

Advanced graph theory: Test 1: CS60047

Autumn 2024

9:10 am to 10:50 am; Time: 100 minutes; Maximum marks: 100

August 30, 2024

1. Do bipartite graphs have the following property? For every induced subgraph H of G , $|V(H)| \leq \alpha(H)\omega(H)$. Establish your claim. Does this property hold for complements of bipartite graphs? Explain.
[10+10 marks]
2. Show that in a triangle-free graph G , the number of edges is at most $\alpha(G)\beta(G)$. [10 marks]
3. Show that in an undirected connected graph G , $\alpha(G) \geq \frac{|V|}{\Delta(G)+1}$ where $\Delta(G)$ is the maximum vertex degree. [15 marks]
4. Show that a tree can have at most one perfect matching. [10 marks]
5. Suppose a tree T has a perfect matching. Then show that $o(T \setminus \{v\}) = 1$ for every vertex v of T . [15 marks]
6. Prove Hall's theorem using Tutte's theorem. [15 marks]
7. Prove the Konig-Egervary theorem using Tutte's theorem. [15 marks]