Computational Complexity (Autumn 2006: CS40007, CS60049)

Assignment 1

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Due on: 16/08/06

Exercise 1 *Show that* 4^{st} *raised to* $S(n)$ *exceeds the number of configurations of machine* M_1*, where* s *is the number of states of* M_1*,* t *is the cardinality of* M_1*'s alphabet and* $S(n)$ *is the space complexity of* M_1*.*

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Exercise 2 *Show that the decision problem of determining whether an* n*-vertex undirected graph has a vertex cover of size* $\lceil n/3 \rceil$ *is* ***NP****-complete.*

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Exercise 3 *Show that the problem of determining whether the language* $L(G)$ *generated by a given context-free grammar* G *is the empty set, is in the class* $\boldsymbol{P}$*. (See HU79). What happens for regular grammars? What is the input in these decision problems? [Note that regular grammars and context-free grammars may be nondeterministic.]*

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Exercise 4 *We simulate a* $k > 1$ *tape TM* M_1 *compressing* m *cells into 1 to get the same language recognized in* $cT(n)$ *time by another TM* M_2*, provided* $T(n)/n \to \infty$ *as* $n \to \infty$*, when we choose* m *such that* $mc \geq 16$*. Show that* M_2 *can indeed simulate* $T(n)$ *steps of* M_1 *as above; show particularly that at least* m *moves of* M_1 *can be simulated in 8 moves of* M_2*.*

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Exercise 5 *Prove : If* $inf_{n\to\infty} T(n)/n = \infty$ *and* $c > 0$*, then* $\boldsymbol{DTIME}(T(n)) = \boldsymbol{DTIME}(cT(n))$*.*

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Exercise 6 *In Theorem 12.3 [HU79], what happens if* $T(n)$ *is a constant multiple of* n*.*

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Exercise 7 *Extend the above two results to nondeterministic Turing machines.*

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Exercise 8 *Study Theorems 12.5 and 12.6 in [HU79].*