Contents

Section outline

Sum of products from sets

Regions

1. $A \cap B \cap C$
2. $A \cap B \cap \overline{C}$
3. $A \cap \overline{B} \cap C$
4. $\overline{A} \cap B \cap C$
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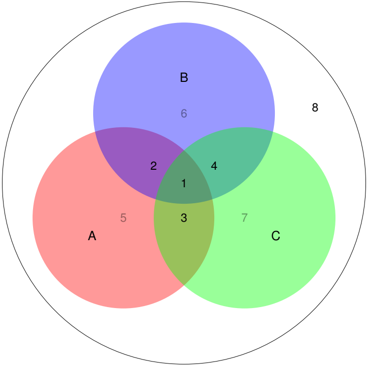

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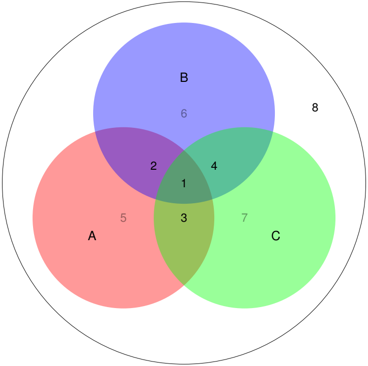

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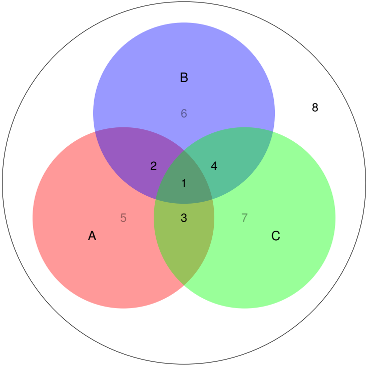

Selections

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$abc + ab\overline{c} = ab$

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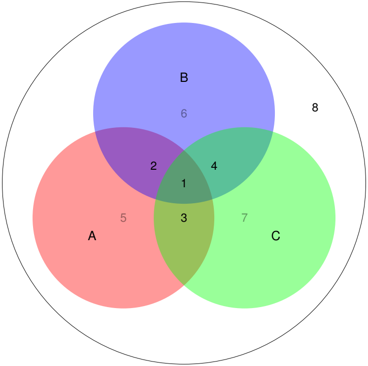

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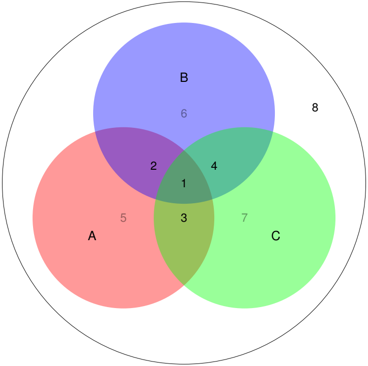

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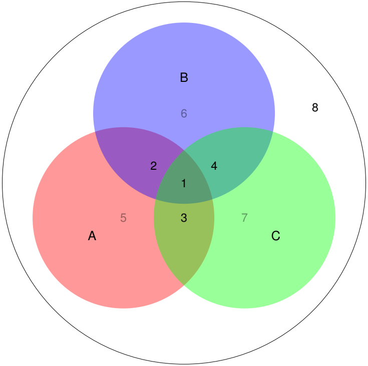

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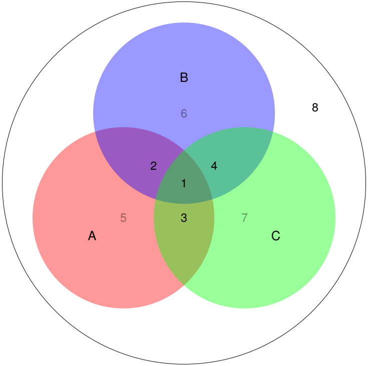

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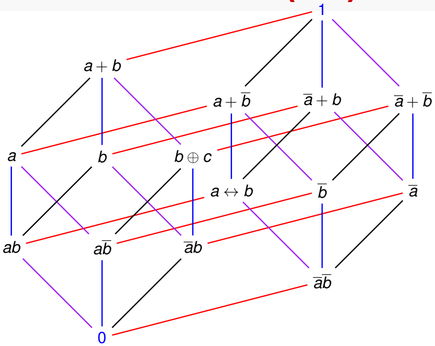
Boolean lattice (BL) for 2 variables



