

1. Nine points in three-dimensional space with integer coordinates are chosen at random. Prove that the midpoint of the line segment connecting some two of them also has integer coordinates.
2. n cars are standing on a circle. Say car i has fuel f_i . Let fuel required one car to travel the whole circle is F . And it is given that $\text{SUM}(f_i) = F$. Prove that there will always be a car which will start travelling and collect fuel from the cars and will complete the circle. (i.e. it will never run out of fuel while travelling)
3. Let n be the number of sections and each section has 2 plots on either sides of the road. Find all possible ways to construct buildings in the plots such that there is a space between any 2 buildings.
4. Let $A = \{a_1, a_2, \dots, a_n\}$ be a finite set, and let ' \leq ' be a partial ordering of A . Given a bipartite graph $G = (U, V, E)$ with no of elements in each partite is n and $(u(i), v(j))$ belongs to E iff $a(i) \leq a(j)$. How does a chain and an anti chain appear in the graph?
5. Prove that $\text{Sum}(i=0, r) (m C i) * (n C (r - i)) = (m + n) C r$. ($n C r$ is the standard combinatorial)