Lecture 2: Vehicle/Driver/Traffic Modeling

Introduction to Traffic Modeling

Prof. Sangyoung Park
Module "Vehicle-2-X: Communication and Control"
Contents

- Vehicle Dynamics
- Traffic Models
  - Microscopic
  - Macroscopic
- Driver Behaviors
Vehicle Dynamics

- Study on vehicles in motion
- How the vehicles react to driver inputs on a given road
- Factors
  - Drivetrain and braking
  - Suspension and steering
  - Distribution of mass
  - Aerodynamics
  - Tires

Source: mathworks
Drive Resistance

- $v(t)$: vehicle velocity
- $a(t)$: vehicle acceleration
- $m_{tot}$: total vehicle mass
Drive Resistance

- $F(t) = F_{air}(t) + F_{c}(t) + F_{r}(t) + F_{acc}(t)$
- $P_{t}(t) = F_{t}(t) \cdot v(t)$

- $F_{t}(t)$: Traction force
- $F_{air}(t)$: Aerodynamic drag
- $F_{c}(t)$: Climbing force
- $F_{r}(t)$: Rolling resistance
- $F_{acc}(t)$: Acceleration force
- $P_{t}(t)$: Traction power
- $v(t)$: Vehicle velocity

Attention: $P_{t}(t) \neq P_{motor}(t)$
Aerodynamic Drag

- \( F_{air} = \frac{1}{2} \rho_{air} C_d A v_{rel}^2 \)

- \( \rho_{air} \): density of air, 1.225 kg/m\(^3\)
- \( C_d \): drag coefficient
- \( A \): frontal area
- \( v_{rel} \): relative velocity \((v_{rel} = v_{vehicle} + v_{wind})\)

Measured Drag Coefficients
Aerodynamic Drag

- Drag coefficients of vehicle types

<table>
<thead>
<tr>
<th></th>
<th>$C_d$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger vehicle</td>
<td>0.28</td>
<td>1.5-2.8</td>
</tr>
<tr>
<td>Transporter</td>
<td>0.35</td>
<td>3.0</td>
</tr>
<tr>
<td>Coach (long distance bus)</td>
<td>0.4</td>
<td>7.5</td>
</tr>
<tr>
<td>Bus 12 m</td>
<td>0.6</td>
<td>8.3</td>
</tr>
<tr>
<td>ICE 3</td>
<td>0.2</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Source: Prof. Voß (2016), Vorlesung Alternative Antriebssysteme und Fahrzeugkonzepte
Drag Resistance vs Velocity

- Power to overcome aerodynamic drag
- Again, $P = F \cdot v$, so what is the relationship between $F$ and $v$ then?

Larminie (2003), Electric Vehicle Technology Explained
Drag Resistance

- Vehicles’ shapes have become more aerodynamic over time

![Graph showing the change in drag resistance over time with car images representing different eras and years.](image)
Rolling Resistance

- Force resisting the motion when a body “rolls” on a surface
  - Deformation of the tire: Tire gets hot because tire is not perfectly elastic
  - Air circulation: Work is done on the air around the tire
  - Sliippage: Tire gets hot due to friction

<table>
<thead>
<tr>
<th>What</th>
<th>Surface of tire and air</th>
<th>Tire tread</th>
<th>Sidewall and bottom part</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Air circulation</td>
<td>Deformation hence dissipation of energy</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Slippage on ground</td>
<td>bending</td>
<td>bending</td>
</tr>
<tr>
<td></td>
<td></td>
<td>compression</td>
<td>shearing</td>
</tr>
<tr>
<td>How</td>
<td></td>
<td>shearing</td>
<td></td>
</tr>
<tr>
<td>Contri-</td>
<td>&lt; 15%</td>
<td>60 to 70%</td>
<td>20 to 30%</td>
</tr>
<tr>
<td>bution</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: http://thetiredigest.michelin.com/michelin-ultimate-energy-tire
Rolling Resistance

- $F_r(\alpha) = C_{rr} m_{tot} g \cdot \cos(\alpha)$, where
  - $C_{rr}$: Coefficient of rolling resistance
  - $m_{tot}$: Total vehicle mass
  - $g$: Standard gravity
  - $\alpha$: slope angle
Rolling Resistance

