Analysis and Modeling of Lowest Unique Bid Auctions

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Abstract

We study a special kind of online auction known as *Lowest Unique Bid Auction* (LUBA) which allocates a good to the agent who submits the lowest bid that is not matched by any other bid (i.e., the least and the unique bid). We exhaustively analyze this kind of auction mechanism by constructing various networks and studying their properties from the weblogs available as bid histories. We also study how the top winners differ from others in their bidding strategies by analyzing various parameters. Profit analysis of all bidders reveals that top winners need not be high profit bidders, and there might exist a few "crazy" bidders who frequently participate in various auctions irrespective of their profit or loss. Finally, we propose a memory-driven agent based dynamical model where both bidder participation and bidding choices are decided preferentially based on the previous participation and winning histories of the individual bidders. Remarkably, our model accurately predicts the participation and winning distribution of the bidders.

1 Introduction

Owing to the rapid expansion and the importance of online auctions, researchers recently have begun to pay attention to the various aspects of online auctions. One of the greatest contributions of online auctions is their ability to provide detailed bidding records in the form of weblogs often termed as *bid history*. In such online transactions, the pattern of bidding activity is more complex than traditional offline transactions. The representations of online auctions in the form of complex systems have gained momentum recently [14, 4]. In this paper, we focus on one of the typical and unconventional online auctions called *Lowest Unique Bid Auction* (LUBA) [12] which allocates a good to the agent who submits the lowest bid that is not matched by any other bid. In other words, rather than the bidder with the highest bid (as in the case of traditional auctions), in LUBA, the winner of an auction is decided by the lowest and the unique bid submitted by a bidder among all the bids placed so far in the auction, i.e., the offer which is not made by any other bidder and is lowest (see Figure 1).

Lowest Unique Bid Auctions (LUBAs): Websites offering LUBAs first appeared in Scandinavian countries in early 2006 [2]. Since then, they rapidly developed in many other European countries (France, Germany, Holland, Italy, Spain, and the UK). Gradually, this auction format is gaining increasing media attention. Let us introduce in more detail the functioning of a LUBA. As a first step, agents must register to one of these websites and transfer an amount of money of their choice to a personal deposit. Users can then browse through the items on sale and submit as many bids as they want on the items in which they are interested. A maximum bid amount (generally lower than ten hundreds of cents) is associated with each bidding item. Bids are generally expressed in cents and are private. Every time that a user places a bid, a fixed amount of money (typically 2 Euros) is deducted from her deposit. The auctioneer justifies this cost as a price for a "packet of information" that she sends to the bidder. In fact, as soon as a bid is submitted, the user receives one of the three following messages: (i) your bid is currently the unique lowest bid; (ii) your bid is unique but is not the lowest; or (iii) your bid is not unique. Note that, each agent knows only what she has bid, without any information on which values the other agents have bid. In general, there is no restriction for the number of bids that a particular agent may place. During the bidding period, which usually lasts for a few days, users can at any time log into their account in order to check the current status of their bids, to add new ones, or to refill the deposit. Once the auction closes (decided previously either by time stamp limit or by the total number of bids), the object is sold to the bidder who submitted the lowest unique bid.

This allocation mechanism is, therefore, considerably different with respect to traditional auction formats. In particular, it is the requirement about the uniqueness of the winning bid that represents a novelty. On the one hand, this requirement undermines key objectives that lie at the core of standard auction theory like, for instance, the efficiency of the final allocation. Moreover, LUBA adds some new strategic elements. In fact, from a strategic point of view, a LUBA is more similar to other well-known games like Guessing Game [5] than to a standard auction. LUBAs are particularly profitable for both the auctioneers and the winners of the auctions and lead to winning items at an impressively low cost, generally far less than 2% of the market value of the item in the auction [2]. Regarding the auction policy and success/failure of LUBA, different people have different points of view: while some groups say LUBA is a game of strategy, some say that it is just a lottery; even few

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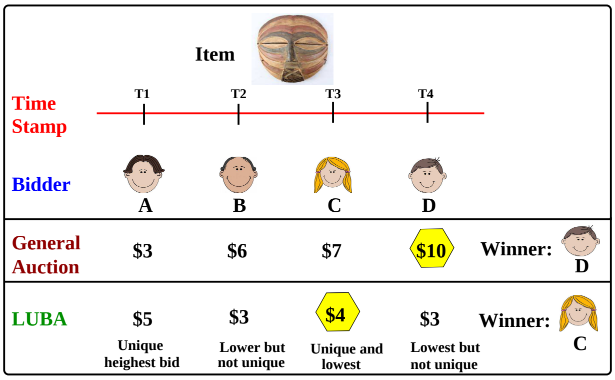


Figure 1: (Color online) An example comparing general auction and LUBA. Four bidders participate in the auction. In each time stamp, every bidder is allowed to submit the bid. In general auction, the participants can see other bids and accordingly decide the bid values. At the end, the bidder submitting the highest bid is selected as the winner. In contrast, in LUBA no participant can see bids placed by other participants. Finally, whoever bids the lowest unique positive amount wins the auction.

suspect it is a plain scam [1].

Contributions of the paper: In our work, we attempt to investigate several questions related to the gaming strategy and individual role of the participants in LUBAs. In particular, the questions are as follows: (i) when a bidder joins a new auction, does she use any experience of the past when submitting a new bid? (ii) is this auction mechanism a game, lottery or a scam? (iii) are winning bidders rational? (iv) whether bidders follow any participation or bidding pattern? We have collected a reasonable amount of auction history in the period of 2007-2010 from `www.uniquebidhomes.com`. We generate various networks and try to answer the above questions by studying various properties and patterns of the networks, constructed from the dataset. *The most important component of this study is to unfold the relationship between the number of auctions that a bidder won and her net profit. To the best of our knowledge, this aspect has not been systematically investigated in spite of a lot of literature on this topic. Remarkably, we observe that the bidders are highly addictive to winning while being almost indifferent to the net profit gained.* Finally, we attempt to represent LUBA using a memory-driven agent based model that can accurately reproduce the real-world bidding phenomenon of LUBA. The simulation results of the agent-based model show a reasonable resemblance with the real-world LUBA results in terms of different probability distributions such as the participation degree distribution (unweighted and weighted) and the winning distribution of bidders.

