Online bipartite matching

$G = (V, E)$. $V = L \cup R$. $E \subseteq L \times R$.

L is known in advance.

In each time step, a vertex in R appears, along with the edges incident on it.

Task: To maintain a large matching of this graph. When a vertex arrives, the algorithm may match it to an unmatched neighbor.

Deterministic algorithm:

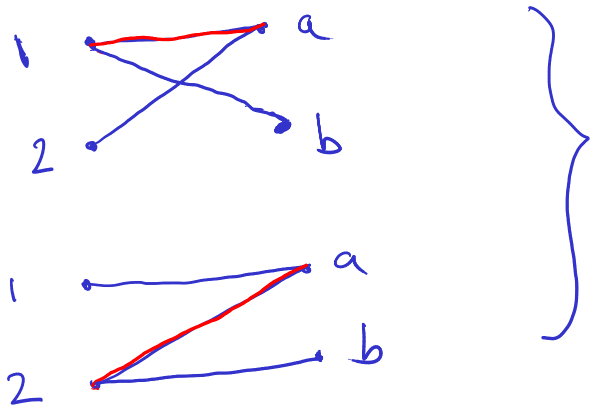
When a vertex arrives, match it to an arbitrary unmatched neighbor (if exists).

Verify: The computed matching is a maximal matching.

$\frac{1}{2}$-competitive (since any maximal matching is of size at least $\frac{1}{2} *$ |maximum matching|).

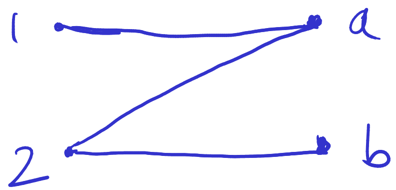
Claim: No deterministic algorithm is better than $\frac{1}{2}$-competitive.

<u>Proof</u>: Let A be a det. algo.



OPT $= 2$

Complete the argument by taking many copies of this basic graph and ensuring that OPT is a growing quantity.



: G_1

Rand. alg.: Batch an arriving vertex with a uniform random unmatched neighbour.

Expected cost on $G_1 = \frac{1}{2} \times 2 + \frac{1}{2} \times 1$
$= \frac{3}{2} > 1.$