Mid-semester examination: CS60064
Department of Computer Science and Engineering
IIT Kharagpur

Computational Geometry: Instructor: S P Pal: Time 2 hours: Maximum marks 100
February 22, 2023: 2 pm to 4 pm
Write precisely, clearly and with rigour.
You may make assumptions and state your assumptions, if necessary.

1. Design a linear time algorithm for computing the union of two convex polygons in the plane. The convex polygons are given in two separate lists. Each list has the coordinates of the vertices of the corresponding convex polygon in counterclockwise order around its boundary. You have to argue in your design discussion how the algorithm runs in linear time.

   [10 marks]

2. How is a "circle" event detected in Fortune's Voronoi diagram computation by plane-sweep? What actions are required when a "site" event occurs? (10 marks)

3. Let S be a set of n line segments in the plane with a total of k intersections between the line segments in S. Consider the set S_j of the first j line segments in a random permutation of the set S of n line segments, where $j < n$. Let k_j be the number of intersections between the line segments in S_j. Derive an upper bound on the expected value of the random variable k_j in terms of j, n and k.

   [10 marks]

4. For axis-parallel rectangular queries, given an arbitrary rectangle R, we wish to determine and report the k points from a given set S that lie inside the given query rectangle R. Here the rectangle has sides parallel to the axes. The set S of the n given points is preprocessed and a kd-tree is already constructed and provided. Outline the query procedure for reporting all the k points of S inside R, using the kd-tree in $O(f(n) + k)$ time where $f(n) = o(n^2)$.

   [10 marks]

5. Given a set S of n line segments in the plane, there may or may not be any intersections between the n line segments. Show that it is possible to decide whether there is at all any intersection in $O(n \log n)$ time.

   [10 marks]

6. Consider the intersection P of a set S of n halfspaces in $\mathbb{R}^3$, where each halfspace is defined by a bounding $2d$-plane in $\mathbb{R}^3$.

   How would you specify the halfplanes bounding the halfspaces in S? (2 marks)

   Show that the common intersection of the boundaries defined by the three halfplanes for any subset of three halfspaces in S is a point in space. You may assume that no four of the n halfplanes defining the halfspaces in S meet at a point, and no two of the halfplanes are parallel to each other. (2 marks)

   Show that P is convex and is made of $2d$-faces, edges and vertices defined by halfplanes in $3d$-space. (3 marks)

   Can P have faces and edges of infinite sizes? Why? (3 marks)

   Estimate the number of edges, faces and vertices of P in terms n? (3 marks)

   Design an incremental algorithm that processes halfspaces in S one at a time to compute M in $O(n^2)$ time. (5 marks)

   Show that we can achieve a better expected running time that is $o(n^2)$ using a random sequence for processing the halfspaces in S. (7 marks)