- $F_r(\alpha) = C_{rr} m_{tot} g \cdot \cos(\alpha)$

<table>
<thead>
<tr>
<th>$C_{rr}$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0003 to 0.0004</td>
<td>Railroad steel on steel rail</td>
</tr>
<tr>
<td>0.0022 to 0.0050</td>
<td>Bicycle tires</td>
</tr>
<tr>
<td>0.0100 to 0.0150</td>
<td>Ordinary car tires on concrete</td>
</tr>
<tr>
<td>0.3000</td>
<td>Ordinary car tires on sand</td>
</tr>
</tbody>
</table>

- How much force is required for rolling a 1000 kg car on concrete?
  - $F_r = 0.01 \times 1000 \times 9.8 = 98 \text{ N}$

- On sand?
  - $F_r = 0.3 \times 1000 \times 9.8 = 2,940 \text{ N}$
Rolling Resistance

- **Other factors**
  - Vehicle speed: But not as much as it affects drag
  - Tire pressure: low pressure means more deformation
Climbing Resistance

- $F_c(\alpha) = mg \cdot \sin(\alpha)$
- What is 10% in the sign?
- Slope $[\%] = \frac{dh}{dx} = \tan(\alpha)$
- 45° is 100% and 5.7° is 10%

The steepest roads in the world are Baldwin Street in Dunedin (38%) , New Zealand and Canton Avenue in Pittsburgh (37%) , Pennsylvania.
Acceleration Force

- \( F_{acc} = (m_{vehicle} + m_{acc}) \cdot \dot{v} \)
- \( m_{vehicle} \): Vehicle mass
- \( m_{acc} \): Equivalent acceleration mass
- Force is being applied to change the motion status of vehicle
- Not all energy is \( \frac{1}{2}mv^2 \), but also rotational energy in vehicles and engines are there
- The rotational speed should also be changed

\[
\omega_{Motor} = \omega_{wheel} \cdot i
\]

Mass inertia of typical wheels
- 235/65 R17 = 1.7 kgm²
- 245/55 R18 = 1.9 kgm²

Mass inertia of PSM E-Motor
- HVH250 – 115 = 0.086 kgm²
- HVH250 – 090 = 0.067 kgm²
Roughly How Much Power?

- Acceleration from 0 to 100 kph? \( m = 1600 \text{ kg} \)

- Cruising at 60 kph with \( C_D A = 0.3 \cdot 2.2 \text{ m}^2 = 0.66 \text{ m}^2 \) and \( \rho = 1.2 \frac{\text{kg}}{\text{m}^3} \)
  - What is the share of aerodynamic drag?

- Cruising at 120 kph?
  - What is the share of aerodynamic drag?
Powertrain

- Powertrain
  - Main components that generate power and deliver it to the road surface, water or air
  - Engine
  - Transmission
  - Drive shafts
  - Differentials

D. Steckberg, „Development of an internal combustion engine fuel map model based on on-board acquisition”
Side Note: Model-Based Design (MBD)

- A mathematical and visual method of addressing problems associated with designing complex control, signal processing, and communication systems (from Wikipedia)

- A system model is at the center of the development process from requirements development, through design, implementation, and testing

- Steps
  - Step 1: modeling a plant
  - Step 2: Analyzing and synthesizing a controller for a plant
  - Step 3: Simulating the plant and controller
  - Step 4: Integrating all these phases by deploying the controller
V-Model

- Graphical representation of a systems development lifecycle
- **Left-side:** decomposition of requirements, creation of system specifications,
- **Right side:** integration of parts and validation
- Correct model is essential in such life cycle!
Powertrain Modeling

- MATLAB/Simulink example
  - Vehicle with four-speed transmission

Source: Mathworks
Powertrain Modeling: Generic Engine

- Generic Engine Model
  - Programmed relationship between **torque** and **speed**
  - Controlled by the throttle signal

- Throttle valve controls the amount of air fed into the engine

**Generic engine**
Source: mathworks

**Throttle valve**
Source: W. Ribbens, “Understanding automotive electronics”
Powertrain Modeling: Generic Engine

- Rough outline
  - Air inflow is controlled by throttle plate
  - Fuel is mixed with air
  - Electronic engine control controls the ignition

(Gasoline) Engine control diagram

Source: W. Ribbens, “Understanding automotive electronics”
Engine power demand

- Maximum power available $g(\Omega)$ for a given engine speed $\Omega$
- Third order polynomial model is often used

- Normalized throttle input signal $T$ specifies the actual engine power $P$
  - A fraction of the maximum power in a steady-state engine speed
  - $P(\Omega, T) = T \cdot g(\Omega)$
  - Engine torque is $\tau = P/\Omega$

- There is minimum speed
  - Stall speed usually 500 RPM
Fuel consumption model?

