Test 1: February 10, 2023: 11 am

Computational Geometry: LTP 3-0-0: Credits 3. Marks: 100
Time: 100 minutes
CS60064

1. Given a set S of n points in the plane, we wish to find the nearest neighbour in S for each of these n points in S. Show that we can do such a computation in $O(n \log n)$ time. [15 marks]

2. We are given n circles in the plane with varying radii and centres. We wish to compute the "convex hull" of these circles, the smallest convex region of the plane enclosing these circles. Design your best solution for this computation in terms of running time. [15 marks]

3. We are given a set S of n point sites in the plane. Given a query point p that is not one of these n sites, we wish to determine the site in S nearest to p in $O(\log^2 n)$ time using a preprocessed data structure that requires $O(n^2)$ space for preprocessing the set S. What preprocessing is suitable for this purpose? Can we improve the space and query complexity? If so, briefly state the better design that uses $o(n^2)$ space but logarithmic or poly-logarithmic query time. [10+10 marks]

4. Why is the space complexity of a 2d-range search tree $O(n \log n)$? [10 marks]

5. We preprocess n points in the plane for constructing a 2d-range tree. We are given a query with m axis-aligned rectangles. We wish to report the set of the k (distinct) points covered by these m rectangles, each point reported just once. Let q be the number of intersections between the edges of the m rectangles. Determine the best query time complexity you can achieve in terms of n, m, q and k. [15 marks]

6. For the 2d- linear programming problem, show that the worst-case time complexity for an incremental algorithm that processes one constraint at a time is $O(n^2)$. [10 marks]

7. In the planar partition incremental construction algorithm where the input has n line segments and the number of intersectings pairs of segments is k, what is the worst-case running time? Why? [15 marks]