Formal Languages and Automata Theory

First Short Test

Time: 20 minutes | 29–January–2021 | Maximum marks: 20

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1. Let L be a language over the alphabet Σ. It is given that $L^2 = L$. Which of the following statements must be true?

(A) $L = \Sigma^*$.
(B) All strings in L are of even lengths.
(C) $L^* = L$.
$\boxed{\textbf{(D)}}$ $L^+ = L$.

Solution (A) The set of all strings of even lengths satisfies the given condition, and is not equal to Σ^*.

(B) Σ^* satisfies the given condition, but contains strings of odd lengths.

(C) False if $\varepsilon \notin L$.

(D) Show by induction on n that $L^n = L$ for all $n \geqslant 1$.

2. Recall that a string x is called a *prefix* (resp. *suffix*) of a string w if $w = xy$ (resp. $w = yx$) for some string y. A string x is called a *border* of a string w if x is both a prefix and a suffix of w. A prefix/suffix/border x of w is called *proper* if $x \neq w$. For a language L over the alphabet $\{a, b\}$, define the following languages.

$$\begin{aligned} \mathrm{PP}(L) &= \{x \mid x \text{ is a proper prefix of some } w \in L\}, \\ \mathrm{PS}(L) &= \{x \mid x \text{ is a proper suffix of some } w \in L\}, \\ \mathrm{PB}(L) &= \{x \mid x \text{ is a proper border of some } w \in L\}. \end{aligned}$$

Which of the following statements must be true?

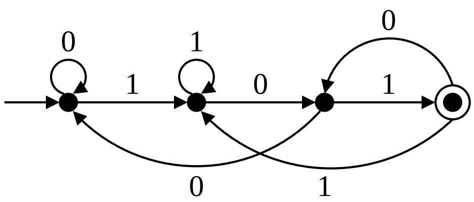
$\boxed{\textbf{(A)}}$ $\mathrm{PB}(L) \subseteq \mathrm{PP}(L) \cap \mathrm{PS}(L)$.
(B) $\mathrm{PP}(L) \cap \mathrm{PS}(L) \subseteq \mathrm{PB}(L)$.
(C) $\mathrm{PB}(L) \subseteq L$.
(D) $L \subseteq \mathrm{PB}(L)$.

Solution (A) Any member $x \in \mathrm{PB}(L)$ is both a proper prefix and a proper suffix of some string in L.

(B) Take $L = \{ab, ba\}$.

(C) and (D) Take $L = \{aba\}$.

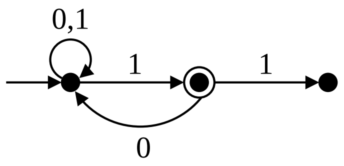
3. What is the language of the following DFA over the alphabet $\{0, 1\}$?



(A) The set of all strings containing an even number of 1's.
(B) The set of all strings containing the pattern 101.
$\boxed{\textbf{(C)}}$ The set of all strings ending with the pattern 101.
(D) The set of all strings not containing the pattern 010.

Solution (A) Accepts 1101. (B) Does not accept 1010. (D) Accepts 0101.

4. What is the language of the following NFA over the alphabet $\{0,1\}$?



(A) The set of all strings not containing the pattern 11.
$\boxed{\textbf{(B)}}$ The set of all strings ending with 1.
(C) The set of all strings ending with 01.
(D) The set of all strings not containing the pattern $(01)^*1$.

Solution (A), (C) and (D) Accepts 011.

5. Let $N = (Q, \Sigma, \Delta, S, F)$ be an NFA. If $x \in \Sigma^*$ is *not* accepted by N, which of the following statements must be true?

(A) There may exist a path labeled by x from a state in S to a state in F.
$\boxed{\textbf{(B)}}$ All paths labeled by x starting from a state in S end in states not in F.
(C) There exists a unique path labeled by x from a state in S to a state not in F.
(D) There exists at least one path labeled by x from a state in S to a state not in F.

Solution (B) is vacuously true if there are no paths labeled by x from S to F.

6. Let A, B be languages over an alphabet Σ, and $C = A - B$. Which of the following statements must be true?

$\boxed{\textbf{(A)}}$ If A and B are regular, then C is regular.
(B) If A and C are regular, then B is regular.
(C) If B and C are regular, then A is regular.
(D) If C is regular, then A and B are regular.

