Locality-Sensitive Orderings

In this talk, we present an overview of a very recent result of Timothy Chan, Sariel Har-Peled, and Mitchell Jones, that appeared in the Innovations in Theoretical Computer Science Conference in January 2019.

Their result is as follows: Consider points in a unit cube in d-dimensions. For $\epsilon > 0$, there is a family of $O(\frac{1}{\epsilon^d}\log(\frac{1}{\epsilon}))$ orderings of points in $[0,1)^d$ with the following property: For any $p, q \in [0,1)^d$, there is an ordering in the family such that all the points lying between p and q are within a distance of at most $\epsilon||p-q||_2$ from p or q.

One of the key components in obtaining the above result is the following interesting result of Walecki from the 19th century: For n elements $\{0, 1, 2, \ldots, n-1\}$, there is a set of $\lceil \frac{n}{2} \rceil$ permutations of the elements, such that, for all $i, j \in \{1, 2, \ldots, n-1\}$, there is a permutation in which i and j are adjacent.

This result, in combination with partitioning of a quadtree in so called $\epsilon-$quadtrees, is used to derive the main result. We will also look into how to compute approximate nearest neighbors from these orderings. If time permits, we will present high-level overview of how one can maintain approximate nearest neighbors under insertion and deletion of points.