CS60047 Advanced Graph Theory Autumn 2024
Homework 1

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Name ... (Roll No. ...)

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Each question carries 10 marks	Submission deadline: Before midsems
$*$ **For creating figures if required, it is advised to use ipe editor**	

1. Is it true that $\chi(G) + \chi(\overline{G}) \leq n + 1$, where n is the number of vertices of the perfect graph G ? Why? Does this inequality hold for general graphs as well? Explain.

   Response: LaTeX your response here.

2. Suppose $|V(G)| = n$ and $V(G)$ can be partitioned into k sets $\{V_1, V_2, \ldots, V_k\}$, such that for each $1 \leq i < j \leq k, \exists x \in V_i$ and $\exists y \in V_j$, so that x and y are non-adjacent. Then show that $\chi(G) \leq n - k + 1$.

   Response: LaTeX your response here.

3. Show that for an n vertex graph G, $\chi(G) \cdot \chi(\overline{G}) \geq n$.

   Response: LaTeX your response here.

4. The edge chromatic number of a graph G is denoted by $\chi'(G)$ and the maximum degree of G is denoted by Δ. Show that $\chi'(G) \leq 2\Delta - 1$ and for $\Delta \geq 3$, the bound can be improved as $\chi'(G) \leq 2\Delta - 2$.

   Response: LaTeX your response here.

5. For a natural number x, $[x]$ denotes the set $\{1, \ldots, x\}$. Let T be a tree and let τ be a finite family of subtrees of T such that each $t \in \tau$ has at least 2 vertices and $\forall i, j \in [|\tau|]$, $t_i \cap t_j \neq \emptyset$. Then show that $\cap_\tau \neq \emptyset$. This is called the Helly property of trees.

   Response: LaTeX your response here.

6. Prove that *extending* perfect graphs at vertices, by replacing vertices by respective (may be different) perfect graphs, preserves perfectness.

   Response: LaTeX your response here.

7. Show that for a connected graph G which is not $\Delta(G)$-regular, $\chi(G) \leq \Delta(G)$, where $\Delta(G)$ is the maximun vertex degree in G.

   Response: LaTeX your response here.