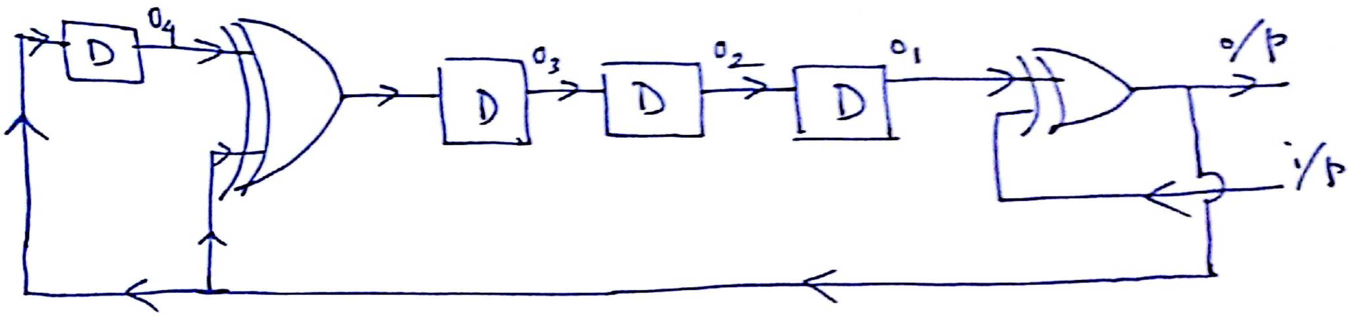
1. End Sem FTS'16 Spring Solution.1

a) Decoding circuit for (15, 11) cyclic code with generator polynomial X^4+X^3+1. (5)



b) RAID level (1) ⟶ 2 disks for 1. (10)
When both works
↳ read access partitioned for fast executio[n]

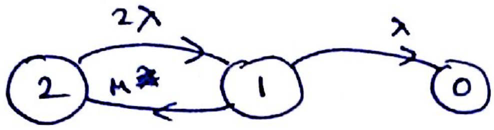
level (2) ⟶ data disks + Hamming coded disks.

(3) ⟶ Each data disk has per sector error code.
d data disks + 1 parity disk.
i'th position of all disks give a (d+1)-bit word ⟶ data + code.

(4) ⟶ Every data word is not interleaved among disks. (for faster access).
A block (stripe) of all disks is coded as a parity block.

(5) ⟶ No seperate disk for parity.
Instead, parity ~~data~~ blocks are interleaved among data blocks.

1) c)



(5)

$$\dot{P}_2 = -2\lambda P_2 + \mu P_1$$
$$\dot{P}_1 = -(\lambda+\mu) P_1 + 2\lambda P_2$$
$$\dot{P}_0 = 1 - P_1 - P_2$$

$T_{2\to 0}(n)$: time to enter state 0 after n visits to state 1.

$$= n\left(\frac{1}{2\lambda} + \frac{1}{\lambda+\mu}\right) = n\cdot\frac{3\lambda+\mu}{2\lambda(\lambda+\mu)}$$

Time spent in state 1 & 2 in n visits

↳ ~~prob~~ probability $= \left(\frac{\mu}{\mu+\lambda}\right)^{n-1} \cdot \left(\frac{\lambda}{\mu+\lambda}\right)$

↑ q ↑ p

$$MTTDL = \sum_{n=1}^{\infty} q^{n-1} \cdot p \cdot T_{2\to 0}(n)$$

$$= \ldots\ldots = \frac{3\lambda+\mu}{2\lambda^2}$$

2) a) At time t, probability of individual processor being faulty $q(t) = 1 - e^{-\lambda t}$

(5)

Probability that n_1 out of N processors are faulty

$$= {}^{N}C_{n_1}\, q^{n_1} \cdot (1-q)^{N-n_1}$$

The overall output is incorrect if majority ($> m$) agree on an incorrect o/p.

$\binom{a}{b} = {}^{a}C_{b}$

↳ Prob(~~n_2 out of n_1, ($n_2 < n_1$)~~ exactly n_2 out of n_1 faulty processors have same o/p)

$$= {}^{n_1}C_{n_2} \cdot 2^{-8n_2} \cdot (1-2^{-8})^{n_1-n_2}$$

Prob(majority with same o/p)

$$= x = \sum_{n_1=m+1}^{N} \sum_{n_2=m+1}^{n_1} \binom{N}{n_1} \cdot q^{n_1} (1-q)^{N-n_1} \cdot \binom{n_1}{n_2} \cdot 2^{-8n_2} \times (1-2^{-8})^{n_1-n_2}$$

In any run, no. of possible incorrect o/p $= 2^8 - 1 = 255$.