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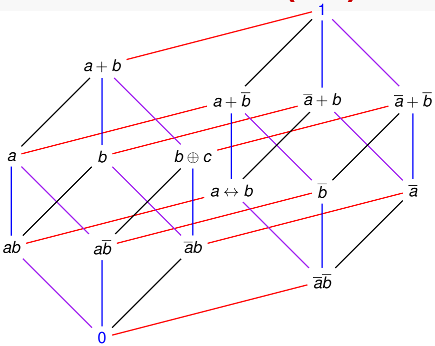

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- Grouping with parentheses is permitted

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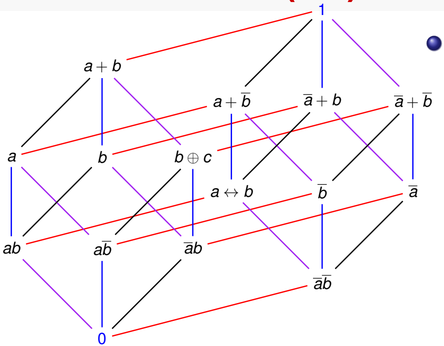

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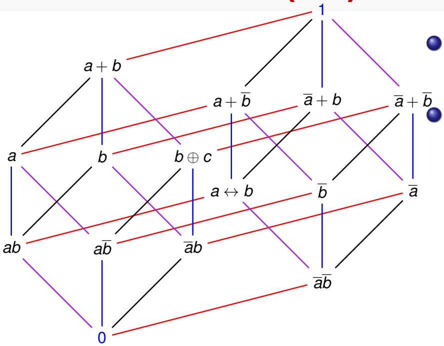

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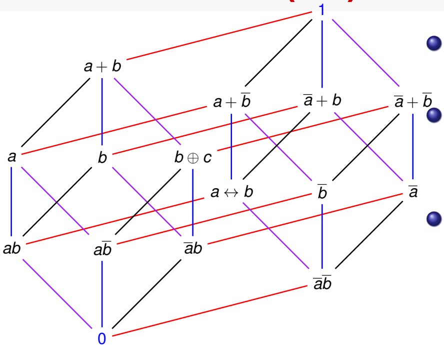

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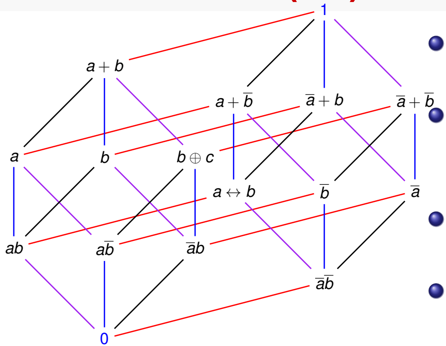

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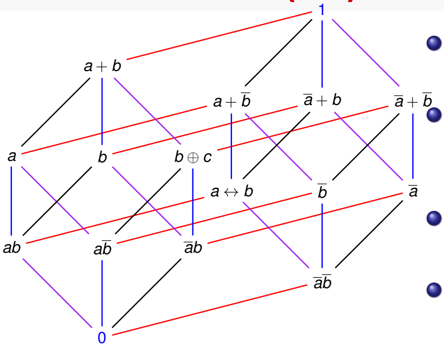

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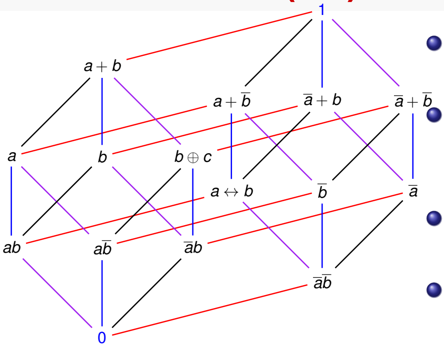

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- DeMorgan's laws, distributivity, commutativity, idempotence, involution may be used to transform a Boolean expression to SOP

Functional completeness

- May be derived from the Boolean lattice

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x	y	$\overline{x}$	$x \cdot y$	$x + y$
0	0	1	0	0
0	1	1	0	1
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RAM Random access memory

