Discrete Structures - Tutorial 9

Error Correcting Codes

1. An encoding function $E : Z_2^2 \to Z_2^5$ is given by the generator matrix

$$G = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Determine all the code words. What can be said about the error detection capability of this code?

Soln: $E(0\ 0) = [0\ 0]G = [0\ 0\ 0\ 0\ 0]$.
Similarly, $E(0\ 1) = [0\ 1\ 0\ 1\ 1]$, $E(1\ 0) = [1\ 0\ 1\ 1\ 0]$, $E(1\ 1) = [1\ 1\ 1\ 0\ 1]$.

From these, we find that
$d(E(00), E(01)) = 3$, $d(E(00), E(10)) = 3$, $d(E(00), E(11)) = 4$,
$d(E(01), E(10)) = 4$, $d(E(01), E(11)) = 3$, $d(E(10), E(11)) = 3$.
Thus $min(E) = 3$. Therefore, the code can detect all errors of weight ≤ 2.

Ring Theory

2. Show that if x and y are members of a ring R then $(-x)y = -xy$ and $(-x)(-y) = xy$.

Soln: The first problem is to show that $(-x)y$ is the additive inverse of xy; that is, we have to show that $(-x)y + xy$ is 0. We have
$((-x)y + xy) = (-x + x)y$ (distributive law)
$= 0y$
$= 0$.

The second property can be deduced from the first, as follows. Replacing y by $-y$ we have $(-x)(-y) = -(x(-y))$. Using the commutative property, $-(x(-y)) = -((-y)x)$. Using the first property with x and y interchanged, this is equal to $-(-yx)$, which is $yx = xy$.

3. Show that if x is an element of a ring R and u, v are elements of R such that $ux = xu = 1, vx = xv = 1$, then $u = v$.

Soln: This is a consequence of the associative law. The neatest way to set out the argument is follows.
$u = u1 = u(xv) = (ux)v = 1v = v$.
(The result says that the inverse of x is unique.