$\therefore$ Ans $= x \times 255$.

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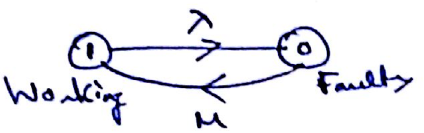
2(b)

$E_1 \to$ execution 1 fault free] $\to$ event
$E_2 \to$ " 2 . . .] variables.

We want $P(E_1 \,\&\, E_2)$ (10)

~~$= P(E_2 \ldots).\ P(E$~~

$= P(E_2 / E_1) \cdot P(E_1)$

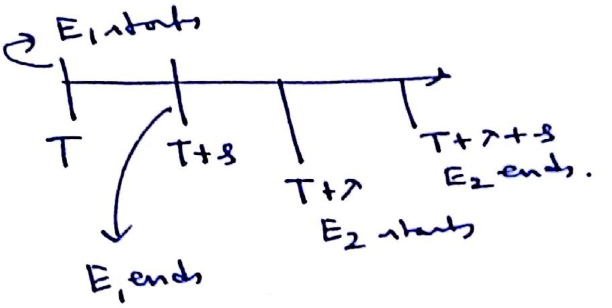


$P_1' = -\lambda P_1 + \mu P_0$

$= -\lambda P_1 + (1 - P_1)\cdot\mu \quad , \quad P_1(0) = 1$

$$P_1(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} \cdot e^{-(\lambda+\mu)t}.$$

↑ Probability of processor working correctly at time 't'.



$\therefore P(E_1)$

$= P($processor is working from $t = T$ to $t = T + s)$

$= P_1(T) \cdot e^{-\lambda s}$

$P(E_2 / E_1)$

$= P($processor is working at $t = T + r$ & " works for an interval of size 's' starting from $t = T + r$ / processor was working at $t = T + s)$

$= P_1(r - s) \cdot e^{-\lambda s}.$

[Note $P(E_2) = P_1(T + r) \cdot e^{-\lambda s}.$]

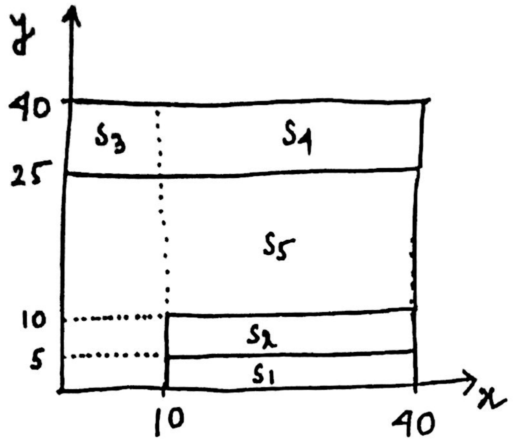
$\therefore$ Ans $= P(E_1 \,\&\, E_2)$

$= P_1(r - s) \cdot e^{-\lambda s} \cdot P_1(T) \cdot e^{-\lambda s}.$

3) a) In book. (3)   2(c) In book, $s = \sqrt{n}$. (5)

2 + 2 + 2

3(b)



S_1 : $x > a$ && $y < b$
S_2 : $x > a$ && $a > y \geq b$
S_3 : $y > 25$ && $x <= a$
S_4 : $y > 25$ && $x > a$
S_5 : Rest

$P(S_i)$, $i = 1, 2, \ldots, 5$: ~~indicates~~ variables x and y can take values from region S_i with probability $P(S_i)$.

