CS60021: Scalable Data Mining

Stream Mining

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Data Streams

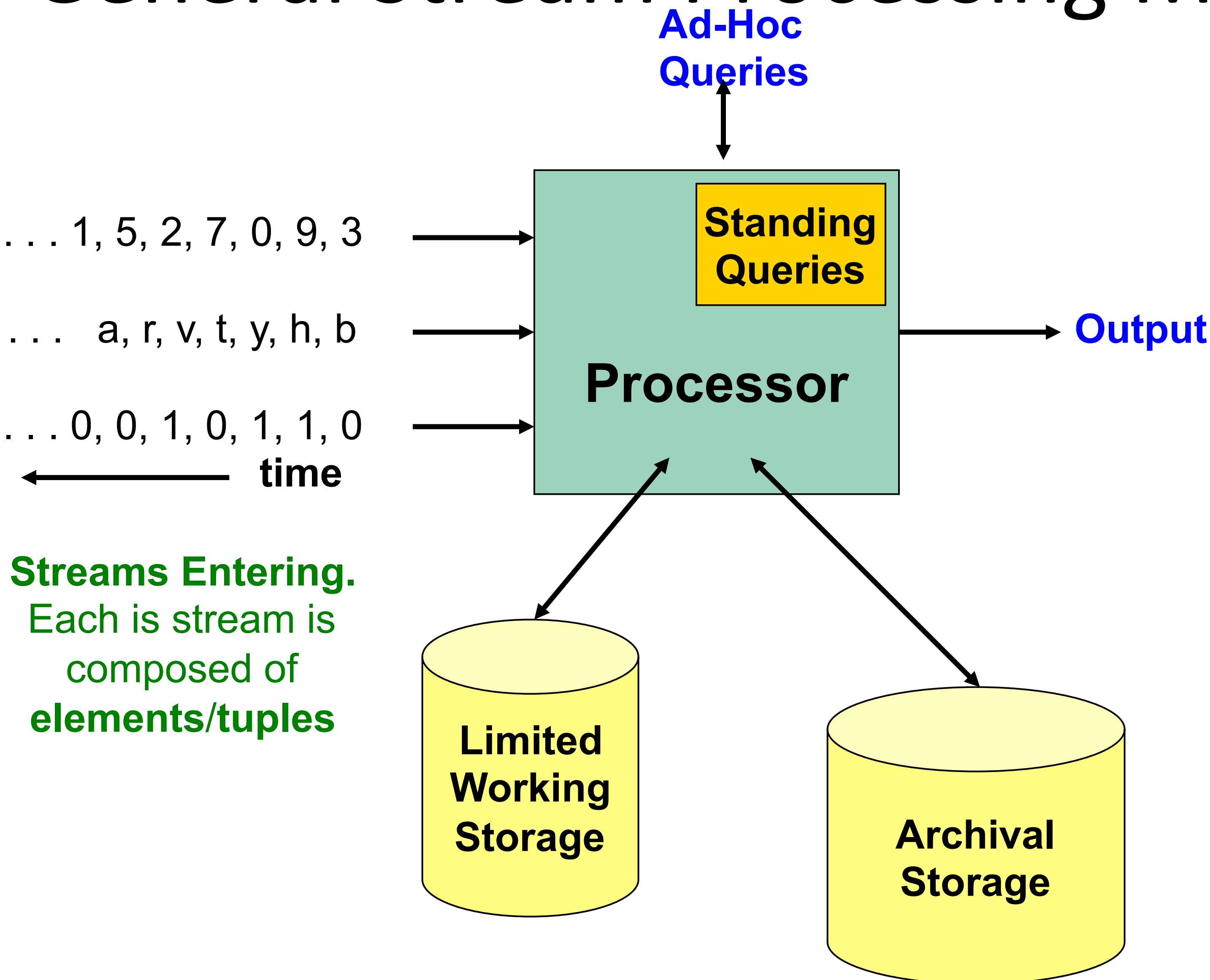
- **In many data mining situations, we do not know the entire data set in advance**

- **Stream Management** is important when the input rate is controlled **externally:**
  - Google queries
  - Twitter or Facebook status updates
- We can think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)

