Discrete Structures (CS21001)

Quiz 1

20 marks

1. A number $p \in N$ is perfect if it equals the sum of its positive divisors less than itself. Examples:
   - The number 6 is perfect since $6 = 1+2+3$.
   - The number 28 is perfect since $28 = 1+2+4+7+14$

a. Prove that if $A = \{ 2^{n-1} (2^n - 1) : n \in N$, and $2^n - 1$ is prime $\}$ and $P = \{ p \in N : p$ is a perfect number $\}$, then $A \subseteq P$. (4 marks)

b. Using 31 as a prime, generate a perfect number. (1 mark)

c. Using part (a) as result, prove that if $A = \{ 2^{n-1} (2^n - 1) : n \in N$, and $2^n - 1$ is prime $\}$ and $E = \{ p \in N : p$ is perfect and even $\}$, then $A = E$. (5 marks)

2. An isomorphism from a poset (S_1,R_1) to a poset (S_2,R_2) is a bijection $f:S_1 \rightarrow S_2$ such that, for all $x,y \in S_1 : (x,y) \in R_1$ iff $(f(x),f(y)) \in R_2$

   When such an isomorphism exists, we say that (S_1,R_1) is isomorphic to (S_2,R_2).

   Prove that isomorphism relation is an equivalence relation. (6 marks)

3. Let $(L, \leq)$ be a lattice in which $\wedge$ and $\vee$ denote the operations of meet and join, respectively. Prove that for any $a,b \in L$,

   $a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b = b$ (4 marks)