SOME DECIDABLE PROBLEMS

ABOUT TURING MACHINES

Abhijit Das

Department of Computer Science and Engineering Indian Institute of Technology Kharagpur

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Given *M*, decide whether *M* contains at least 2020 states.

- A Turing machine looks at the encoding of *M*, and finds out the answer.
- This machine runs in finite time for every input.

Given *M*, decide whether *M* halts within 2020 steps on input ε .

- Simulate *M* on ε for (at most) 2020 steps.
- If the simulation halts (after accepting/rejecting), accept.
- If the simulation does not halt after 2020 steps, reject.
- This machine is also a decider.

Problem 3

Given *M*, decide whether *M* takes more than 2020 steps on some input.

- *M* takes more than 2020 steps on some input ⇐⇒
 M takes more than 2020 steps on some input of length ≤ 2020.
- Suppose that *M* takes ≤ 2020 steps on all inputs of length ≤ 2020 . Supply an input *w* of length > 2020 to *M*.



- Within 2020 steps, *M* cannot see more than 2020 symbols from the input.
- This initial behavior of *M* on *w* is the same as its behavior on the prefix of *w* of length 2020. *M* is deterministic. *M* halts on *w* within 2020 steps.
- A decider simulates *M* on all inputs of length ≤ 2020, each for 2020 steps.
- If some simulation takes more than 2020 steps, accept, else reject.

Given *M*, decide whether *M* takes more than 2020 steps on all inputs.

- *M* takes more than 2020 steps on all inputs ⇐⇒
 M takes more than 2020 steps on all inputs of length ≤ 2020.
- It suffices to simulate M on all inputs of length ≤ 2020 , each for 2020 steps.

Problem 5

Given *M*, decide whether *M* ever moves to the right of the 2020-th cell on input ε .

- Let m = |Q| (number of states).
- Let $k = |\Gamma|$ (number of symbols in the tape alphabet).
- Suppose *M* never goes to the right of the 2020-th cell.
- Total number of configurations possible is $2021mk^{2020}$.
- Simulate *M* on ε for $2021mk^{2020}$ steps.
- If the head ever moves to the right of the 2020-th cell, accept.
- Otherwise, some configuration is repeated (pigeon-hole principle).
- Thus the machine must have entered an infinite loop, and will never go beyond the 2020-th cell. Reject.

- 1. Prove that the following problems on a TM *M* are decidable.
 - (a) Decide whether *M* halts on some input within 2020 steps.
 - (b) Decide whether M halts on all inputs within 2020 steps.
 - (c) Decide whether *M* runs for at least 2020^{2020} steps for input a^{2020} .
 - (d) Decide whether *M* on input ε moves left at least ten times.
 - (e) Decide whether M on a given input w moves left at least ten times.
- **2.** Is the problem whether a Turing machine on any input reenters the start state decidable or not? Prove.