- Constant per revolution?
- As a function of speed and torque? Brake-specific fuel consumption (BSFC)]

\[ BSFC = \frac{r}{P}, \text{ where } r \text{ is the fuel consumption rate (gram/sec), and } P = \tau \Omega \]
Powertrain Modeling: Transmission

- Simpler to model
  - Dog clutch, cone clutch, disk friction clutch
- Efficiency?
  - $\eta_c = C_{sr}C_{tr}$, where the RHS are speed ratio and torque ratios
Differentials

- Gear arrangement that permits power from engine to be transmitted to a pair of driving wheels dividing the force equally between them.
- Gear train with three shafts that has the property that the rotational speed of one shaft is the average of the others.
- Allows the wheels to follow paths of different lengths when turning a corner or traversing an uneven road.
- [https://www.youtube.com/watch?v=rxHjKoB2vn4](https://www.youtube.com/watch?v=rxHjKoB2vn4)

Planetary gear
Brake Modeling

- Band brakes
  - High torque at cost of low precision (chain-saw, go-kart)
- Disc brakes
  - Braking torque
    - $T_{br} = F_{br}R_m = \mu_kPA_{tot}R_m = \mu_kP\frac{\pi D_b^2 N}{4}R_m$, when $\Omega \neq 0$
    - $T = \frac{\mu_sP\pi D_b^2 R_m N}{4}$, when $\Omega = 0$
  - Where
    - $D_b$ is the area of an oil piston
    - $N$ is the number of pistons
    - $\mu_k$ kinetic friction coef.
    - $P$ brake oil pressure
    - $R_m$ mean effective radius (axle-midline of brake calipers)
### Tires

- **Non-slipping**
  - \( V_x = r_w \Omega \), where \( V_x \) is velocity, \( r_w \) is tire radius, and \( \Omega \) is angular velocity

- **Slip**
  - \( V_{sx} = r_w \Omega - V_x \), where \( V_{sx} \) is the wheel slip velocity
  - Wheel slip is \( k = \frac{V_{sx}}{|V_x|} \), \( k = -1 \) for perfect sliding, 0 for perfect rolling

- **Deformation**
  - Because of the deformation, tire-road contact turns at slightly different angular velocity \( \Omega' \)
NEDC

Fig. A.1  Vehicle speed and acceleration versus time of the European NEDC

<table>
<thead>
<tr>
<th>Distance [m]</th>
<th>11,000</th>
<th>Duration [s]</th>
<th>1180</th>
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</thead>
<tbody>
<tr>
<td>Idling time [%]</td>
<td>24</td>
<td>Average speed [km/h]</td>
<td>34</td>
</tr>
<tr>
<td>Cruising time [%]</td>
<td>40</td>
<td>Maximum speed [km/h]</td>
<td>120</td>
</tr>
<tr>
<td>Acceleration time [%]</td>
<td>21</td>
<td>Number of stops</td>
<td>14</td>
</tr>
</tbody>
</table>

Source: Giarkoumis
Drive cycles - Passenger Cars and Light-duty Trucks

- **WLTC**

<table>
<thead>
<tr>
<th>Distance [m]</th>
<th>23266</th>
<th>Duration [s]</th>
<th>1800</th>
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<tbody>
<tr>
<td>Idling time [%]</td>
<td>13</td>
<td>Average speed [km/h]</td>
<td>47</td>
</tr>
<tr>
<td>Cruising time [%]</td>
<td>4</td>
<td>Maximum speed [km/h]</td>
<td>131</td>
</tr>
<tr>
<td>Acceleration time [%]</td>
<td>44</td>
<td>Number of stops</td>
<td>8</td>
</tr>
</tbody>
</table>

- WLTC = Worldwide Harmonized Light-Duty Vehicles Test Cycle
- WLTP = Worldwide Light-Duty Vehicles Test Procedure