The Stream Model

- Input **elements** enter at a rapid rate, at one or more input ports (i.e., **streams**)
  - **We call elements of the stream tuples**
- **The system cannot store the entire stream accessibly**
- **Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?**

General Stream Processing Model



Streams Entering. Each is stream is composed of **elements/tuples**

Problems on Data Streams

- **Types of queries one wants on answer on a data stream:**
  - **Sampling data from a stream**
    - Construct a random sample
  - **Queries over sliding windows**
    - Number of items of type ***x*** in the last ***k*** elements of the stream

Problems on Data Streams

- **Types of queries one wants on answer on a data stream:**
  - **Filtering a data stream**
    - Select elements with property ***x*** from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last ***k*** elements of the stream
  - **Estimating moments**
    - Estimate avg./std. dev. of last ***k*** elements
  - **Finding frequent elements**

Applications

- **Mining query streams**
  - Google wants to know what queries are more frequent today than yesterday

- **Mining click streams**
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour

- **Mining social network news feeds**
  - E.g., look for trending topics on Twitter, Facebook

Applications

- **Sensor Networks**
  - Many sensors feeding into a central controller
- **Telephone call records**
  - Data feeds into customer bills as well as settlements between telephone companies
- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks

Sampling from a Data Stream: Sampling a fixed proportion

Sampling from a Data Stream

- Since **we can not store the entire stream**, one obvious approach is to store a **sample**
- **Two different problems:**
  - **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
    - At any "time" ***k*** we would like a random sample of ***s*** elements
      - **What is the property of the sample we want to maintain?** For all time steps ***k***, each of ***k*** elements seen so far has equal prob. of being sampled

Sampling a Fixed Proportion

- **Problem 1: Sampling fixed proportion**
- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Answer questions such as: How often did a user run the same query in a single days**
  - Have space to store **1/10th** of query stream
- **Naïve solution:**
  - Generate a random integer in **[0..9]** for each query
  - Store the query if the integer is **0**, otherwise discard

Problem with Naïve Approach

- **Simple question: What fraction of queries by an average search engine user are duplicates?**
  - Suppose each user issues ***x*** queries once and ***d*** queries twice (total of ***x*+2*d*** queries)
    - **Correct answer: *d*/(*x*+*d*)**
  - **Proposed solution: We keep 10% of the queries**
    - Sample will contain ***x*/10** of the singleton queries and **2*d*/10** of the duplicate queries at least once
    - But only ***d*/100** pairs of duplicates
      - **d/100 = 1/10 · 1/10 · d**
    - Of ***d*** "duplicates" ***18d/100*** appear exactly once
      - **18d/100 = ((1/10 · 9/10)+(9/10 · 1/10)) · d**
  - **So the sample-based answer is** $\frac{\frac{d}{100}}{\frac{x}{10}+\frac{d}{100}+\frac{18d}{100}} = \boldsymbol{\frac{d}{10x+19d}}$

Problem with Naïve Approach

- Question: what fraction of queries are duplicate?
- Suppose there are x single queries and d duplicate queries; (x+2d) stream items
- Fraction of duplicate queries: d/(x+d)
- Proposed approach: keep 10% sample
  - x/10 single queries, 2d/10 duplicate queries
  - 2d/100 queries marked duplicate, 18d/100 not.
  - Answer: (d/100) / ((x/10 +18d/100) + d/100)
  - Or: d / (10 x +19 d)

Solution: Sample Users

Solution:

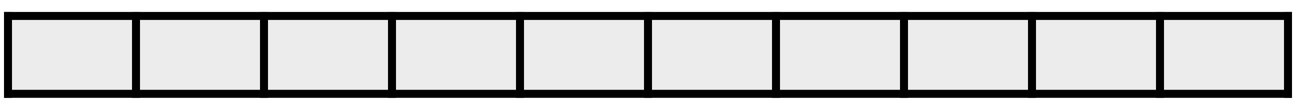
- Pick **1/10th** of **users** and take all their searches in the sample

- Use a hash function that hashes the user name or user id uniformly into 10 buckets