2 Related work

Apart from the classical contributions discussed in [2], few researchers have recently studied the dynamics of unique bid auctions. Houba et al. [3] and Rapoport et al. [10] analyze the equilibrium of a LUBA in which bidders submit a unique bid, there is a non-negative bidding fee, and the winner pays her bid. Both these studies find that in the symmetric mixed equilibrium, bidders randomize with decreasing probabilities over a support that comprises the lowest possible bid and is made of consecutive numbers. Rapoport et al. [10] also analyze HUBAs, i.e., unique bid auctions in which the winner is the bidder who submits the highest unmatched offer. Such a mechanism is also studied by Raviv and Virag [11]. Östling et al. [6] obtain a similar result for what they call a LUPI (Lowest Unique Positive Integer) game in which players can again submit a single bid, but there are no bidding fees, and the winner does not have to pay for her bid. Finally, Eichberger and Vinogradov [1] analyze a LUBA (that they call LUPA, i.e., Least Unmatched Price Auction) where bidders can submit multiple costly bids, and the winner must pay her winning bid. Given that no information about other bidders behavior is disclosed during the auction, they model the game as a simultaneous game. For some special ranges of the parameters, they show the existence of a unique Nash equilibrium in which agents mix over bidding strings that comprise the minimum allowed bid and are made of consecutive numbers. Gallice [2] studies the influence or the role played by signals that the seller sends to the bidders about the status of their bids.

Recently, Radicchi and Baronchelli [8] observe agents adopting “Levy flight search strategy” in their exploration of bid space and develop a model revealing agents using rational bidder strategies. In extension to this paper, Radicchi et al. [9] again rationalize these findings by adopting an evolutionary perspective which is able to account for the observed empirical measurements. Pigolotti et al. [7] use a grand canonical approach to derive an analytical expression for the equilibrium distribution of strategies. They also study the properties of the solution as a function of the mean number of players, and compare them with a large dataset of internet auctions.

3 Description of the dataset

Many online auction sites provide bid histories and make them publicly available for the bidders. Some of the sites where datasets could be extracted include `auctionair.co.uk`, `bidster.com`, `bidbudgie.co.uk`, `bidson.com`, `bidandclick.com`, `bidson.com`, `lowbids.com.au`, `cashop.co.uk` etc. We collected the data from `www.uniquebidhomes.com` which was also used by Radicchi et al. [9] for their analysis. The data includes all auctions organized during 2007, 2008, 2009 and part of 2010. We collected the following information concerning the auctions: the value of the goods, the amount of fee, the maximum bid amount, duration of the auction or the required number of bids. The dataset also made available information about each single bid, getting information about its value, the time when it was made and the agent who made it. Note that, 14% of the auctions do not have any maximal bid value allowed for the auction; 14% of auctions use duration to expire an auction, while the remaining use the total number of bids to expire an auction; 48% of the auctions have limit on the maximal number of bids allowed per bidder.

Total number of auctions	189
Total number of bidders	3740
Total number of bids	55041
Average number of agents involved in an auction	50
Average number of bids made by a single agent in an auction	5.81

Table 1: Description of the auction dataset.

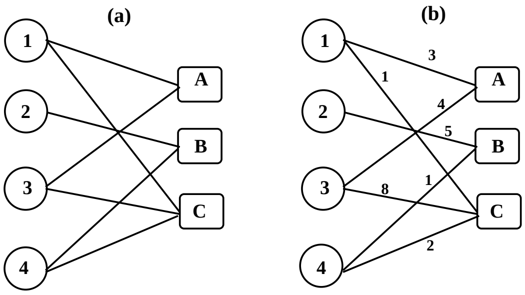


Figure 2: (Color online) A schematic diagram of (a) unweighted and (b) weighted bipartite networks of an online auction. Bidders and auctions are represented by circles and squares respectively.

Detailed description of the dataset is noted in Table 1.

4 Bidder-auction bipartite network

The data contains the information of the auctions where the bidders (uniquely identified by the bidder IDs) submit their bid. Thus, we can construct a bipartite network comprising two disjoint sets of vertices, i.e., bidders and auctions with an edge between bidder i and auction j if i bids at least once in the auction j, as shown in Figure 2(a). Similarly, a weighted bipartite network is also constructed with edges having weights where weight of the edge joining bidder i and auction j represents the number of bids placed by i in an auction j as shown in Figure 2(b).

The degree of a bidder node in Figure 2(a) signifies the number of auctions she has participated so far. Similarly, the degree of an auction node in Figure 2(a) represents the number of distinct bidders participating in that particular auction. On the other hand, the weight of an edge in Figure 2(b) represents the activeness of the bidders, i.e., number of bids submitted by a bidder in that auction. We notice that the unweighted (weighted) average degree of a bidder node and an auction node are 2.53 (5.81) and 50.81 (98.29) respectively. The cumulative degree distribution of bidder nodes follows a power-law behavior asymptotically in both the networks as shown in Figure 3.

One-mode projection of the bidder partition: A bidder network can be constructed from the bipartite network by projecting it on to the bidder nodes. The projected network consists of all bidders as the set of vertices, and there exists an edge between two bidders i and j if i and j participate in the same auction at least once. A weighted projection can also be constructed by edges having weights representing the number of common auctions in which both i and j participated. Figure 4 shows the binary and weighted one-mode projection of the bipartite network of Figure 2(a).