Solution (A) Use the product construction with $F_C = F_A \times (Q_B - F_B)$.

(B) Take $A = \mathscr{L}(b^*a^*)$ and $B = \{a^n b^n \mid n \geqslant 0\}$, so $C = \mathscr{L}(a^+b^+)$.

(C) and (D) Take $A = \{a^n b^n \mid n \geqslant 0\}$ and $B = \mathscr{L}(a^*b^*)$, so $C = \emptyset$.

7. Consider the two regular expressions $\alpha = (0^*10^*)^*$ and $\beta = (\,(11^*) + (00^*)\,)^*$. Which of the following statements is true?

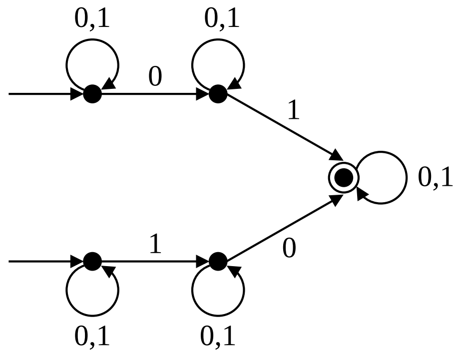
(A) α and β are equivalent.
(B) α and β are not equivalent because 010101 is matched to α but not to β.
(C) α and β are not equivalent because 101010 is matched to β but not to α.
$\boxed{\textbf{(D)}}$ α and β are not equivalent because 000000 is matched to β but not to α.

Solution (B) $0\,1\,0\,1\,0\,1 \in \mathscr{L}(\beta)$.

(C) $10\;10\;10 \in \mathscr{L}(\alpha)$.

(D) Any nonempty string in $\mathscr{L}(\alpha)$ contains 1.

8. Consider the NFA given below.



To which of the following regular expressions is this NFA *not* equivalent?

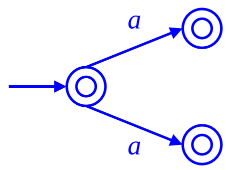
(A) $(0+1)^*(\ (0(0+1)^*1)+(1(0+1)^*0)\ )(0+1)^*$
$\boxed{\textbf{(B)}}$ $(0+1)^*(0+1)(0+1)^*(0+1)(0+1)^*$
(C) $(\ (00^*11^*)+(11^*00^*)\ )(0+1)^*$
(D) $(0+1)^*(\ (01)+(10)\ )(0+1)^*$

Solution The NFA accepts w if and only if w contains both 0 and 1. (A) is a direct conversion from the NFA. (C) and (D) follow from the observation mentioned above for the language of the NFA. (B) The regular expression can generate 00 and 11.

9. Let $N = (Q, \Sigma, \Delta, S, F)$ be an NFA with $|Q| = k$ and $S \neq \emptyset$. Suppose that all the states in Q are accessible. Which of the following statements must be true?

(A) If $F = Q$, then N accepts all strings in Σ^*.
(B) If $F = Q$, then N accepts exactly k strings in Σ^*.
$\boxed{\textbf{(C)}}$ N accepts at least one string in Σ^* if and only if $F \neq \emptyset$.
(D) If N accepts a string, then N accepts infinitely many strings in Σ^*.

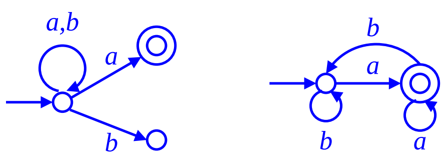
Solution For each of (A), (B) and (D), consider the following NFA.



10. Let L be a regular language having an NFA with k states. Which of the following statements is *false*?

(A) L may have an NFA with $< k$ states.
(B) L may have a DFA with $< k$ states.
(C) There exists a DFA for L with $\leqslant 2^k$ states.
$\boxed{\textbf{(D)}}$ Every DFA for L contains $\geqslant 2^k$ states.

Solution For each of (A), (B) and (D), consider the following NFA.



An NFA with 3 states

An equivalent NFA/DFA with 2 states

(C) Use the subset construction.