Generalized Solution

- **Stream of tuples with keys:**
  - Key is some subset of each tuple's components
    - e.g., tuple is (user, search, time); key is **user**
  - Choice of key depends on application

- **To get a sample of *a/b* fraction of the stream:**
  - Hash each tuple's key uniformly into ***b*** buckets
  - Pick the tuple if its hash value is at most ***a***



Hash table with **b** buckets, pick the tuple if its hash value is at most **a.**
How to generate a 30% sample?
Hash into b=10 buckets, take the tuple if it hashes to one of the first 3 buckets

Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of fixed size

Maintaining a fixed-size sample

- **Problem 2: Fixed-size sample**
- **Suppose we need to maintain a random sample *S* of size exactly *s* tuples**
  - E.g., main memory size constraint
- **Why?** Don't know length of stream in advance
- **Suppose at time *n* we have seen *n* items**
  - **Each item is in the sample *S* with equal prob. *s/n***

How to think about the problem: say s = 2

Stream: a x c y z k c d e g…

At **n= 5,** each of the first 5 tuples is included in the sample **S** with equal prob.

At **n= 7,** each of the first 7 tuples is included in the sample **S** with equal prob.

Impractical solution would be to store all the *n* tuples seen so far and out of them pick *s* at random

Solution: Fixed Size Sample

- **Algorithm (a.k.a. Reservoir Sampling)**
  - Store all the first ***s*** elements of the stream to ***S***
  - Suppose we have seen ***n-1*** elements, and now the n^{th} element arrives ($n > s$)
    - With probability ***s/n***, keep the n^{th} element, else discard it
    - If we picked the n^{th} element, then it replaces one of the ***s*** elements in the sample ***S***, picked uniformly at random
- **Claim:** This algorithm maintains a sample ***S*** with the desired property:
  - After ***n*** elements, the sample contains each element seen so far with probability ***s/n***

Proof: By Induction

- **We prove this by induction:**
  - Assume that after n elements, the sample contains each element seen so far with probability s/n
  - We need to show that after seeing element $n+1$ the sample maintains the property
    - Sample contains each element seen so far with probability $s/(n+1)$
- **Base case:**
  - After we see **n=s** elements the sample **S** has the desired property
    - Each out of **n=s** elements is in the sample with probability $s/s = 1$

Proof: By Induction

- **Inductive hypothesis:** After ***n*** elements, the sample ***S*** contains each element seen so far with prob. ***s/n***
- **Now element *n+1* arrives**
- **Inductive step:** For elements already in ***S***, probability that the algorithm keeps it in ***S*** is:

$$\underbrace{\left(1-\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ discarded}}+\underbrace{\left(\frac{s}{n+1}\right)}_{\substack{\text{Element } \mathbf{n+1} \\ \text{not discarded}}}\underbrace{\left(\frac{s-1}{s}\right)}_{\substack{\text{Element in the} \\ \text{sample not picked}}}=\frac{n}{n+1}$$

- So, at time ***n,*** tuples in ***S*** were there with prob. **s/n**
- Time ***n→n+1,*** tuple stayed in ***S*** with prob. **n/(n+1)**
- So prob. tuple is in ***S*** at time ***n+1*** = $\boldsymbol{s/n \cdot n/n+1 = s/n+1}$

Queries over a (long) Sliding Window

Sliding Windows

- A useful model of stream processing is that queries are about a ***window*** of length ***N*** – the ***N*** most recent elements received

- **Interesting case:** ***N*** is so large that the data cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows for all cannot be stored
- **Amazon example:**
  - For every product **X** we keep 0/1 stream of whether that product was sold in the **n**-th transaction
  - We want answer queries, how many times have we sold **X** in the last **k** sales

Sliding Window: 1 Stream

- **Sliding window on a single stream:** **N = 6**

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past Future →

Counting Bits (1)

- **Problem:**
  - Given a stream of **0**s and **1**s
  - Be prepared to answer queries of the form **How many 1s are in the last *k* bits?** where $k \leq N$

- **Obvious solution:**
  Store the most recent ***N*** bits
  - When new bit comes in, discard the ***N*+1**st bit