**Introduction Sept. 2017**
### Comparison of NEDC and WLTC (NEFZ und WLTP)

<table>
<thead>
<tr>
<th></th>
<th>NEDC</th>
<th>WLTC</th>
<th>Modification</th>
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<tbody>
<tr>
<td>Distance [m]</td>
<td>11,000</td>
<td>23,266</td>
<td>+ 100%</td>
</tr>
<tr>
<td>Duration [s]</td>
<td>1,180</td>
<td>1,800</td>
<td>+ 50%</td>
</tr>
<tr>
<td>Idling time [%]</td>
<td>24</td>
<td>13</td>
<td>- 50%</td>
</tr>
<tr>
<td>Cruising time [%]</td>
<td>40</td>
<td>4</td>
<td>More dynamic</td>
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<tr>
<td>Acceleration time [%]</td>
<td>21</td>
<td>44</td>
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</tr>
<tr>
<td>Number of stops</td>
<td>14</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Average speed [km/h]</td>
<td>34</td>
<td>47</td>
<td>+ 40%</td>
</tr>
<tr>
<td>Maximum speed [km/h]</td>
<td>120</td>
<td>131</td>
<td>+ 10%</td>
</tr>
</tbody>
</table>

**Consequences**
- Closer to real driving cycle
- Higher CO2 emissions
- Higher energy consumption
- Lower electric range

![Graph showing comparison between NEDC and WLTC driving cycles](Image)
Drive cycles - Passenger Cars and Light-duty Trucks

- FTP 75

**Fig. A.9** Vehicle speed and acceleration versus time of the U.S. FTP-75

<table>
<thead>
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<th>Distance [m]</th>
<th>17769</th>
<th>Duration [s]</th>
<th>1877</th>
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</thead>
<tbody>
<tr>
<td>Idling time [%]</td>
<td>18</td>
<td>Average speed [km/h]</td>
<td>47</td>
</tr>
<tr>
<td>Cruising time [%]</td>
<td>8</td>
<td>Maximum speed [km/h]</td>
<td>91</td>
</tr>
<tr>
<td>Acceleration time [%]</td>
<td>39</td>
<td>Number of stops</td>
<td>19</td>
</tr>
</tbody>
</table>
Drive cycles – Heavy Duty Vehicles

- Braunschweig

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
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<tbody>
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<td>Distance [m]</td>
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<td>Duration [s]</td>
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<tr>
<td>Idling time [%]</td>
<td>24</td>
<td>Average speed [km/h]</td>
<td>23</td>
</tr>
<tr>
<td>Cruising time [%]</td>
<td>6</td>
<td>Maximum speed [km/h]</td>
<td>58</td>
</tr>
<tr>
<td>Acceleration time [%]</td>
<td>40</td>
<td>Number of stops</td>
<td>8</td>
</tr>
</tbody>
</table>
Microscopic traffic modeling
  
  „Single vehicle-driver units, so the dynamic variables of the models represent microscopic properties like the position and velocity of single vehicles“ – Wikipedia

Macroscopic traffic modeling
  
  It is a mathematical traffic model that formulates the relationships among traffic flow characteristics like density flow, mean speed of a traffic stream, etc.
Fundamental diagram of traffic flow

- Relationship between traffic flux (vehicles/hour) and the traffic density (vehicles/km)
- Primary tool for graphically displaying traffic flow information
- Comprises three different graphs
  - Flow-density
  - Speed-flow
  - Speed-density

- Flow: cars/h
- Speed: km/h
- Density: ?

\[ Q = D \cdot V \]

Flow = Speed * Density
Macroscopic Traffic Model

- **Speed-density**
  - The denser the traffic (cars/km), slower the speed
    - Could you drive fast at a very small inter-vehicle distance?
  - $V_f$: Free flow speed
  - $D_{max}$: Jam density
Macroscopic Traffic Model

- **Flow-Density**
  - If the car density is small, flow is small because the number of cars is small.
  - If the density is large, flow is small because flow velocity (km/h) is small.
  - The „apex“ is the capacity of the segment of the road.
  - There exists an optimal traffic density.
  - „Wave speed“ (w): slope of the stable region.

![Graph showing traffic flux and density relationship]
Macroscopic Traffic Model

- Flow-speed graph
  - There exist two flows
  - $V_C$: Critical speed
Some key terms used in the model

Stability?
- If one of the vehicles brake, does this result in persistent stop-and-go?
- **Free**: less than 12 vehicles per mile are on a road
- **Stable**: between 12 and 30 vehicles per mile per lane
- **Unstable**: more than 30 vehicles per miles per lane
- **Jam density**: Traffic stops! (more than 185-250 vehicles per mile per lane)
- Remember the congestion in the ring road from the first lecture?
- The numbers are „empirical“ (not causal from mathematical derivations)
Microscopic Traffic Model