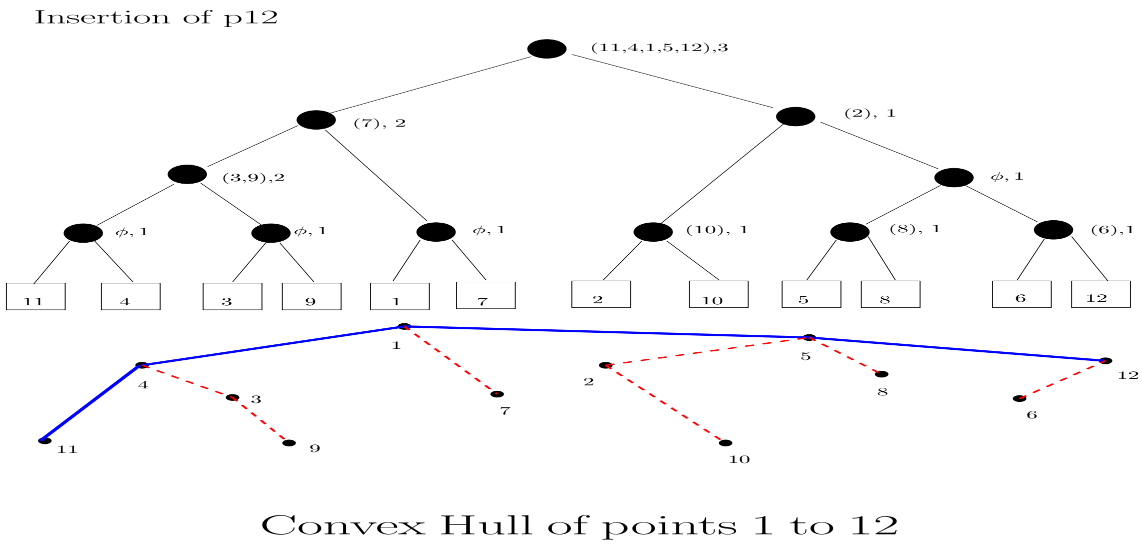

Figure 1: Upper hull of 12 points

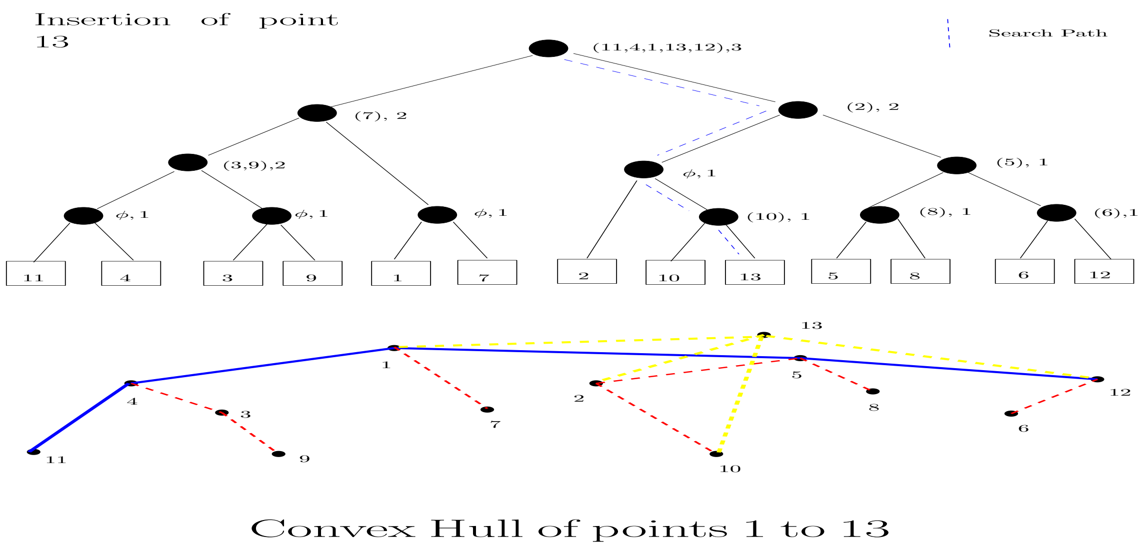

Figure 2: Insertion of point 13

7. Starting with 12 points, the upper hull is seen in the first figure below. Point 13 is then inserted. Finally, point 1 is deleted. The upper hull is maintained and represented at each step.

   (a) Show that we can maintain the main skeletal tree of the n leaf nodes and $n-1$ internal nodes in a balanced binary search tree along with the concatenable queues attached to the internal nodes in a total of $O(n)$ space. (7 marks)

   (b) Explain the representation of the partial and full hulls in concatenable queues in the diagrams. (8 marks)

   (c) Explain the insertion and deletion process and the maintenance of the whole data structure in these operations. (5+5 marks)