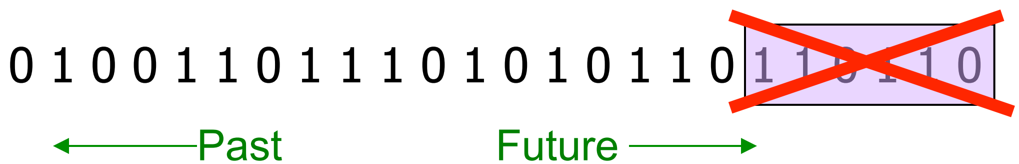
0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 $\boxed{1\ 1\ 0\ 1\ 1\ 0}$ Suppose N=6

← Past Future →

Counting Bits (2)

- **You can not get an exact answer without storing the entire window**

- **Real Problem:**
  What if we cannot afford to store *N* bits?
  – **E.g.**, we're processing 1 billion streams and **N = 1 billion**



- **But we are happy with an approximate answer**

An attempt: Simple solution

- **Q: How many 1s are in the last *N* bits?**
- A simple solution that does not really solve our problem: **Uniformity assumption**

N

0 1 0 0 1 1 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 0 0 1 0 1 0 1 1 0 0 1 1 0 1 0

Past Future

- **Maintain 2 counters:**
  - ***S***: number of 1s from the beginning of the stream
  - ***Z***: number of 0s from the beginning of the stream
- **How many 1s are in the last N bits?** $N \cdot S / S + Z$
- **But, what if stream is non-uniform?**
  - What if distribution changes over time?

DGIM Method

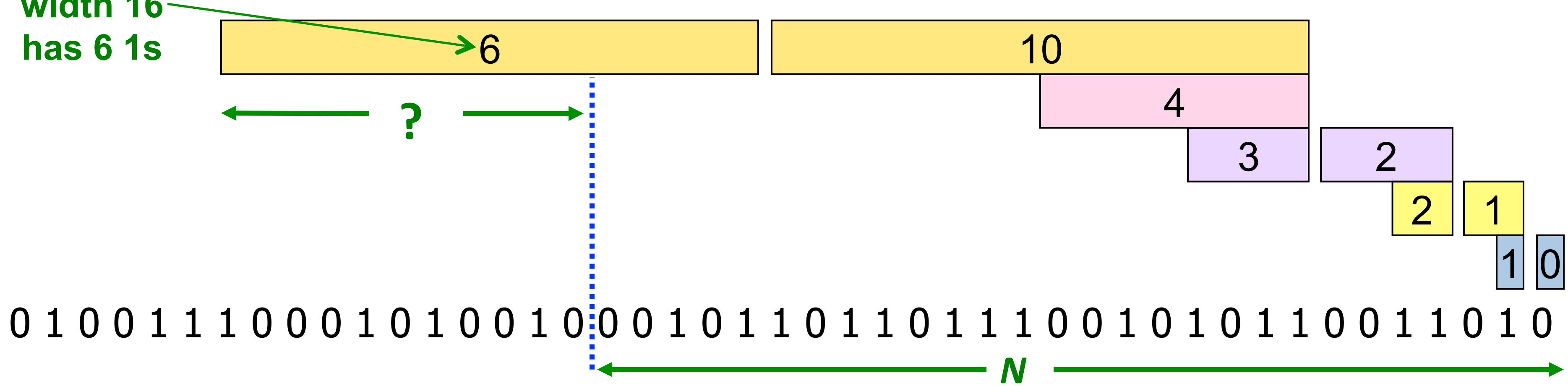
- **DGIM solution that does <u>not</u> assume uniformity**
- We store $O(\log 2N)$ bits per stream
- **Solution gives approximate answer, never off by more than 50%**
  - Error factor can be reduced to any fraction > 0, with more complicated algorithm and

DGIM Method

- DGIM solution does not assume uniformity
- Store $O(\log N)^2$ bits per stream.
- Solution gives and answer which is never off by more than 50%
  - Can be improved to arbitrary factor *1/r, r>0*.