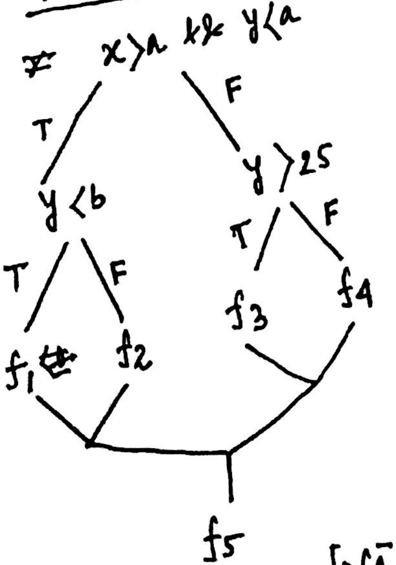
$P(\bar{f_i})$, $i = 1, 2, \ldots, 5$: indicates the failure probability of function f_i

Ver 1	Ver 2
$P(\bar{f_1}) = 0.01$	$P(\bar{f_1}') = 0.005$
$P(\bar{f_2}) = 0.02$	$P(\bar{f_2}') = 0.001$
$P(\bar{f_3}) = 0.001$	$P(\bar{f_3}') = 0.003$
$P(\bar{f_4}) = 0.003$	$P(\bar{f_4}) = 0.003$
$P(\bar{f_5}) = 0.01$	$P(\bar{f_5}) = 0.01$

$P(S_1)$: $0.25 \times 0.3 = 0.075$
$P(S_2)$: $0.6 \times 0.3 = 0.18$
$P(S_3)$: $0.7 \times 0.075 = 0.0525$
$P(S_3)$: $0.075 - 0.0525 = 0.0225$
$P(S_4)$: 0.075
$P(S_5)$: $1 - (0.075 + 0.18 + 0.0525 + 0.0225)$
$= 0.67$

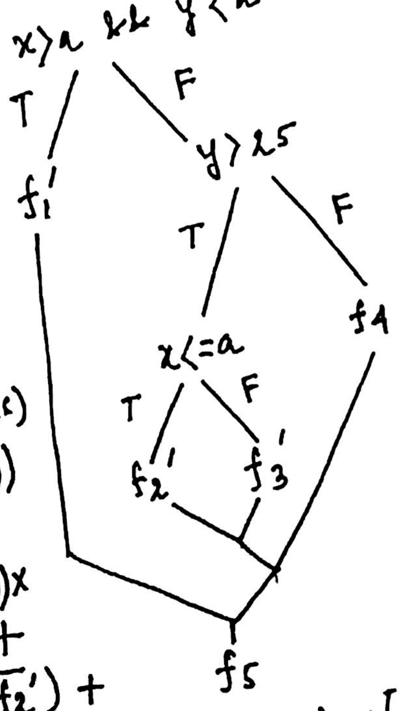
Ver 1

Paths



$P(\text{Ver 1 fails}) = P(S_1) \times [P(\bar{f_1}) + (1 - P(\bar{f_1})) \times P(\bar{f_5})] +$
$P(S_2) \times [P(\bar{f_2}) + (1 - P(\bar{f_2})) \times P(\bar{f_5})] +$
$(P(S_3) + P(S_4)) \times [P(\bar{f_3}) + (1 - P(\bar{f_3})) \times P(\bar{f_5})] +$
$P(S_5) \times [P(\bar{f_4}) + (1 - P(\bar{f_4})) \times P(\bar{f_5})]$
= CALCULATE !!

Ver 2



$P(\text{Ver 2 fails})$
$= (P(S_1) + P(S_2)) \times [P(\bar{f_1}') + (1 - P(\bar{f_1}')) \times P(\bar{f_5})] +$
$P(S_3) \times [P(\bar{f_2}') + (1 - P(\bar{f_2}')) \times P(\bar{f_5})] + P(S_4) \times [P(\bar{f_3}') + (1 - P(\bar{f_3}')) \times P(\bar{f_5})] + P(S_5) \times [P(\bar{f_4}) + (1 - P(\bar{f_4})) \times P(\bar{f_5})]$
= CALCULATE !!

Joint Failure Probability =

$$P(S_1) \times \left[\left(P(\bar{f_1}) + (1 - P(\bar{f_1})) \times P(\bar{f_5}) \right) \times \left(P(\bar{f_1}') + (1 - P(\bar{f_1}')) \times P(\bar{f_5}) \right) \right] +$$

$$+ P(S_2) \times \left[\left(P(\bar{f_2}) + (1 - P(\bar{f_2})) \times P(\bar{f_5}) \right) \times \left(P(\bar{f_1}') + (1 - P(\bar{f_1}')) \times P(\bar{f_5}) \right) \right]$$