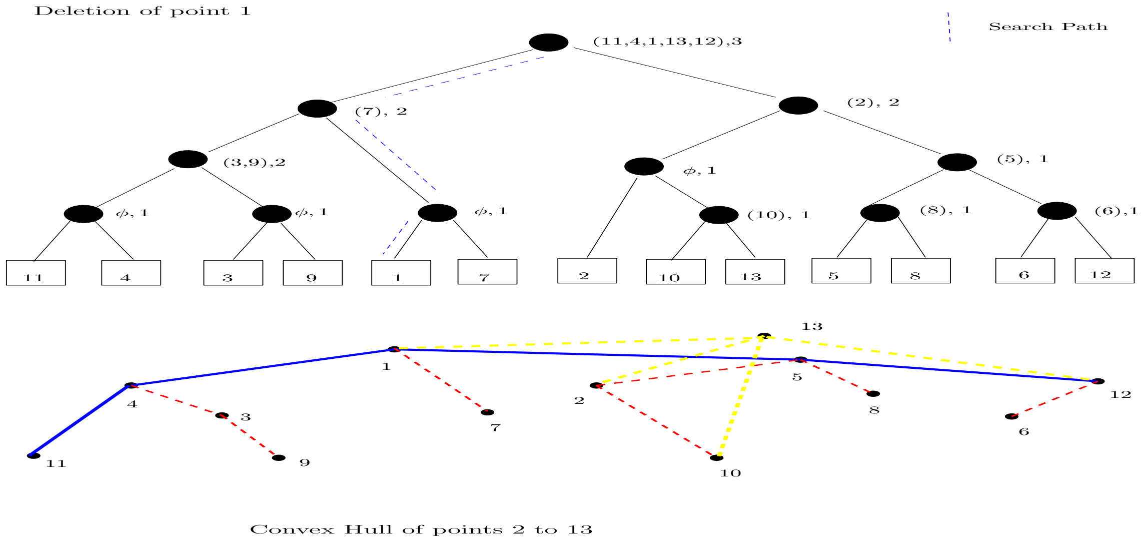

Figure 3: Deletion of point 1

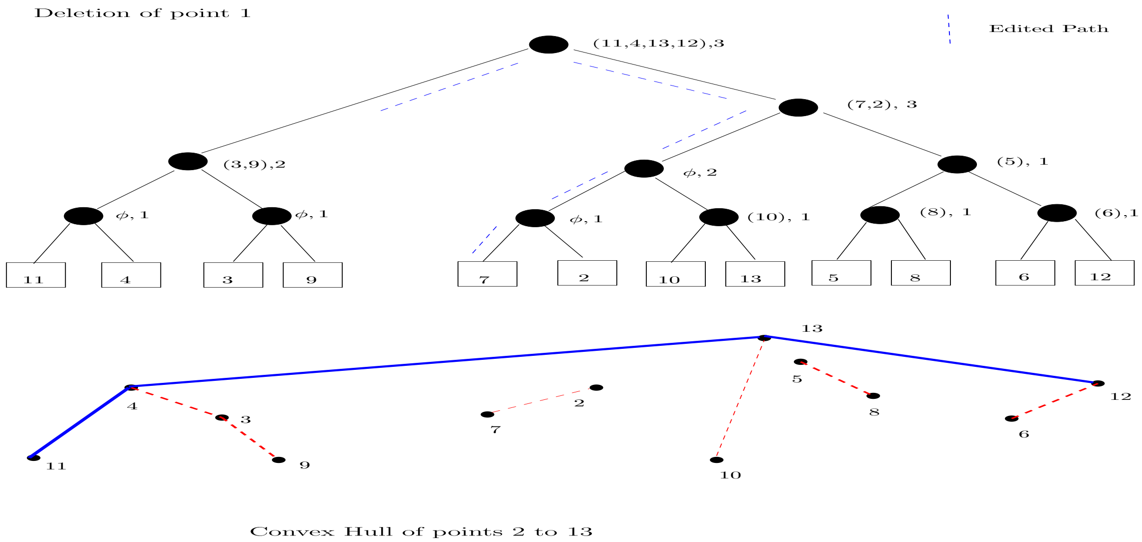

Figure 4: Upper hull of points 2 through 13