Idea: Exponential Windows

- **Solution that doesn’t (quite) work:**
  – Summarize **exponentially increasing** regions of the stream, looking backward
  – Drop small regions if they begin at the same point as a larger region



We can reconstruct the count of the last ***N*** bits, except we are not sure how many of the last **6 1s** are included in the ***N***

What's Good?

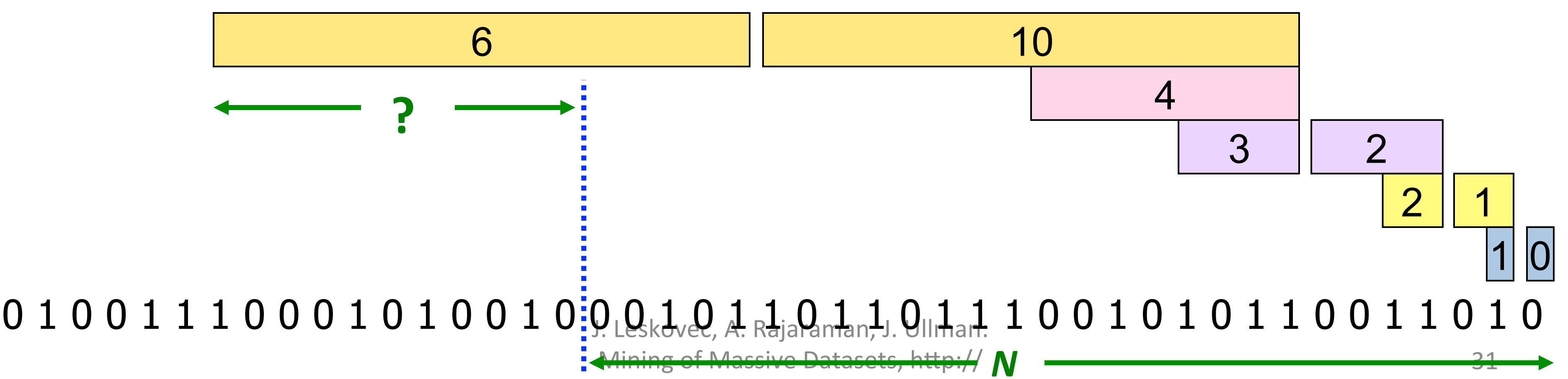
- **Stores only $O(\log^2 N)$ bits**
  - $O(\log N)$ counts of $\log_2 N$ bits each

- **Easy update as more bits enter**

- Error in count no greater than the number of **1s** in the "**unknown**" area

What's Not So Good?

- As long as the **1s** are fairly evenly distributed, the error due to the unknown region is small – **no more than 50%**
- But it could be that all the **1s** are in the unknown area at the end
- In that case, **the error is unbounded!**



Fixup: DGIM method

- **Idea:** Instead of summarizing fixed-length blocks, summarize blocks with specific number of **1s**:
  - Let the block ***sizes*** (number of **1s**) increase exponentially

- **When there are few 1s in the window, block sizes stay small, so errors are small**

1001010110001011 0 101010101010101 1 0 10101010101011 1 0 1010101 110101 000 101 1 00 1 0

N

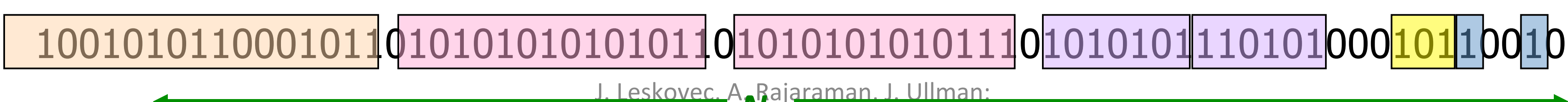
DGIM: Timestamps

- Each bit in the stream has a ***timestamp***, starting **1**, **2,** …
- Record timestamps modulo ***N*** (**the window size**), so we can represent any **relevant** timestamp in $\boldsymbol{O}(\boldsymbol{\log_2 N})$ bits

DGIM: Buckets

- A ***bucket*** in the DGIM method is a record consisting of:
  - **(A) The timestamp of its end [O(log *N*) bits]**
  - **(B) The number of 1s between its beginning and end [O(log log *N*) bits]**