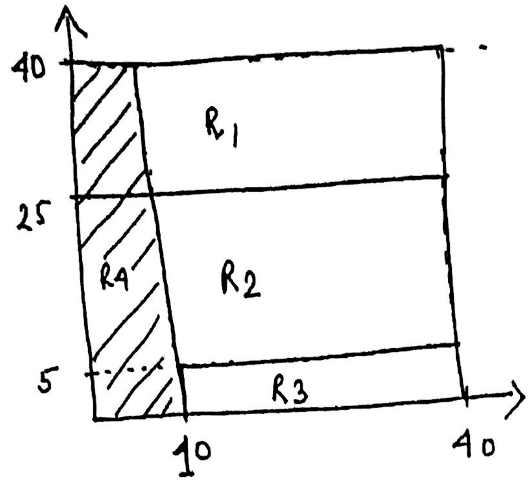
....

$$+ P(S_5) \times \left[P(\bar{f_4}) + (1 - P(\bar{f_4})) \times P(\bar{f_5}) \right]$$

[The last path is common for both the versions. If this path fails for ver 1, it will also fail for ver 2]

= CALCULATE !!

3. c) (i)



7+4

$R_1 : y > 25$

$R_2 : y \leq 25$

$R_3 : x > 10$ && $y < 5$

$R_4 : x <= 10$ && $y >= 5$

$P(\bar{f_1}) = 0.10$

$P(\bar{f_2}) = 0.09$

$P(\bar{f_3}) = 0.05$

$P(\bar{f_4}) = 0.07$

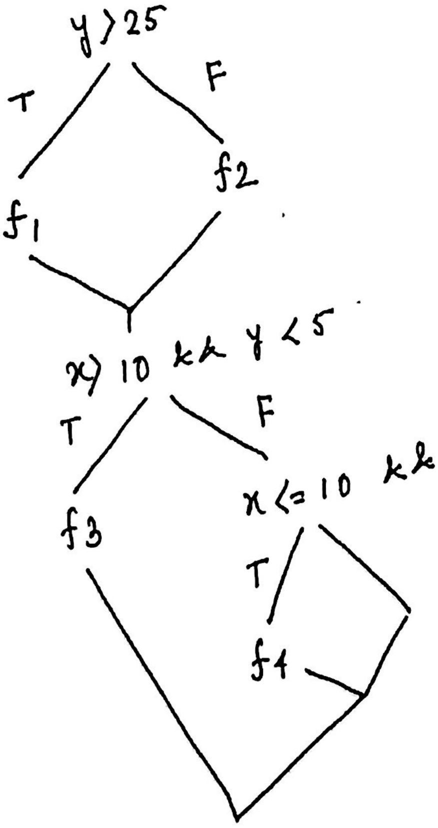
$P(R_1) = 1 \times 0.10 = 0.10$

$P(R_2) = 1 - 0.10 = 0.90$

$P(R_3) = 0.35 \times 0.25 = .0875$

$P(R_4) = 0.65 \times (0.35 + 0.30 + 0.10) = 0.65 \times 0.75 = 0.715$

Paths:



P (Program fails)

$= P(R_1)\left[P(\bar{f_1}) + P(R_3) \times (1 - P(\bar{f_1})) \times P(\bar{f_3}) + (1 - P(R_3)) \times P(R_4) \times (1 - P(\bar{f_1})) \times P(\bar{f_4})\right]$

$+ P(R_2)\left[P(\bar{f_2}) + P(R_3) \times (1 - P(\bar{f_2})) \times P(\bar{f_3}) + (1 - P(R_3)) \times P(R_4) \times (1 - P(\bar{f_1})) \times P(\bar{f_4})\right]$

$= 0.10 \times [0.10 + 0.0875 \times (1 - 0.10) \times 0.05 + (1 - 0.0875) \times 0.715 \times (1 - 0.10) \times 0.07] +$

$0.90 [0.09 + 0.0875 \times (1 - 0.09) \times 0.05 + (1 - 0.0875) \times 0.715 \times (1 - 0.09) \times 0.07]$

$= 0.13649$

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Reliability = 1 − 0.13649
= 0.86351

(ii) f_1 executed 4 times.
f_2 executed 2 times.

