Exercise set 3 (Finite automata and regular expressions)

1. Repeat Exercise 11 of Exercise set 2 using the RE $\rightarrow$ ϵ-NFA conversion procedure discussed in the lecture.

2. Convert the DFA's of Exercise 7 in Exercise set 2 to equivalent regular expressions. (**Remark:** You may skip the third DFA, if you think that it involves too much manual labor. Try the first two.)

3. Prove that the following languages are not regular:

(a) The set of all valid arithmetical expressions over the alphabet $\{0, 1, \ldots, 9, +, -, \times, /, (,)\}$. (**Hint:** The left and right parentheses appearing in a valid arithmetical expression must be balanced.)

(b) The language of palindromes over any alphabet Σ with $|\Sigma| \geqslant 2$.

(c) The language $\{a^k b^l a^k \mid k, l \in \mathbb{Z}_+\}$ over $\Sigma := \{a, b\}$.

(**Remark:** We shall see later that all these languages are context-free, i.e., have finite representations using context-free grammars.)

4. Which of the following languages are regular? (Unless otherwise stated you may take $\Sigma := \{a, b\}$.)

(a) $\{(ab)^k (ba)^l \mid k, l \in \mathbb{N}\}$.

(b) $\{(ab)^k (ba)^k \mid k \in \mathbb{N}\}$.

(c) $\{\alpha \in \Sigma^* \mid |\alpha| \text{ is a perfect square}\}$.

(d) $\{\alpha \in \Sigma^* \mid |\alpha| \text{ is not a perfect square}\}$. (**Hint:** Look ahead.)

* **(e)** $\{\alpha \in \Sigma^* \mid |\alpha| \text{ is square-free}\}$. (**Remark:** You may need some elementary number theory.)

(f) $\{\alpha \in \Sigma^* \mid |\alpha| \text{ is divisible by } 25\}$.

(g) A finite language over some Σ.

(h) The union of finitely many regular languages over some Σ.

(i) The union of infinitely many regular languages over some Σ.

5. Let L, L_1 and L_2 be regular languages over a given alphabet Σ. Show that each of the following is also a regular language:

(a) $L_1 \cup L_2$

(b) $\bar{L} := \Sigma^* \setminus L$

(c) $L_1 \cap L_2$ (**Hint:** De Morgan's law.)

(d) $L_1 \setminus L_2$

* **(e)** $L^R := \{\alpha^R \mid \alpha \in L\}$

6. Let L be a language over an alphabet Σ and let $a \in \Sigma$. Define the quotient language

$$L/a := \{\alpha \in \Sigma^* \mid \alpha a \in L\}.$$

* **(a)** Show that if L is a regular language, then so is L/a.

(b) Prove or disprove: $(L/a)a = L$.

(c) Prove or disprove: $(La)/a = L$.

7. Let L and L' be languages over an alphabet Σ. Assume that L is regular. Show that the following languages are regular:

* **(a)** The (generalized) quotient language $L/L' := \{\alpha \in \Sigma^* \mid \alpha\beta \in L \text{ for some } \beta \in L'\}$. (Note that L' can be any language, not necessarily regular).

(b) $\operatorname{prefix}(L) := \{\alpha \in \Sigma^* \mid \alpha\beta \in L \text{ for some } \beta \in \Sigma^*\}$.

(c) $\operatorname{suffix}(L) := \{\beta \in \Sigma^* \mid \alpha\beta \in L \text{ for some } \alpha \in \Sigma^*\}$.

(d) $\operatorname{substring}(L) := \{\beta \in \Sigma^* \mid \alpha\beta\gamma \in L \text{ for some } \alpha, \gamma \in \Sigma^*\}$.