- **Constraint on buckets:**
  Number of **1s** must be a power of **2**
  - That explains the **O(log log *N*)** in **(B) above**



Representing a Stream by Buckets

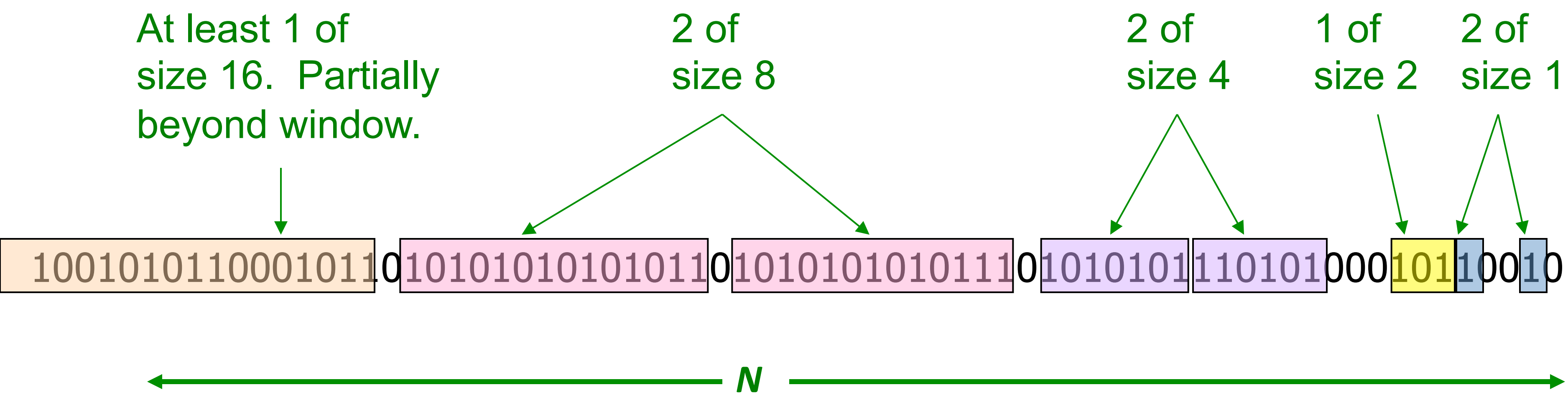
- Either **one** or **two** buckets with the same **power-of-2 number** of **1s**

- **Buckets do not overlap in timestamps**

- **Buckets are sorted by size**
  – Earlier buckets are not smaller than later buckets

- Buckets disappear when their end-time is **> *N*** time units in the past

Example: Bucketized Stream



Three properties of buckets that are maintained:

- Either **one** or **two** buckets with the same **power-of-2** number of **1s**
- Buckets do not overlap in timestamps
- Buckets are sorted by size

Updating Buckets (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to ***N*** time units before the current time

- **2 cases:** Current bit is **0** or **1**

- **If the current bit is 0:**
  no other changes are needed

Updating Buckets (2)

- **If the current bit is 1:**
  - **(1)** Create a new bucket of size **1**, for just this bit
    - **End timestamp = current time**
  - **(2)** If there are now **three buckets of size 1**, **combine the oldest two into a bucket of size 2**
  - **(3)** If there are now **three buckets of size 2**, **combine the oldest two into a bucket of size 4**
  - **(4) And so on …**

Example: Updating Buckets

Current state of the stream:

1001010110001011 0 1010101010101011 0 101010101010111 0 1010101 1101010 00 1011 0 010

Bit of value 1 arrives

001010110001011 0 1010101010101011 0 101010101010111 0 1010101 1101010 00 1011 0 0 10 1

Two orange buckets get merged into a yellow bucket

001010110001011 0 1010101010101011 0 101010101010111 0 1010101 1101010 00 1011 0010 1

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

010110001011 0 1010101010101011 0 101010101010111 0 1010101 1101010 00 1011 0010 1 1 0 1

Buckets get merged…

010110001011 0 1010101010101011 0 101010101010111 0 1010101 1101010 00 1011 0010 1 10 1