failure ~~of~~ probability of f_1 =

$P(\bar{f_1})$ = 0.1 × 0.15 + 0.1 × 0.85 × 0.1 × 0.15 + 0.1 × 0.85 × 0.1 × 0.85 × 0.1 × 0.15 + 0.1 × 0.85 × 0.1 × 0.85 × 0.1 × 0.85 × 0.1 × 0.15

= 0.0164.

failure ~~Propab~~ probability of f_2 =

$P(\bar{f_2})$ = 0.09 × 0.2 + 0.09 × 0.8 × 0.09 × 0.2
= 0.01813

P(Program fails)
= 0.10 × [0.0164 + 0.0875 × (1 − 0.0164) × 0.05 + (1 − 0.0875) × 0.715 × (1 − 0.0164) × 0.07] + 0.90 × [0.01813 + 0.0875 × (1 − 0.01813) × 0.05 + (1 − 0.0875) × 0.715 × (1 − 0.01813) × 0.07]

= 0.067104.

Reliability = 1 − 0.067104
= 0.932896.

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4) a) In book (3 + 2 + 2)

b) $k = -1$ Advantage → force 2's complement operations leading to large no. of bit flips.

(2)

Disadvantage
↳ potential of overflow if variable = largest -ve int.

c) In book. $\dfrac{\left[1-\left(\frac{sf}{\sigma}\right)^n\right]\left[(1-f)-(1-\sigma)\frac{sf}{\sigma}\right]}{\left[1-\frac{sf}{\sigma}\right]}$ (3)

d) We list all possible ways of putting out incorrect value. (8)

i) incorrect val in stage1 $= q_1 \cdot \dfrac{\alpha_1}{L}$ — ①

Probability of activating stage 2
= stage 1 fail & o/p out of range $[0, \alpha_1]$
+ " " pass . . . .

$$= q_1\left(1-\frac{\alpha_1}{L}\right) + (1-q_1)\cdot\int_{\alpha_1}^{L}\frac{\mu_1 \cdot e^{-\mu_1 Z}}{1-e^{-\mu_1 L}}\, dZ$$

$$= q_1\cdot\left(1-\frac{\alpha_1}{L}\right) + (1-q_1)\cdot\frac{e^{-\mu_1\alpha_1}-e^{-\mu_1 L}}{1-e^{-\mu_1 L}} = s_2 \text{ say}$$

ii) probability of o/p being incorrect in stage 2

$$= s_2 \cdot q_2 \cdot \frac{\alpha_2}{L}$$

$$\therefore \text{penalty} = \pi_{bad}\cdot\left[q_1\frac{\alpha_1}{L} + s_2 q_2\frac{\alpha_2}{L}\right] = \Pi_1$$

Probability of no o/p at all

$$= s_2\left[q_2\left(1-\frac{\alpha_2}{L}\right) + (1-q_2)\int_{\alpha_2}^{L}\frac{\mu_2 e^{-\mu_2 x}}{1-e^{-\mu_2 L}}\, dx\right]$$

$$= s_2\cdot\left[q_2\left(1-\frac{\alpha_2}{L}\right) + (1-q_2)\cdot\frac{e^{-\mu_2\alpha_2}-e^{-\mu_2 L}}{1-e^{-\mu_2 L}}\right]$$

$$= s_2 \cdot s_3 \text{ say}$$

Expected Penalty $= \Pi_1 + \Pi_{stop}\cdot s_2 \cdot s_3$

p_r^i → o/p line in stage i carries a mem req.

5) a) $BW = N \cdot p_r(0)$ where, $p_r^{(i-1)} = p_r^{(i)} - \frac{(p_r^{(i)})^2}{4}$

Connectability $Q = 2^{2K} \cdot p_\ell^{K+1} \cdot p_s^K$ ↳ switch box is fault free

↑ link is fault-free.

3+3+3

Accessability $A_c = 2^K \cdot p_\ell \cdot \phi(K)$

$\phi(i) = 1 - (1 - p_\ell \, \phi(i-1))^2$

$\phi(0) = 1 - q_\ell^2 = (1 - (1 - p_\ell)^2)$

$\phi(i)$ = probability that at least one ~~line~~ path out of a switch box at stage i leads to the o/p of network in a fault free manner.

CCC → (2) [Scalability]

b) optimal rejuvenation period $= \left(\frac{C_p}{(n-1)\lambda C_e}\right)^{1/n}$

(5)

c) Explained in class.

(2+1+1)

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