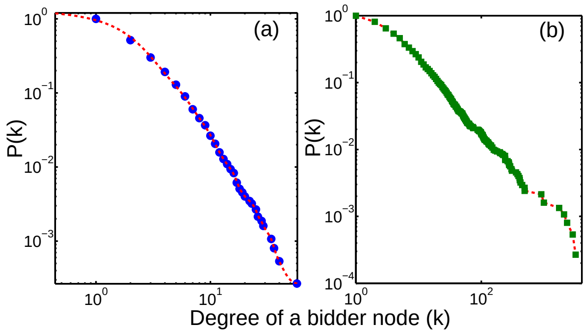


Figure 3: (Color online) Cumulative degree distributions of bidder nodes in the (a) unweighted and (b) weighted bipartite networks.

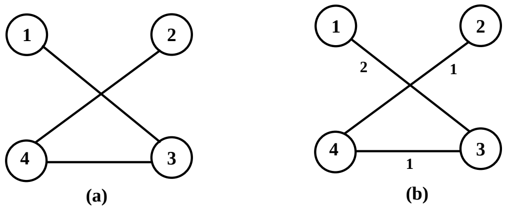


Figure 4: (Color online) The (a) unweighted and (b) weighted one-mode projections of the bidder-auction bipartite network (shown in Figure 2(a) on to the bidder partition.

5 Winners under the lens

One of the most critical components of such a LUBA system is its winners. The strategies adopted by the winners in turn actually reflect most of the important characteristic features of a LUBA system. Therefore, in this section, we specifically focus on the behavioral dynamics of the winners of the auctions. We notice that there are only 51 unique bidders who won at least one of the 189 auctions. We call the bidders "top winners" if they won a large number of auctions. We observe that the top five winners participated in 70% of the auctions and have won 57% of them. Among these, the top three winners participated in 55% of the auctions and won 50% of the auctions. These percentages show that the top winners are highly efficient and cover most of the winning space. This statistics raises the question of "strategic playing" or any possibility of "scam" in the LUBAs. We investigate some general statistics and few properties related to the networks in the following subsections.

5.1 Network properties

We investigate several properties of the top winner nodes in the constructed networks. In particular, we investigate the degree of the winner node in bipartite network signifying her experience or activity, weighted degree of bipartite network signifying her intensity of participation or her total bid space, degree of projected bidder network representing the level of participation in the auctions participated by the winners, degree of weighted projected bidder network representing the co-activity, degree-centrality representing the importance of the winner node, clustering coefficient, average neighbor degree etc.

Property	Bidder ID					
	433B	434B	81B	2016B	1715B	Avg. bidder
Number of wins	37	30	28	7	5	–
Deg. in bipartite network	40	36	34	20	29	2.53
Weighted degree in bipartite network	2333	2083	3192	489	994	14.72
Degree in projected bidder network	822	2230	2494	2008	2060	881.87
Weighted degree in projected bidder network	1170	3081	3410	2456	2608	919.53
Degree centrality	0.22	0.6	0.67	0.54	0.55	0.235
Clustering coefficient	0.27	0.61	0.50	0.74	0.71	0.8
Avg. neighbor degree	877.73	1397.92	1269.34	1539.45	1505.83	1280.78
Avg. neighbor weighted degree	959.26	1448.93	1332.94	1510.55	1522.31	1290.01

Table 2: Network properties of the bidder network for top five winners. The bidder with ID '433B' is the topmost winner and the bidder with id '1715B' is the fifth ranked winner. The last column indicates the average behavior of a bidder in the network.

Table 2 shows the properties of the top five bidders and the corresponding average properties. As shown in the table, the degrees (weighted degrees) of top winners in the bipartite network are very high compared to the average degree and are found to be among top five (six) high degree values (except the fourth ranked bidder, i.e., '2016B') implying top winning bidders aggressively participate in many auctions. It also raises the question whether they really learn from their experience. However, there are also few bidders with high participation but very low number of wins (even zero). For instance, Bidder '435B' has participated in the highest number of auctions (i.e., 57) but did not win any of the auctions. This raises a question whether these auctions are addictive making few bidders play with high losses. We notice that the properties of the topmost winner (i.e, '433B') are quite different relative to other winners. Therefore, we intensively study the properties of the topmost winner in detail in later sections.

5.2 Other properties

5.2.1 Item values

The item value represents the market value of an item as described by the auctioneer prior to the auction. We study the values of the items on which usually top bidders try to concentrate. For this, we average the item values of all the auctions participated by a particular bidder and calculate the *Mean Item Value* (MIV) corresponding to the bidder. We then find out the average item value of any bidder by averaging the MIV of all the bidders and compare this average behavior to the MIV of the top winners as shown in Table 3. The last column of Table 3 shows that the top winners other than topmost bidder participate in auctions with high item values expressed as a percentage increase/decrease from the average case. This seems to be followed in any auction game where the highly rational players usually participate in only high valued items. Interestingly, for the topmost bidder MIV is around 52% less than the average MIV which indicates that she participates in low valued items where competition with rational bidders might be relatively less.

Bidder Id	MIV	% increase w.r.t. average value
Average (AVG)	76421.61	–
433B	36337.5	– 52.45
434B	193905.42	+153.73
81B	237708.82	+211.05
2016B	293379.25	+283.90
1715B	221727.41	+190.14

Table 3: Mean Item Values (MIV) of top five winners along with the percentage increase/decrease of MIV with respect to the average value.

5.2.2 Number of bids

The number of bids placed by a bidder in an auction indicates her intensity of bidding in that auction. To see how aggressive are top bidders in their bidding, we calculate the *Mean Number of Bids* (MNB) for every bidder by averaging the number of bids placed by a bidder in all the auctions she participated. Averaging the MNBs of all bidders indicates the mean behavior of an arbitrary bidder.

Bidder ID	MNB
Average	5.81
433B	58.32
434B	57.86
81B	93.88
2016B	24.45
1715B	34.27

(a)

Table 4: MNB for the top winners.

Bidder ID	MNB	Wins
1632B	904	0
1B	477	2
1642B	239	0
1576B	215	0
1645B	178	0

(b)

Table 5: Top five bidders having largest MNBs and their number of wins.