State of the buckets after merging

010110001011 0 1010101010101011010101010101110 1010101110101000 10110010 1 10 1

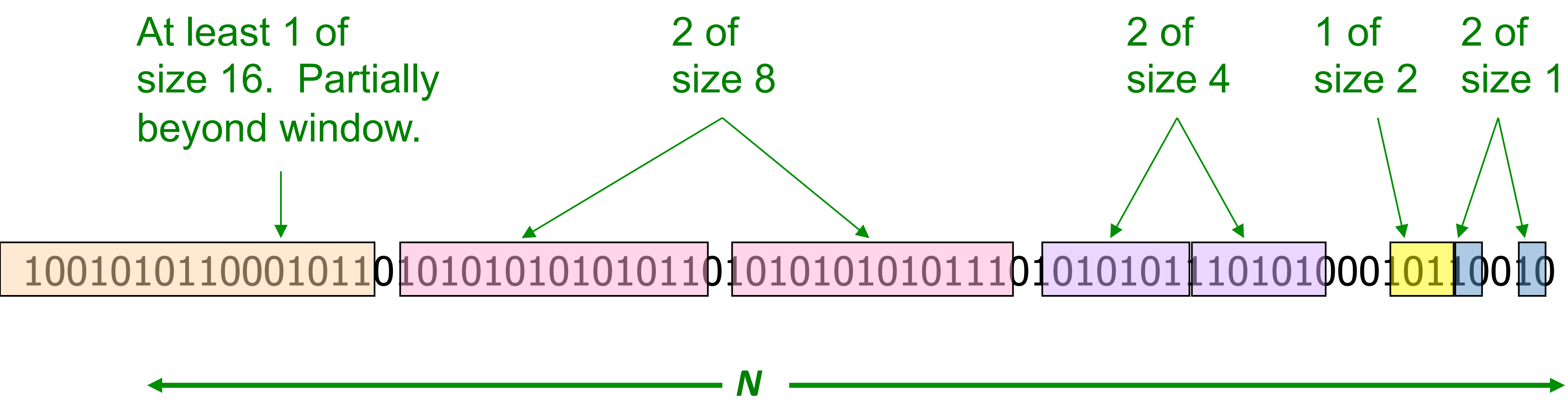
How to Query?

- **To estimate the number of 1s in the most recent *N* bits:**
  1. **Sum the sizes of all buckets but the last**

     (note “size” means the number of 1s in the bucket)
  2. **Add half the size of the last bucket**

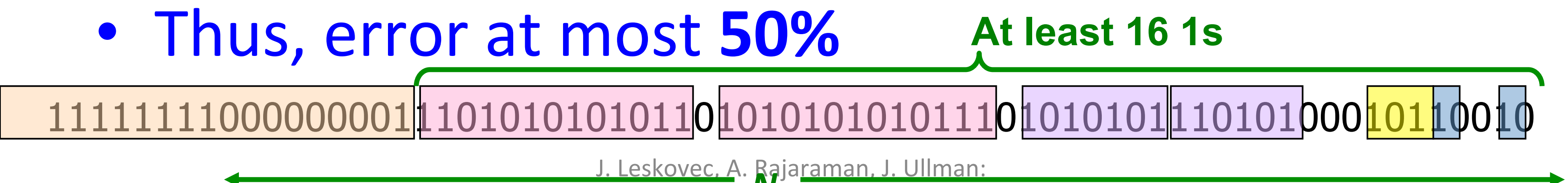
- **Remember:** We do not know how many **1s** of the last bucket are still within the wanted window

Example: Bucketized Stream



Error Bound: Proof

- **Why is error 50%? Let's prove it!**
- Suppose the last bucket has size $\mathbf{2^{j}}$
- Then by assuming $\mathbf{2^{j-1}}$ (i.e., half) of its **1s** are still within the window, we make an error of at most $\mathbf{2^{j-1}}$
- Since there is at least one bucket of each of the sizes less than $\mathbf{2^{j}}$, the true sum is at least $\mathbf{1 + 2 + 4 + .. + 2^{j-1} = 2^{j} -1}$
- Thus, error at most **50%**



