Grammars for Regular Sets

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- If a grammar is right linear or left linear, then it is called a **regular grammar**.
- Note: Suppose $G = (N, \Sigma, P, S)$ is a right linear grammar. $G' = (N, \Sigma, P', S)$ such that $P' = \{A \rightarrow \alpha | A \rightarrow \alpha^{\text{rev}} \in P\}$. Then G' is a left linear grammar.

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- We construct an NFA with ϵ-transitions $M = (Q, \Sigma, \delta, [S], \{[\epsilon]\})$ that simulates the derivations of G.
- States of Q denoted by $[\alpha]$ such that $\alpha = S$ or α is a (not necessarily proper) suffix of an RHS of a production in P.

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  2. If $a \in \Sigma, \alpha \in \Sigma^* \cup \Sigma^* N$, $\delta(a\alpha, a) = \{[\alpha]\}$ (regular grammar)

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- $[\alpha] \in \delta([S], w)$ iff
  either $\alpha = S, w = \epsilon$ (Type 1 transition),
  or $S \to_G^* xA \to_G^1 xy\alpha$ such that $A \to y\alpha \in P$ and $xy = w$.

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- Proof by induction on length of derivation (Try out formal proof).

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- Let $w = w_1 w_2 \ldots w_n$. Second step of simulation is going from $[w_1 w_2 \ldots w_n A]$ to $[w_2 \ldots w_n A]$ on reading w_1.

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- If $A \in N$, then after n steps we are at state $[A]$. Now by ϵ-transition, we reach states corresponding to all possible RHS of productions with A as LHS. And the simulation continues in a similar manner.

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- First, suppose $s \notin F$. Construct right linear grammar $G = (Q, \Sigma, P, s)$ where P contains:
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  $p \rightarrow aq$ when $\delta(p, a) = q$,
  and also $p \rightarrow a$ if $\delta(p, a) \in F$.
- $\hat{\delta}(p, w) = q$ if and only if $p \rightarrow_G^* wq$ (Try out formal proof).
  Note: If $q \in F$ the statement says that w is accepted by M if and only if it is generated by G.

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- Left linear grammar: Use the right linear grammar for L^{rev}, which is also a regular set.