In Table 4, it can be observed that top winners are highly aggressive in their bidding whose values are very high compared to the average MNB. Especially, third ranked winner ('81B') has high MNB, which is also reflected in Table 3. It indicates that she has won many auctions with relatively high item values. Interestingly, highly aggressive bidders having largest MNBs are not top winners in terms of the number of auctions she has won. Precisely, even top 12 bidders having high MNBs do not fall in the list of top five winners. Table 5 shows the top five bidders having largest MNBs along with their total number of wins. The table also indicates that except the bidder with second-largest MNB, none of the others have won even a single auction, though they are the most aggressive bidders. This result indicates that LUBA mechanism may not have the general lottery system characteristic where the probability of winning is directly proportional to the number of bids [6].

5.2.3 Number of other bidders

The number of bidders co-participating in an auction indicates the level of competition that a particular bidder faces in an auction. While the auction is in progress the bidder has no idea about this value. Here, we measure the *Mean number of Other Bidders* (MOB) for a particular bidder by taking average of the number of other bidders for all auctions she participated. Therefore, average MOB is the average of MOBs of all the bidders taken together. Table 6 notes the average MOB of an arbitrary bidder as well as the

Bidder Id	Mean number of Other Bidders (MOB)
Average	50.08
433B	30.25
434B	86.58
81B	101.29
2016B	123.8
1715B	90.93

Table 6: Mean number of Other Bidders (MOB) participated in an auction for the top five winners.

individual MOBs of the top five winners. It can be observed from the table that all the top five winners other than the topmost winner have very high MOB values compared to the average MOB. This indicates that the winners mostly participate in auctions of high-valued items where the competition is usually more than average. For topmost winner, MOB is lower than the average suggesting that she is more interested to participate in auctions where the number of other bidders is relatively low. Since the MIV of the topmost winner is less (see Table 3), the MOB could also be less. The statistics of the second, third, fourth and fifth ranked winners are interesting because they are able to win many high-valued auctions where the pressure of competition is relatively higher than average which should usually be the case.

5.2.4 Range bids

Range bids are performed by selecting a range of bid values within the maximum allowable range set by the auctioneer and then placing a bid on each single value in that interval. This is an opportunity offered by the web site hosting the auctions. Each bid in this case is characterized by the same time stamp. Such a system is beneficial for agents who make a significant number of bids, but becomes less relevant for agents who invest relatively small amount of money. The idea behind this strategy is as follows. The first goal of a bidder (B) is to find a relatively small free number x, i.e., a number that was not bid so far by any bidder. If such a number is found, the second goal is to invalidate all the unique bids of other bidders on numbers lower than x, i.e., to bid on a number that was less than x and was bid by a single bidder (unique bid), thereby removing it from the list of potential winning bids. The third goal is to make it too costly for other bidders to invalidate the bid of B on x. To achieve the second goal, a bid (say, m) should be as small as possible, and, of course, smaller than x. To achieve the third goal, another bid (say, n) should be as large as possible and larger than x. A large n has the additional advantage that other possibly free numbers higher than x receive a bid. Therefore, if some other bidder invalidates x, a different possibility of winning the auction is still preserved. Having a large interval $[m, n]$ is costly, so that a strategic bidder faces a trade-off between increasing the probability of winning and having a lower bidding cost.

To see the usage of range bids in winning, we investigate the usage of range bids in the bid space of the winners. As given in Table 7, the percentage of range bids covering the total bid space of all the bidders in all auctions is 71.38%. Next, we only observe the winning bid in each auction and

% of rangebids in total bidspace	71.38
% of rangebids in win-bids	87.30
% of rangebids in all winners bidspace	92.18
% of rangebids in all non-winners bidspace	58.91

Table 7: Usage of range bids in the bid space by the winners.

find that the percentage of these winning bids that fall in range bids sums up to 87.3%, which is quite high. To see the usability of range bids for the winners, we only consider the bid space of the winners in all the auctions and find that 92.18% of their bid space is covered by range bids and the same test for non-winners shows that only 58.91% of their bid space comprises range bids. All these values together indicate that the winners significantly rely on range bids for their success.

5.2.5 Number of wins per participation

We hypothesize that just the number of wins may not be indicative of the efficiency of a bidder. Rather a more meaningful quantity might be the number of wins per participation (denoted by ρ) which is the ratio of the number of wins and the total number of auctions a bidder participated. We rank bidders based on the ρ value. Note that, the more the value of ρ of a bidder, the more she is placed at the top rank. We observe that top five winners hold the third, fourth, fifth, thirteenth and thirty-first positions respectively based on the value of ρ. We notice that the top two bidders having high ρ values have participated in just one auction, thus making their ρ values equal to 1. However, the next three rankers with high ρ values are also the top three winners which suggests that these bidders have a high success rate. Even fifth high ρ valued bidder has participated only in five auctions and won four out of them.

To observe how the ρ value changes as the bidders participate in the auctions, we plot the ρ values of the top five winners after a certain number of auctions she played where the auctions are time-ordered. Figure 5(a) shows that initially ρ remains zero until a bidder has not started participating in an auction. Once she starts participating, her ρ value rises up or goes down depending on her wins/loses or remains same if she does not participate. It can be observed from Figure 5(a) that the topmost winner initially won all the auctions she participated but lost only three times later thus making the ρ value very high in her complete history. Second and third ranked winners also won many initial auctions and lost only few auctions at various time steps. However, forth and fifth ranked winners started participating later while maintaining a steady behavior in terms of the ρ values. The slope of the plot in Figure 5(b) shows how fast the ρ changes for the top five winners.

