Switching Circuits and Logic Design
CS21002 (section-1)
Tutorial 2

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1. Show that if there is a n-bit single-error correcting code for encoding objects in $\{0, \ldots, m-1\}$, then $m(n+1) \leqslant 2^n$. Conclude that there is no single error correcting 6-bit code for decimal digits.

2. Show that the only sum-of-products representation of the parity function is the canonical sum of minterms representation.

3. Design a two-level code converter from BCD to 2-out-of-5.

4. You are supplied with just one NOT gate and an unlimited amount of AND and OR gates and are required to design a circuit that realizes the expression $T(w, x, y, z) = w'x + x'y + xz'$. Only unprimed variables are available as inputs.

   Hint: You may find the map of T helpful.

5. (a) Define a half subtractor, show its truth tables, and derive logic expressions for difference (D) and borrow (B) outputs.

   (b) Define a full subtractor, show its truth tables, and derive logic expressions for difference (D) and borrow (B) outputs.

6. A communication system is designed to transmit just two code words, $A = 0010$ and $B = 1101$. However, owing to noise in the system, the received word may have as many as two errors. Design a combinational circuit that receives the words and that can correct one error and detect the existence of two errors. The circuit receives a 4-bit word $(X = X_0X_1X_2X_3)$ as input, and produces three switching output A, B and C. On each input, exactly one output bit is 1. Output A will be equal to 1 if the transmitted word is A and there has been at most one error, output B will be equal to 1 if the transmitted word is B and there has been at most one error, and output C will be equal to 1 if the word received has two errors and thus cannot be corrected.

7. With the aid of a four-variable Karnaugh map, derive minimal sum-of- products expressions for each of the following functions:

   (a) $f1(w, x, y, z) = \sum(0, 1, 2, 3, 4, 6, 8, 9, 10, 11)$;

   (b) $f2(w, x, y, z) = \sum(0, 1, 5, 7, 8, 10, 14, 15)$;

   (c) $f3(w, x, y, z) = \sum(0, 2, 4, 5, 6, 8, 10, 12)$.

8. Find the minimal sum-of-products and minimal product-of-sums expressions for $f(w, x, y, z) = \prod(1, 4, 5, 6, 11, 12, 13, 14, 15)$. Is your answer unique?

9. Given the function $T(w, x, y, z) = \sum(1, 2, 3, 5, 13) + \sum(6, 7, 8, 9, 11, 15)$:

   (a) find a minimal sum-of-products expression;

   (b) find a minimal product-of-sums expression;

   (c) compare the expressions obtained in (a) and (b); if they do not represent identical functions, explain why.

10. Figure out the adjacencies of a six-variable map. Show the representation of $T(u, v, w, x, y, z) = u'w'y' + uwy + w'xy'z$.

11. Show maps for four-variable functions with the following specifications. If this is impossible, explain why.

    (a) A function with eight minterms for which

    (i) there are no essential prime implicants.

    (ii) all the prime implicants are essential.

    (b) Repeat (a) for functions with nine minterms.

    (c) A function with an even number of prime implicants, of which exactly half are essential.

    (d) A function with six prime implicants, of which four are essential and two are covered by essential ones.

12. (a) Let f (x 1 , x 2 , . . . , x n) be equal to 1 if and only if exactly k of the variables equal 1. How many prime implicants does this function have?

    (b) Repeat (a) for the case where f assumes the value 1 if and only if k or more of the variables are equal to 1.