Adjacency matrices of complex unit gain graphs

M. Rajesh Kannan[1]

[1]Department of Mathematics,
Indian Institute of Technology Kharagpur,
Kharagpur-721302, India.
email: rajeshkannan1.m@gmail.com, rajeshkannan@maths.iitkgp.ac.in

January 31, 2019

Abstract

Let $G = (V, E)$ be a simple, undirected, finite graph with the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and the edge set $E(G) \subseteq V \times V$. If two vertices v_i and v_j are adjacent, we write $v_i \sim v_j$, and the edge between them is denoted by e_{ij}. The *adjacency matrix* of G is an $n \times n$ matrix, denoted by $A(G) = (a_{ij})$, whose rows and columns are indexed by the vertex set of the graph and the entries are defined by

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \sim v_j, \\ 0 & \text{otherwise.} \end{cases}$$

The adjacency matrix of a graph is one of the well studied matrix class in the field of spectral graph theory [1, 3, 4].

The notion of gain graph was studied in [10, 11]. For a given graph G and a group $\mathfrak{G}$, first orient the edges of the graph G. For each oriented edge e_{ij} assign a value (the *gain* of the edge e_{ij}) g from $\mathfrak{G}$ and assign g^{-1} to the orientated edge e_{ji}. If the group is taken to be the multiplicative group of unit complex numbers $\mathbb{T}$, the graph is called the *complex unit gain graph(or* $\mathbb{T}$*-gain graph)*. In [9], the author defined the notion of adjacency matrices of $\mathbb{T}$-gain graphs, which is a canonical extension of classical adjacency matrices. Particular cases of the adjacency matrix of $\mathbb{T}$-gain graphs were considered with different gains in the literature [2, 5, 6, 7].

In this talk, we shall discuss some of the properties of adjacency matrices of graphs and adjacency matrices of $\mathbb{T}$-gain graphs[8].

References

[1] R. B. Bapat, *Graphs and matrices*, second ed., Universitext, Springer, London; Hindustan Book Agency, New Delhi, 2014. MR 3289036

[2] R. B. Bapat, D. Kalita, and S. Pati, *On weighted directed graphs*, Linear Algebra Appl. **436** (2012), no. 1, 99–111. MR 2859913

[3] Andries E. Brouwer and Willem H. Haemers, *Spectra of graphs*, Universitext, Springer, New York, 2012. MR 2882891

[4] Dragoš Cvetković, Peter Rowlinson, and Slobodan Simić, *An introduction to the theory of graph spectra*, London Mathematical Society Student Texts, vol. 75, Cambridge University Press, Cambridge, 2010. MR 2571608

[5] Krystal Guo and Bojan Mohar, *Hermitian adjacency matrix of digraphs and mixed graphs*, J. Graph Theory **85** (2017), no. 1, 217–248. MR 3634484

[6] Debajit Kalita and Sukanta Pati, *A reciprocal eigenvalue property for unicyclic weighted directed graphs with weights from* $\{\pm 1, \pm i\}$, Linear Algebra Appl. **449** (2014), 417–434. MR 3191876

[7] Jianxi Liu and Xueliang Li, *Hermitian-adjacency matrices and Hermitian energies of mixed graphs*, Linear Algebra Appl. **466** (2015), 182–207. MR 3278246

[8] Ranjit Mehatari, M. Rajesh Kannan, and Aniruddha Samanta, *On the adjacency matrix of complex unit gain graph*, arXiv:1812.03747.

[9] Nathan Reff, *Spectral properties of complex unit gain graphs*, Linear Algebra Appl. **436** (2012), no. 9, 3165–3176. MR 2900705

[10] Thomas Zaslavsky, *Signed graphs*, Discrete Appl. Math. **4** (1982), no. 1, 47–74. MR 676405

[11] ———, *Biased graphs. I. Bias, balance, and gains*, J. Combin. Theory Ser. B **47** (1989), no. 1, 32–52. MR 1007712