6 Profit analysis: winners not gainers

The primary interest of both the auctioneers and the bidders is to increase each of their profit levels. However, in LUBA the winning of a bid for a bidder may not always

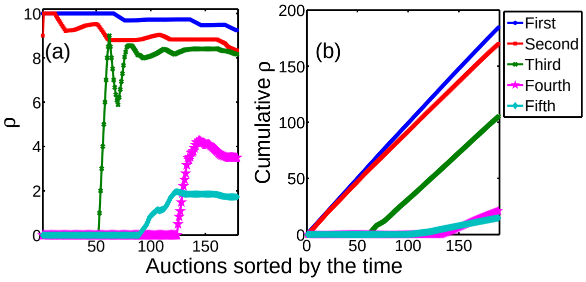


Figure 5: (Color online) (a) Non-cumulative and (b) cumulative distributions of ρ for top five winners with auctions sorted by the time. To smooth the curves, the best sliding window size of five years has been used.

imply a significant profit. Rather, the net profit of a bidder essentially depends on the frequency of bids she has submitted, final bidding amount and success/failure. In this section, we focus on analyzing profit for the bidders with an emphasis on the top five winners and their winning strategies.

6.1 Formulation of profit

Using the real data, profit/loss can be calculated for each bidder in every auction she participated using the number of bids placed, fee amount of the bid, actual value of the auction item put up for auction and the winning bid value. Precisely, the *profit* (Π) of a bidder can be measured by the following equation:

$$\Pi(p, A) = \begin{cases} N - (n_p \times B_A) - N_w & \text{if bidder } p \text{ is a winner} \\ -(n_p \times B_A) & \text{otherwise} \end{cases} \quad (1)$$

where $\Pi(p, A)$ is the profit of the bidder p in auction A, n_p= number of times she has submitted bids in auction A, B_A= predefined bid fee per bid submission, N and N_w are the actual value of the bidding item and the final winning value of the item respectively. If a bidder incurs a loss in an auction, her profit in that auction is negative; whereas if the bidder wins an auction, her profit is estimated as the actual value of the item announced prior to the auction less the total bid fees across the multiple biddings and the final winning value of the item. Once the profits of all the bidders for all the auctions are calculated, the net profit ($\widetilde{\Pi}$) of a bidder can be calculated by summing up the profit in each auction the bidder participated as shown below.

$$\widetilde{\Pi}(p) = \sum_{A \in C_p} \Pi(p, A) \quad (2)$$

where C_p represents the set of auctions where the bidder p participated so far.

6.2 Comparing profits and wins

Once the net profit is calculated for every bidder, we notice a surprising result that 3712 out of 3740 bidders (i.e., 99% of the bidders) are in loss suggesting that LUBA involves high risk and this is even more for newer users. It also raises the question whether this kind of auction mechanism is addictive as it makes participants play even with a negative net profit ($\widetilde{\Pi}$) which is possibly increasing over time. Table 8 shows the net profits of top winning bidders, and Table 9(a) shows the net profits of bidders with high net profit values. One might be tempted to believe that the top winners who won the maximum number of auctions are the bidders who gain high net profits. Interestingly, Table 8 and Table 9(a) indicate that this intuition may not be always true. Only two out of five high net profit bidders (marked in bold font in Table 9(a) belong to the list of top five winners. On the other hand, first, fourth and fifth ranked bidders in terms of net profit won only two, five and two auctions respectively. Table 9(b) shows the net profits of the bidders with high loss. Second most high loss bidder ('134B') won two auctions. Interestingly, fourth ranked bidder with high loss ('1715B') is one of the top five winners. This suggests that winners are not always rational with their bids.

Bidder ID	$\widetilde{\Pi}$	Rank w.r.t. $\widetilde{\Pi}$
433B	– 1127	1223
434B	230701	3
81B	1260639	2
2016B	30793	13
1715B	– 244366	3737

Table 8: Net profit ($\widetilde{\Pi}$) of the top five winners and their rank w.r.t $\widetilde{\Pi}$.

Bidder ID	$\widetilde{\Pi}$	Number of Wins
1B	2713933	2
81B	1260639	28
434B	230701	30
144B	163986	5
580B	117841	2

(a)

Bidder ID	$\widetilde{\Pi}$
1632B	– 90400
134B	– 809695
438B	– 244600
1715B	– 244336
1601B	–243000

(b)

Table 9: (a) Top five bidders with high $\widetilde{\Pi}$ values (top winners are highlighted in bold font); (b) net profit of top five high losers (fifth ranked winner is highlighted in bold font)

6.3 Profit distributions

In the previous subsection, we observed that the net profit of few top winners is negative even after winning many auctions. Therefore, in this section, we look into the profit distributions of the winners over the time scale. Figure 6 shows the distribution of profit for the top two winners with all the auctions arranged in time-order. The curves of rest of the winners follow the similar pattern as in Figure 6. All positive points in the distribution (marked by red circles) indicate that these are the auctions where the bidder won with a positive profit. The negative points show that the bidder incurred losses in the corresponding auctions. Among these, the green points are the ones where the bidders won the auctions but the profit for that auction was negative. From Figure 6(a), it is apparent that the topmost bidder won the initial 23 auctions with significant loss. Since a bidder is supposed to know the actual value of the item and the bid fee prior to joining an auction, it is easy to estimate the maximum number of bids that she would attempt to place or the risk she would wish to take to win with a positive profit even before the auction starts. Therefore, winning the item continuously with loss might indicate that the bidder is crazily addicted to win an auction even while incurring losses. We notice a similar behavior (shown in Figure 6(b)) for the second ranked winner who won first 12 auctions with loss. Hence one can suspect a possibility of scam by the members of the online auction site using these two bidders in expiring the auctions. The reason can be explained in two different ways: first of all, the popularity of the site could have been less in the initial days; secondly, to increase the interest in public, an illusion could have been created among them to portray that the site is very active.