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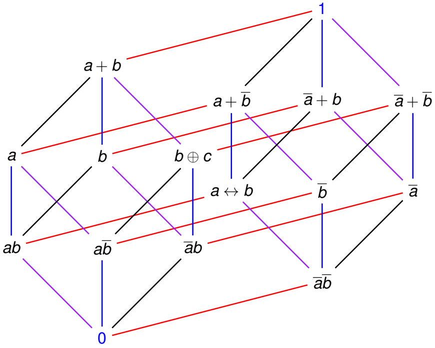
RAM Random access memory

Minority Minority value among given inputs

Boolean expressions



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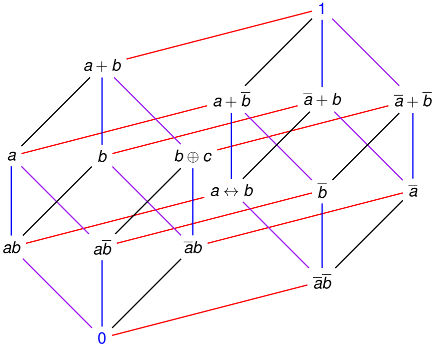

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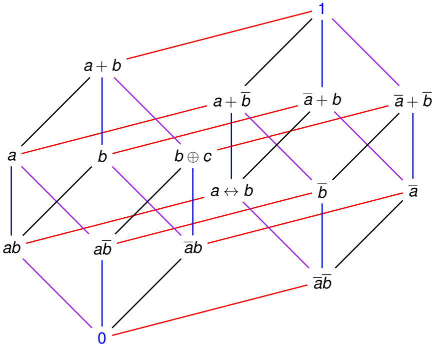

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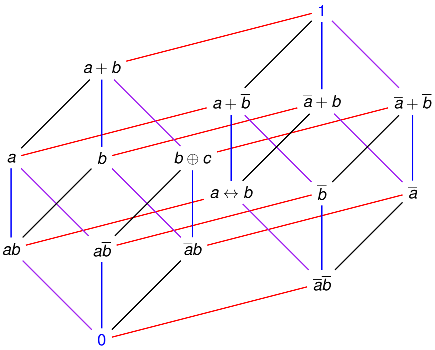

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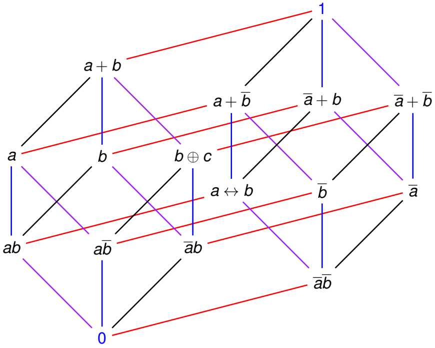

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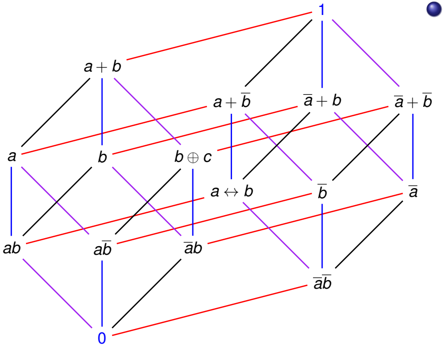

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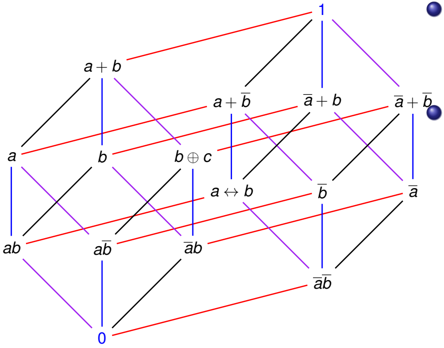

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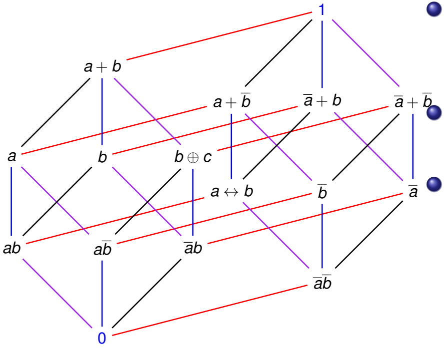

