1. Each of the co-ordinates are odd or even. So if we work mod 2, then each of them are 0 or 1. Now there are 8 such distinct possibilities. But as we took 9 points two points must be same mod 2. Hence their mid point will always be a integer valued.
2. We will prove by induction. Let us fix a direction clockwise. For 2 cars its trivial and let it be the base case. Now assume its true in clockwise for n cars. Now suppose we have n+1 cars. If all the cars can reach the NEXT car in clockwise given the fuel configuration then the trip exists. Now suppose that a car has <= fuel to reach the next car. Now remove this car from the problem and also shorten the path to x length after the removed car(say i th car, x is the path the car i can go before reaching his next neighbour). Now together rest of the cars have >= fuel to cover the whole trip and by induction hypothesis such a trip exists. so now in the shortened circle a car goes from i-1 to i+1. Now if we increase the path there by x amount and also give the fuel in the position of car i, then still the car does not run out of fuel. Hence the from the trip of n, we can create a trip of n+1. Hence such a trip always exists.
3. Two sides are independent. Let the number be Fn. Now if a house exists on first plot the required number is Fn-2. Otherwise its Fn-1. Hence Fn = Fn-1 + Fn-2, Hence the solution is (Fn)^2.
4. Examples are fine for this problem. However following is a constructive method to find chains and anti chains together in a Bipartite Graph.

   For a partial order relation $\leq$ on a set $S = \{a1, a2, . . . , an\}$,
   we define a bipartite graph $G = (U \cup V, E)$, where $|U| = |V| = n$, such that $(ui, vj) \in E$ if and only if $ai \leq aj$, where $i != j$.
   If $A \subseteq S$ forms a chain, then for each pair $(ai, aj) \in A$ such that $ai \leq aj$, the graph G contains an edge (ui, vj).
   In general, for each $ak \in A$, the following set of edges get added to the graph G:

   $E_{S,k} = U_{aj \in A} (\{(uk, vj) : (ak \leq aj) \wedge (j != k)\})$ [U means union]

   Thus, the total set of edges that get added to the graph due to the chain S is as follows:
   $E_S = U_{ak \in A} (E_{S,k})$
   Now, out of all the edges in E_S, let us consider the following subset of edges:
   $E'_S = \{(ui, vj) \in E_S : (\text{there is no } ak \in S) : ((ai \leq ak) \wedge (ak \leq aj))\}$
   So, E'_S is basically the set of edges added due to the relation between only consecutive elements in the chain S. Notice that E'_S forms a matching of size $|A| -1$ in G. . Thus, every chain $A \subseteq S$ contributes a corresponding matching of size $|A| - 1$ to the graph G
   If C is a chain cover of size l, then as each of the chain in disjoint they total correspond to a matching of size n-l. Therefore, if the size of the smallest chain cover is s, then the graph G must have a corresponding maximum matching of size n – s. If the size of the maximum matching in the bipartite graph G is n – s, then by Konig's theorem, we know that the size of the minimum vertex cover in G is also n – s. Thus, the size of the maximum independent set I in G should be $|V| - (n - s) = 2n - (n - s) = n + s$. By pigeonhole principle, this implies that there exists a subset of elements $S' \subseteq S$, where $|S'| = (n+s)-n = s$, such that for each $ai \in S'$, both the vertices ui and vi are part of the maximum independent set I in G. Now this S' forms the largest anti chain. For more on this topic please refer to Konig Theorem and Dilworth's theorem.
5. Multiply (1+x)^m and (1+x)^n and equate it to (1+x)^(m+n). Equate coefficients of r on both side.