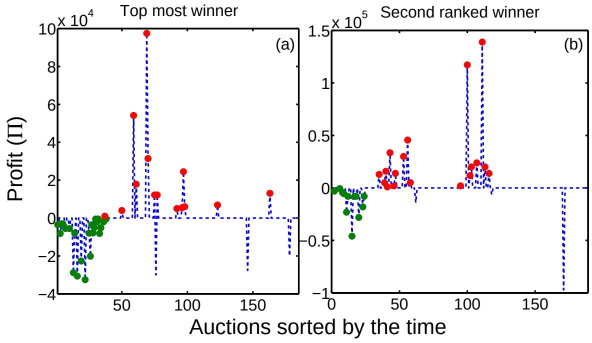


Figure 6: (Color online) Profit distributions of top two winners. The red points indicate the auctions where the bidder won with a positive profit while the green points indicate the auctions where the bidder won with a loss. Rest of the points represent the auctions where the bidder lost the auctions.

We also try to find out how do the bidders end with high losses by observing the profit distributions of high net loss bidders (i.e., the profit is highly negative as shown in Table 9. For this analysis, we concentrate on those bidders who suffered a series of low to medium losses throughout the auction rather than those who just suffered only one or two very high valued losses. Figure 7 shows the profit distribution of second ranked high loss bidder ('134B'). We observe from the figure that the highest proportion of her loss is caused by just one auction which immediately follows a significant win. This behavior is quite consistent for many bidders where high losses generally follow a significant win. The behavior can be explained in the following way: winning an auction can affect people to take higher risks in subsequent auctions making them bid aggressively which might eventually lead to a high loss. This again suggests that the LUBA auctions might be addictive making people play with high loss.

7 Memory-driven agent based dynamical model

As mentioned earlier, one of our primary objectives is to propose a dynamical agent based model in which multiple agents are selected to play a series of auctions so that the simulation results describe the real-world LUBA as accu-

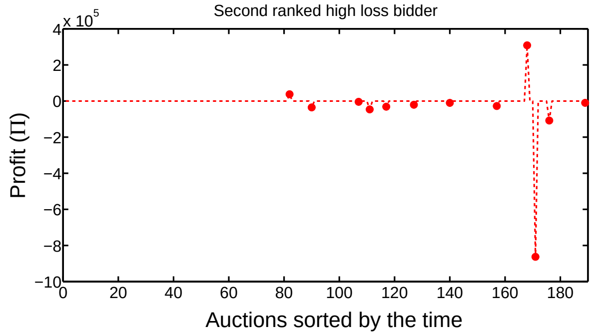


Figure 7: (Color online) The profit distribution of second ranked high loss bidder ('134B').

rately as possible. In this section, we describe a memory-driven model where each agent acts as a bidder. The bidders participating in the subsequent auctions are chosen preferentially based on their previous participation history. The eagerness of a bidder to submit multiple bids in an auction is modeled as a function of her previous success, profit and a randomness factor which is a parameter of the model. Algorithm 1 systematically describes the entire procedure.

The model works as follows. At the beginning, all the auctions in the real-world dataset are chronologically ordered according to the time of occurrences (represented as a set A). Then, the history of the first m auctions (denoted by the set M_0) are directly taken into account in the model in order to avoid the cold-start problem [13]. This serves as the initial database of our model. After this at each step, an auction a is selected from the list of remaining auctions (denoted by $A \setminus M_0$) (step 1); its corresponding bid buffer ($BIDS(a)$) is initialized as empty (step 2); and the auction parameters of a such as the number of bidders participating in the auction (U), the actual value of the item (I), the total number of allowable bids (T), the maximum allowable bid value (M) are taken from the real dataset for fair comparison (step 3). Bidders participating in a particular auction are chosen preferentially based on their extent of previous participations (step 4). Once the bidders are chosen, the actual bidding process starts for the auction a. Each bidder b maintains a memory ($BUFFER(b)$) where all the bids she has placed so far in this auction are stored. To make sure that every bidder participates in this auction, each bidder is initially made to place a random bid (step 6) and this bid is added to her memory as well as to the bid buffer (steps 7 and 8). These bids are treated as the initial bids for that auction. Now for each bidder, an *optimizing value* is calculated (step 9) which indicates the expertise of the bidder in LUBA. We hypothesize that the optimizing value of a bidder depends on the following parameters: proportion of wins (indicating the proficiency of the bidder to win auctions), normalized profit which is calculated using the previous history (indicating how rational the bidder is in playing auctions) and a random factor (probability of a new bidder to win the auction). Individual weights are associated with these factors, and these weights act as the tunable parameters in our model for a particular auction. Once these optimizing values are calculated for all the par-

Algorithm 1 Memory-driven agent based dynamical model

Input: B=a set of bidders, A= list of auctions chronologically ordered by the time of occurrences
Initialization: History of previous m auctions denoted by the set M_0; $BUFFER(b) = \phi, \forall b \in B$

1: **for all** $a \in A \setminus M_0$ **do**
2: $BIDS(a) = \phi$ ▷ Initialize the bid buffer that will contain all the raised bids for the auction a.
3: Select the action parameters: U = number of bidders participating, I= actual value of the item, T= total number of allowable bids, M= maximal allowable bid value (these values are directly taken from the real data for fair comparison).
4: Preferentially select U number of bidders from B (denoted by B_u) with high previous participations. If a bidder b has participated in n_b number of previous auctions, then the probability of selecting b is $p(b) = \frac{n_b+1}{\sum_{b \in B}(n_b+1)}$ (add-one smoothing is used)
5: **for all** $b \in B_u$ **do**
6: Select a bid $S_b = Rand(1, M)$
7: Place the bid: $BIDS(a) = BIDS(a) \cup \{S_b\}$
8: $BUFFER(b) = BUFFER(b) \cup \{S_b\}$
9: Calculate optimizing value (O_b) using the following equation:

$$O_b = [K_1 \times \frac{W_b}{K_2}] + [K_3 \times \sum_{a_b \in AUC(b)} \frac{\Pi_{a_b}(b)}{I_{a_b}}] + [K_4 \times Rand(0,1)] \quad (3)$$

