Exercise set 7 (Undecidability)

1. Show that the following problems are undecidable.

(a) Given a C program P and an input I to P, does P print anything at all?

(b) Given a C program P and an input I to P, does P eventually halt?

2. Show that the following languages over $\{0,1\}$ are not R.E.

(a) $\{\alpha_i \mid \alpha_i$ is not accepted by $M_{2i}\}$.

(b) $\{\alpha_i \mid \alpha_{2i}$ is not accepted by $M_i\}$.

3. Write a code for the TM $M = (\{q_1,q_2,q_3\},\{0,1\},\{0,1,\sqcup\},\delta,q_1,\{q_2\},\sqcup)$, where δ consists of the following five transitions $\delta(q_1,0) = (q_1,0,\rightarrow)$, $\delta(q_1,1) = (q_3,0,\rightarrow)$, $\delta(q_3,0) = (q_1,0,\rightarrow)$, $\delta(q_3,1) = (q_3,0,\rightarrow)$, $\delta(q_3,\sqcup) = (q_2,\sqcup,\rightarrow)$. Write a code $\beta = \langle M,\alpha\rangle$ for the pair (M,α), where $\alpha = 011100101$. Does β belong to $\mathcal{L}(U)$, where U is the universal TM?

4. Show that $\bar{L}_d$, the complement of the diagonal language L_d (over $\{0,1\}$), is R.E., but not recursive.

5. [The halting problem] Show that the language consisting of encodings of all (M,α), M a TM with the input alphabet $\{0,1\}$ and $\alpha \in \{0,1\}^*$, such that M halts on input α (with or without accepting), is RE, but not recursive.

6. Under which of the following operations are R.E. languages closed? Recursive languages?

(a) Union.

(b) Intersection.

(c) Concatenation.

(d) Kleene star.

7. Let $L_1,\ldots,L_n$ constitute a partition of Σ^*. That is, $L_1,\ldots,L_n$ are pairwise disjoint languages with union Σ^*. If all the L_i are R.E., show that each L_i is recursive.

8. Show that the following languages *are* decidable. Here $\langle M\rangle$ denotes the code for a Turing Machine M with input alphabet $\{0,1\}$.

(a) $\{\langle M\rangle \mid M$ has hundred states $\}$.

(b) $\{\langle M\rangle \mid M$ has more than hundred states $\}$.

(c) $\{\langle M\rangle \mid M$ never makes a move left $\}$.