Further Reducing the Error

- Instead of maintaining **1** or **2** of each size bucket, we allow either ***r*-1** or ***r*** buckets (***r* > 2**)
  - Except for the largest size buckets; we can have any number between **1** and ***r*** of those
- **Error is at most *O(*1/*r)***
- By picking ***r*** appropriately, we can tradeoff between number of bits we store and the error

Analysis

- Actual count is c.
- There are at least d and at most d+1 buckets of each size.
- Answer is returned from 2^j size bucket.
- **Case 1**: estimate < c
  - Final bucket: 2^{j-1}. c is at least $(d * 2^j)$.
    diff / c < 1 / 4d
- **Case 2**: estimate > c
  - Final bucket: 2^{j-1}. c is at least $(d * 2^j)$.
    diff / c > 1 / 4d
- Fractional error = $(2^{j-1} - 1) / (1 + d*2^j - 1)$

Extensions

- Can we use the same trick to answer queries **How many 1's in the last *k*?** where ***k* < *N***?
  - **A:** Find earliest bucket **B** that at overlaps with ***k***. Number of **1s** is the **sum of sizes of more recent buckets + ½ size of B**

10010101100010110101010101010101101010101010101111010101011101010001011001 0

← *k* →

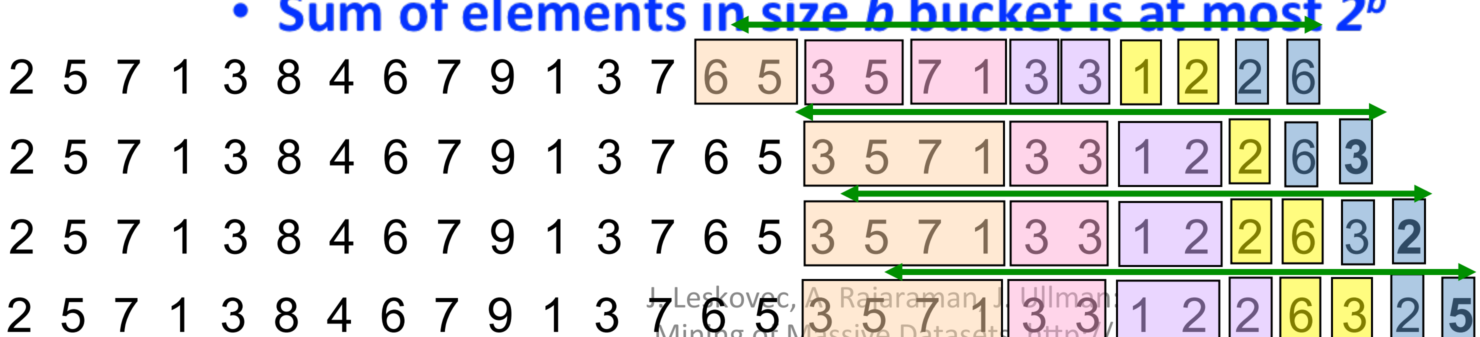
- **Can we handle the case where the stream is not bits, but integers, and we want the sum of the last *k* elements?**

Extensions

- **Stream of positive integers**
- **We want the sum of the last *k* elements**
  - **Amazon:** Avg. price of last **k** sales
- **Solution:**
  - **(1) If you know all have at most *m* bits**
    - Treat ***m*** bits of each integer as a separate stream
    - Use DGIM to count **1s** in each integer
    - The sum is $=\sum_{i=0}^{m-1} c_i 2^i$

c_i …estimated count for **i-th** bit

  - **(2) Use buckets to keep partial sums**
    - **Sum of elements in size *b* bucket is at most 2^b**



Idea: Sum in each bucket is at most 2^b (unless bucket has only **1** integer)

Bucket sizes:

16	8	4	2	1

Summary

- **Sampling a fixed proportion of a stream**
  - Sample size grows as the stream grows
- **Sampling a fixed-size sample**
  - Reservoir sampling
- **Counting the number of 1s in the last N elements**
  - Exponentially increasing windows
  - Extensions:
    - Number of 1s in any last k (k < N) elements
    - Sums of integers in the last N elements