where W_b=number of bids that b has won; $AUC(b)$=a set of auctions where b has participated; $\Pi_{a_b}(b)$=profit of b in the auction a_b; I_{a_b}=actual item value in the auction a_b; $K_1, K_3, K_4 \in \{0, 1\}$; $K_2 \in \{1, |AUC(b)|\}$, i.e., K_1, K_2, K_3 and K_4 are tunable parameters of the system; $Rand(0, 1)$ function generates a number between 0 to 1. Note that, $n_b = |AUC(b)|$.
Endfor
10: $i = 0$ ▷ Starting of sub-steps
11: **while** $i <= (T - U)$ **do**
12: Preferentially select a bidder b using optimizing value (O_b), i.e., probability of selecting a bidder b is determined by $p(b) = \frac{O_b}{\sum_{b_j \in B_u} O_{b_j}}$
13: Select a bid $S_b = Rand(0, M)$ such that $S_b \notin BUFFER(b)$
14: Place the bid $BIDS(a) = BIDS(a) \cup \{S_b\}$
15: $BUFFER(b) = BUFFER(b) \cup \{S_b\}$
Endwhile
16: Find the lowest and unique bid $S_j \in BIDS(a)$ and its corresponding bidder b_j. Declare her as the winner of the auction a
17: Set $BUFFER(b) = \phi, \forall b \in B$
Endfor

ticipating bidders in a particular auction, the rest of the bidding process continues. Now, placing a bid is considered as a *sub-step*. In each sub-step, the bidder who places a bid is chosen preferentially based on the high optimizing value (step 12). Once a bidder is selected, her bid value is randomly chosen within the auction limit such that it has not been placed by her before (step 13). This bid is added into the bid buffer (step 14) and appended to her own memory (step 15). If the number of sub-steps reaches the expiring limit of bids predefined by the auction, the auction stops. The winner of this auction is found by choosing the lowest and the unique bid from the bid buffer ($BIDS(a)$) (step 16). Once all these tasks are completed, the next auction is added into the system and the above steps are repeated. When all the auctions are completed, the model is ready for the evaluation. An illustrative example of the execution of our model for a particular bid is shown in Figure 8.

8 Evaluation of the model

In our model, the number of auctions ($|A|$), the number of bidders ($|B|$) and the number of initial auctions whose statistics are known to the model (m) are taken as 189, 3740 and 20 respectively. After completing the simulation, we compare the performance of our proposed model with the real-world LUBA data using Pearson correlation coefficient (ρ). In particular, our comparison is based on the following two parameters: (i) degree distribution and (ii) winning distribution of the bidders.

8.1 Comparing degree distributions

Evaluation of the model can be done by comparing the unweighted degree distribution of the bidder nodes of the real data with that obtained from our model. As mentioned earlier, the unweighted degree of a bidder indicates the number

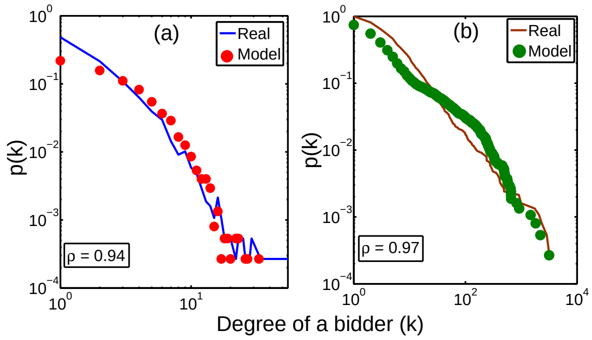
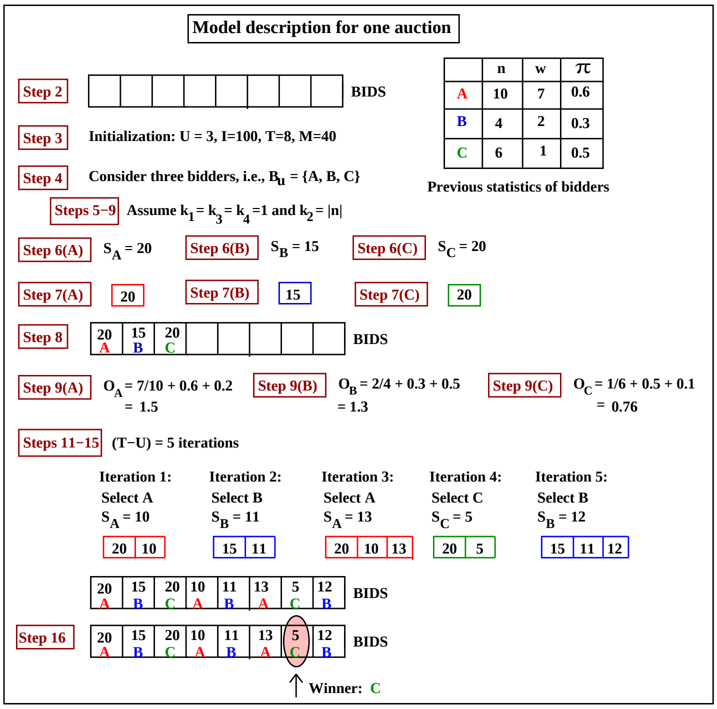


Figure 9: (Color online) The (a) unweighted and (b) weighted degree distributions of the bidders obtained from the model (circles) and from the real data (line).

of auctions she has participated so far. We observe that the simulation results match quite well ($\rho = 0.94$) with the real data for any permitted choices of parameters (K_1, K_2, K_3 and K_4) which are used to calculate the optimizing function of each bidder as mentioned in equation 3 of the algorithm. Figure 9(a) depicts the unweighted degree distributions of the bidders obtained from real-world data and from the model.

