CS60003 Algorithm Design and Analysis, Autumn 2010–11

Class test 2

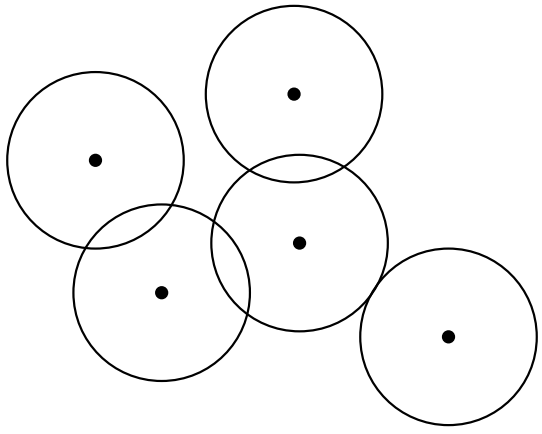
Maximum marks: 20 Time: 08-Nov-2010 Duration: 1 hour

Roll no: _____ **Name:** _____

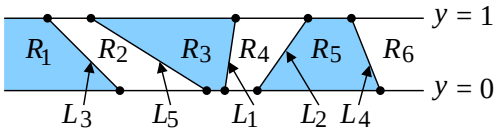
[*Write your answers in the question paper itself. Be brief and precise. Answer <u>all</u> questions.*]

1. [*Flattest Pair*] You are given n points $P_1, P_2, \ldots, P_n$ in general position in the plane. Your task is to find out the pair P_i, P_j (with $i \neq j$) such that the absolute slope of the straight line segment P_iP_j is as small as possible. Propose an $O(n \log n)$-time algorithm to do this. Explain the correctness of your algorithm. **(4)**

2. The following figure shows five circles (of the same radius) in the plane. Draw the (boundary of the) smallest convex *region* enclosing these circles. Also indicate the convex hull of the centers (shown as solid dots) of the five circles. **(4)**



3. You are given n straight line segments $L_1, L_2, \ldots, L_n$ each connecting a point on $y = 0$ to a point on $y = 1$. It is given that these lines are non-intersecting with one another. These segments partition the strip between $y = 0$ and $y = 1$ into $n + 1$ regions $R_1, R_2, \ldots, R_{n+1}$. The following figure illustrates a case of $n = 5$ segments. Each region is specified by the left and the right bounding segments. The first and the last regions are unbounded on one side, denoted by $-$. The six regions (shaded alternately) in the following figure are $R_1 = (-, L_3)$, $R_2 = (L_3, L_5)$, $R_3 = (L_5, L_1)$, $R_4 = (L_1, L_2)$, $R_5 = (L_2, L_4)$ and $R_6 = (L_4, -)$.



(a) Write an $O(n \log n)$-time algorithm to output the regions $R_1, R_2, \ldots, R_{n+1}$ from left to right. **(4)**

(b) Describe an $O(n \log n)$-time algorithm to convert the sorted output of Part (a) to a binary search tree. **(4)**

(c) You are given a point P in the strip between $y = 0$ and $y = 1$, but not on any of the input segments L_i. Describe an $O(\log n)$-time algorithm to identify the region R_k to which P belongs. Should you use the BST of Part (b), explain how the search for P is exactly carried out. **(4)**