Pushdown Automata: Acceptance Issues

$M = (Q, \Sigma, \Gamma, \perp, \delta, s, F)$

- F — a subset of Q, possibly empty, the set of final states
- s — the start state
- $\perp$ — The initial bottom-of-stack symbol
- Γ — Stack alphabet
- Σ — input alphabet
- Q — a finite set of states

$\delta \rightarrow$ transitions

$\delta \rightarrow$ binary relation from $Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma$ to $Q \times \Gamma^*$

$\delta \subseteq (Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma) \times (Q \times \Gamma^*)$

$\boxed{((p, a, A), (q, \gamma)) \in \delta}$

$\boxed{\delta \rightarrow \text{finite}}$

Transition

Inputs: ① the current state (p) $\in Q$

② the input symbol (a) $\in \Sigma \cup \{\epsilon\}$

③ the current top of (A) stack $\in \Gamma$

Effects: ⓐ new state $\in Q$ (q)

ⓑ A replaced by $\gamma \in \Gamma^*$

Configuration of M $\rightarrow$ current snapshot of M

↳ current state p

↳ the portion of the input yet to be read (left to right) $\in \Sigma^*$

↳ the current content of the stack $\in \Gamma^*$ (read from top to bottom)

$$C \in Q \times \Sigma^* \times \Gamma^*$$

Initial configuration

$$(s, x, \perp)$$

Final configuration (Accept configuration)

Acceptance by empty stack $\quad (q, \epsilon, \epsilon),\ q \in Q$

Acceptance by final state $\quad (q, \epsilon, \gamma),\ q \in F,\ \gamma \in \Gamma^*$

$$((p, a, A), (q, \gamma)) \in \delta$$

$$(p, ay, A\beta) \xrightarrow[M]{1} (q, y, \gamma\beta)$$ one-step change of configuration

$$((p, \epsilon, A), (q, \gamma)) \in \delta$$

$$(p, y, A\beta) \xrightarrow[M]{1} (q, y, \gamma\beta)$$ one-step (ϵ-transition)

<u>Def</u>: $C \xrightarrow[M]{0} D \iff C = D.$

$C \xrightarrow[M]{n+1} D \iff \exists$ a configuration E such that $C \xrightarrow[M]{n} E$ and $E \xrightarrow[M]{1} D$

$C \xrightarrow[M]{*} D \iff C \xrightarrow[M]{n} D$ for some $n \geq 0$.

Language of M $\longrightarrow$ $\mathcal{L}(M)$

Acceptance by <u>empty stack</u>

$$\mathcal{L}(M) = \left\{ x \in \Sigma^* \;\middle|\; (s, x, \perp) \xrightarrow[M]{*} (q, \epsilon, \epsilon) \text{ for some } q \in Q \right\}$$

Acceptance by <u>final state</u>

$$\mathcal{L}(M) = \left\{ x \in \Sigma^* \;\middle|\; (s, x, \perp) \xrightarrow[M]{*} (q, \epsilon, \gamma) \text{ for some } q \in F \text{ and for some } \gamma \in \Gamma^* \right\}$$

Equivalence of these two modes of acceptance

$M = (Q, \Sigma, \Gamma, \perp, \delta, s, F) \longrightarrow$ a PDA that accepts by empty stack / final state

$M' = (Q', \Sigma, \Gamma', \perp\!\!\!\perp, \delta', s', \{t\})$, $t \notin Q \cup \{s'\}$
$s' \notin Q$

$\hookrightarrow$ t is the only accept state

$\hookrightarrow$ accepts both by empty stack and by final state

(i) M' is in state t

(ii) The stack of M' is empty.

$s' \to$ new start state

$$((s', \epsilon, \perp\!\!\perp), (s, \perp \perp\!\!\perp)) \in \delta'$$

$$\perp\!\!\perp \notin \Gamma, \quad \Gamma' = \Gamma \cup \{\perp\!\!\perp\}$$

M' simulates M

Keep every transition of δ in δ'.

M accepts by <u>empty stack</u>

(q, ϵ, ϵ)

Add the transition

$$((q, \epsilon, \perp\!\!\perp), (t, \epsilon)) \text{ to } \delta'$$

M accepts by <u>final state</u>

(q, ϵ, γ), $q \in F$, $\gamma \in \Gamma^*$

<u>Add to δ'</u>

$((q, \epsilon, A), (t, A))$, $q \in F$, $A \in \Gamma \cup \{\perp\!\!\perp\}$

$((t, \epsilon, A), (t, \epsilon))$

$\mathcal{L}(M') = \mathcal{L}(M)$ M accepts by <u>empty stack</u>

$x \in \mathcal{L}(M)$ $(s, x, \perp) \xmapsto[M]{*} (q, \epsilon, \epsilon)$

You design a PDA
- mention what kind of acceptance your PDA does

$$(s, x, \perp) \xmapsto[M']{*} (q, \epsilon, \epsilon)$$

$$(s', x, \perp\!\!\perp) \xmapsto[M']{1} (s, x, \perp \perp\!\!\perp) \xmapsto[M']{*} (q, \epsilon, \perp\!\!\perp) \xmapsto[M']{*} (t, \epsilon, \epsilon)$$

$\mathcal{L}(M) \subseteq \mathcal{L}(M')$

$\mathcal{L}(M') \subseteq \mathcal{L}(M)$ $x \in \mathcal{L}(M')$ $(s', x, \perp\!\!\perp) \xmapsto[M]{*} (t, \epsilon, \epsilon)$

in s', t $\rightarrow$ no input symbol is consumed

$$(s, x, \perp) \xmapsto[M]{*} (q, \epsilon, \epsilon).$$