Evaluation can also be performed by comparing the weighted degree distribution of bidders obtained from the sum of all bids placed by a bidder in all the auctions she played so far. We compare the weighted degree distribution of the bidders obtained from the model and from the real data. We tune the parameters of the optimizing function to obtain the best similarity of the model output with the real distribution ($\rho = 0.97$), and the observed parameter values are: $K_1 = 1$, $K_4 = 0$ and for any permitted values of K_2 and K_3. Therefore, the optimizing function becomes $O_b = \frac{W_b}{K_2} + K_3 \times \sum_{a_b \in AUC(b)} \frac{\Pi_{a_b}(b)}{I_{a_b}}$ (the repre-



	n	w	π
A	10	7	0.6
B	4	2	0.3
C	6	1	0.5

Figure 8: (Color online) An illustrative example of the working principle of our model for a particular auction. n and w represent the number of auctions a bidder has participated and the number of auctions a bidder has won respectively. For the sake of clarity, in step 4 all three bidders are taken without considering any preferential selection. The buffers for individual bidders are represented in different colors.

sentative symbols are similar as mentioned in equation 3). Figure 9(b) depicts the weighted degree distributions of the bidders obtained from real-world data and from the model with $K_1 = 1, K_2 = |AUC(b)|, K_3 = 1$ and $K_4 = 0$.

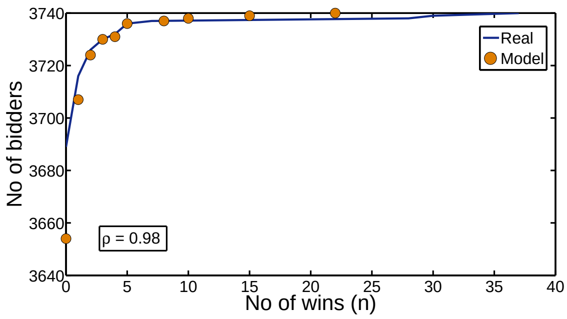


Figure 10: (Color online) The cumulative winning distribution of the bidders obtained from the model (circles) and from the real data (line). The value in y-axis corresponding to a particular value (say, m) in x-axis indicates the number of bidders who have won less than or equals to m number of auctions.

8.2 Comparing winning distributions

We further perform the evaluation by comparing the winning distribution of the bidder, i.e., number of bidders having a particular number of wins in real data and that generated by the model. Here also, we tune the parameters for gaining maximum similarity between the two distributions. We obtain the maximum similarity ($\rho = 0.98$) for $K_1 = 1, K_2 = 1$ and $K_4 = 0$ and for any permitted value of K_3; and therefore, the optimizing function becomes $O_b = W_b + K_3 \times \sum_{a_b \in AUC(b)} \frac{\Pi_{a_b}(b)}{I_{a_b}}$ (the representative symbols are similar as mentioned in equation 3). The results are most accurate with $K_3 = 0$, i.e., when the optimizing value completely depends on the number of wins of a bidder in all her participated auctions till that step. A boundary condition is carefully handled when $O_b = 0$, i.e., when a bidder has not won a single auction she participated so far. In such a situation, a small random value is assigned to O_b and therefore the immediate next selection is not preferential. Figure 10 shows the cumulative winning distribution of the real data and that generated from the model for the following choice of parameters: $K_1 = 1, K_2 = 1, K_3 = 0$ and $K_4 = 0$.

8.3 Discussions

From the two subsections discussed above, it is quite evident that the proposed model quite remarkably mimics the real-world behavior of LUBA. The values of different parameters are chosen in such a way that it can capture the real-world phenomenon as efficiently as possible. The final values of the parameters reported in the evaluation section lead us to infer few interesting insights mentioned below.
(i) We have noticed that while in Figure 9(a) the best correlation is achieved by combining all three factors involved in calculating optimizing function, the contribution of the random factor may be neglected in Figure 9(b) for the best accuracy measurement. This indicates that though the ran-

dom factor is partially involved in modeling the distribution of the number of auctions the bidders have participated, the average number of times that a bidder tends to bid in a single auction seems to be mostly determined by the two other factors, namely the proportion of wins and the net profit.
(ii) In Figure 10, we have noticed that the highest accuracy is achieved by considering only the proportion of wins. It indeed indicates that the bidders tend to bid only based on their past number of wins. They generally tend to avoid the factor that they are either in profit or loss. It again supports our earlier observation that these auctions might be so addictive that people would tend to participate without bothering much about investments.
(iii) From the two evaluation results, one might believe that the random factor does not seem to be quite important for calculating the optimizing function since most of the times the weight corresponding to the random factor has to be set to zero to obtain the most accurate match with the real data. However, we retain this random factor so that the model is generally applicable to other similar dataset where the random factor might be necessary to improve the accuracy of the results.

9 Conclusion

The paper has introduced and analyzed a peculiar selling mechanism that is becoming increasingly popular over the Internet: Lowest Unique Bid Auctions (LUBAs) that allocate valuable goods to the agent who submits the lowest bid that is not matched by any other bid. We conclude by summarizing our main observations, few important remarks and outlining some of the possible future directions. We observe that:
(i) the degree distribution (both unweighted and weighted) of the bidders follows power law behavior,
(ii) there are only 51 unique winners who won all the 189 auctions; among them 57% of the auctions are won by the top five winners (probably they learn from the previous wins),
(iii) top five winners aggressively participate in most of the auctions,
(iv) surprisingly, the bidder who participated in maximum number of auctions did not win a single one (this might be an outcome of addiction),
(v) top winners except the topmost winner participate in auctions with high item values which is quite expected in any kind of strategic endeavor – highly efficient players generally participate for only high returns,
(vi) the aggressive bidders are not always top winners,
(vii) range bid plays a key role in winning and the aggressive winners tend to heavily use this strategy,
(viii) most surprisingly, about 99% of the bidders are in loss in terms of the net profit,
(ix) the stochastic agent-based model efficiently captures two fundamental characteristics of LUBAs, i.e., the degree distribution and winning distribution of the bidders.

Sealed bid and range bid concepts have not been included in our model. In future, besides incorporating these two frequently used strategies in the model, we plan to design more sophisticated ways of evaluation that would strengthen the basic hypothesis on which our model is based. Moreover, we would also like to investigate other types of auctions games like HUBA, LUPI, LUPA etc.

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