- Newell’s car following model
  - It assumes that the vehicles will maintain the minimum time and space gap
  - But why?
  - If you assume each vehicle follows the same trajectory, you can move the trajectory of a vehicle in parallel in distance (by $\delta$) and time (by $\tau$)

- In time-space diagram,
  - $s_A = v_A \tau + \delta$, where $\tau$ is time separation and $\delta$ is space separation
  - Why is it called time and space gap?
  - Imagine large $v_A$ and 0
  - Shockwave speed $w = \frac{\delta}{\tau}$
  - But why?
What exactly are shockwaves?
- Shock wave is basically the movement of the point that demarcates the two stream conditions: Hence the red and blue slopes

Typical shockwaves propagation
- Forward wave speed
- Backward wave speed
Microscopic Modeling

- Shockwave is also equivalent to the slope between two points in flow-density diagram

- Why?
  - Let’s say there’s a shockwave demarcated by two different streams $v_A, q_A$, and $k_A$ (velocity, flow, and density), $v_B, q_B$, and $k_B$
  - Let’s assume shockwave speed is $w$
  - Relative speeds of two streams to the shockwave are $v_A - w$ and $v_B - w$
  - The number of vehicles passing through the demarcation line are $(v_A - w)k_A$ and $(v_B - w)k_B$, which of course have to be the same as cars don’t disappear or appear at the demarcation line
    \[ (v_A - w)k_A = (v_B - w)k_B \]
  - If you substitute $q = v \cdot k$, arrange by $w$
    \[ w = \frac{q_A - q_B}{k_A - k_B} \]
In time-space diagram,

\[ s_A = v_A \tau + \delta, \]

where \( \tau \) is time separation and \( \delta \) is space separation

- \( k_A = 1/s_A \), where \( k_A \) is the density at traffic state A and \( s_A \) is spacing

- From flow-density graph, \( w = \frac{(q_A-0)}{(k_j-k_A)} = \frac{k_A v_A}{k_j-k_A} \), if you re-arrange

- \( k_A = \frac{(k_j w)}{(v_A + w)} \), where \( k_j \) is the jam density, \( w \) is the wave speed

- So, \( \tau = \frac{1}{(w k_j)} \) and \( \delta = \frac{1}{k_j} \)

- Separation is independent of the speed of the leading vehicle
Then, the location of vehicle $i$ at time $t$ will be

$$x_i(t) = \min(x_A^F(t), x_A^C(t)),$$

Where

$$x_A^F(t) = x_i(t - \tau) + v_f \cdot \tau,$$

is the position of vehicle under free-flow conditions

$$x_i^C(t) = x_{i-1}(t - \tau) - \delta,$$

is the position of vehicle under congested conditions.
However, in reality, the spacing of vehicles is not perfectly maintained by human drivers

Car following models

- Use of partial differential equations describing the complete dynamics of the vehicles’ positions
- Simplest model determines the acceleration of the vehicle $\alpha$ considering the velocity of the preceding vehicle $\alpha-1$
  
  \[ x_\alpha(t) = v_\alpha(t) = F(v_\alpha(t), s_\alpha(t), v_{\alpha-1}(t)) \]

  The simplest control would be
  
  \[ v_\alpha(t + T) = \kappa_i[v_{i-1}(t) - v_i(t)] \]
  
  Which means you adjust acceleration proportional to the speed difference with the preceding vehicle every time period $T$
Microscopic Modeling

- Driver aggressiveness
- More on this in the „control“ part later
Microscopic Modeling

- **Intelligent driver model (IDM)**
  - Free road behavior + behavior at high approaching rates

- \[ \ddot{x}_\alpha = \frac{dx_\alpha}{dt} = v_\alpha \]

- \[ \dot{v}_\alpha = \frac{dv_\alpha}{dt} = a(1 - \left(\frac{v_\alpha}{v_0}\right)^\delta - \left(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha}\right)^2) \]

- With \( s^*(v_\alpha, \Delta v_\alpha) = s_0 + v_\alpha T + \frac{v_\alpha \Delta v_\alpha}{2\sqrt{ab}} \),

- \( v_\alpha \) is the desired velocity at free traffic
- \( s_0 \) is the minimum spacing
- \( T \) is the desired headway
- \( a \) is the desired acceleration
- \( b \) is the comfortable braking deceleration
Microscopic Modeling

- Example result for IDM
References

- W. Ribbens, Understanding Automotive Electronics
- Simscape Driveline Documentation