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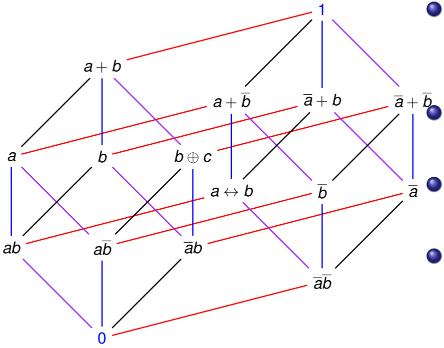

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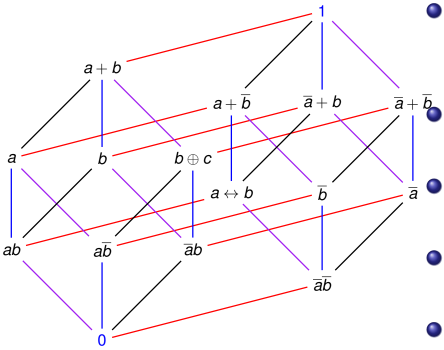

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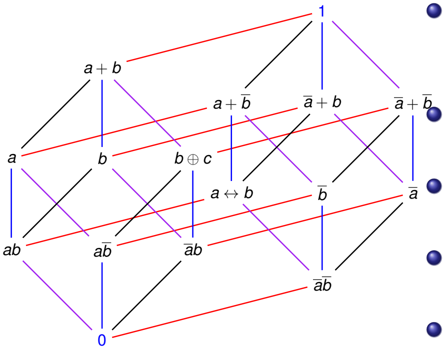

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- Similar to SOP, *product of sums* (POS) may be defined
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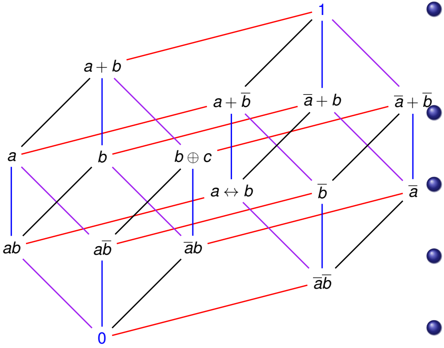
Boolean expressions



- $E = x\overline{z} + \overline{y}z + xy\overline{z}$
- $E = \overline{((\overline{xy})z)}\,\overline{((\overline{x}+z)(\overline{y}+\overline{z}))}$
- $E = x(\overline{\overline{y}z})$

- A SOP expression where each FP is a minterm is said to be in *disjunctive normal form* (DNF)
- The DNF of any SOP is unique (why?) – cannonical SOP
- An element x in a BL is *maxterm* if it has 1 as its only successor
- A maxterm is a sum of literals involving all the variables
- Similar to SOP, *product of sums* (POS) may be defined
- A Boolean expression which is a product of maxterms is said to be in *conjunctive normal form* (CNF)
- The CNF of any POS is unique (why?) – cannonical POS

Alternate argument for minterm expansion

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- This ensures that the minterm expansion is unique

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- Each choice denotes a distinct Boolean function

Boolean expression manipulation

- $xy + \overline{x}z + yz = xy + \overline{x}z$
- $(x + y)(\overline{x} + z)(y + z) = (x + y)(\overline{x} + z)$
- $T = (x + y)\overline{[\overline{x}(\overline{y} + \overline{z})]} + \overline{x}\,\overline{y} + \overline{x}\,\overline{z}$
- $xy + \overline{x}\,\overline{y} + yz = xy + \overline{x}\,\overline{y} + \overline{x}z$

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- if $a \oplus b = c$ then $\begin{cases} a \oplus c = b \\ b \oplus c = a \\ a \oplus b \oplus c = 0 \end{cases}$

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- CMOS NAND, NOR

Shannon decomposition

- $f(x_1, x_2, \ldots, x_n) = x_1 \cdot f(1, x_2, \ldots, x_n) + \overline{x_1} \cdot f(0, x_2, \ldots, x_n)$

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- Multiplexer realisation by Shannon decomposition or Shannon expansion
- Repeated application to obtain CNF or DNF of a given Boolean function